Appendix A: Calculation procedures of largest Lyapunov Exponent

To reconstruct the state space, a state vector was created from the joint angle time series. This vector is composed of mutually exclusive information about the dynamics of the system (23), (40) (Eq. (1)).

\[ y(t) = [x(t), x(t-T_1), x(t-T_2), \ldots] \quad \text{Equation (1).} \]

where \( y(t) \) is the reconstructed state vector, \( x(t) \) is the original joint angle data and \( x(t-T_i) \) is the time delay copies of \( x(t) \). The time delay \( T_i \) for creating the state vector was determined by estimating when information about the state of the system at \( x(t) \) is different from the information contained in its time-delayed copy. If the time delay is too small then no additional information about the dynamics of the system would be contained in the state vector.

Conversely, if the time delay is too large then information about the dynamics of the system may be lost and can result in random information (23). Selection of the appropriate time delay was performed by using an average mutual information algorithm (23) (Eq. (2)).

\[
I_{x(t),x(t+T)} = \sum P(x(t), x(t+T)) \log_2 \left[ \frac{P(x(t), x(t+T))}{P(x(t))P(x(t+T))} \right] \quad \text{Equation (2).}
\]

where \( T \) is the time delay, \( x(t) \) is the original joint angle data, \( x(t+T) \) is the time delay data, \( P(x(t), x(t+T)) \) is the joint probability for measurement of \( x(t) \) and \( x(t+T) \), \( P(x(t)) \) is the probability for measurement of \( x(t) \), \( P(x(t+T)) \) is the probability for measurement of \( x(t+T) \). The probabilities were constructed from the frequency of \( x(t) \) occurring in the joint angle time series.

Average mutual information was iteratively calculated for various time delays and the selected time delay occurred at the first local minimum of the iterative process (9, 23). This selection is based on previous investigations that have determined that the time delay at the first local
minimum contains sufficient information about the dynamics of the system to reconstruct the state vector \(^{(23)}\).

It was also important to determine the number of embedding dimensions to unfold the dynamics of the system in an appropriate state space. An inappropriate number of embedding dimensions may result in a projection of the dynamics of the system that has orbital crossings in the state space that are due to false neighbors and not the actual dynamics of the system \(^{(23)}\). To unfold the state space we systematically inspected \(x(t)\) and its neighbors in various dimensions (e.g. dimension = 1, 2, 3,…etc.). The appropriate embedding dimension occurs when neighbors of the \(x(t)\) stop being un-projected by the addition of further dimensions of the state vector (Eq. \(^{(3)}\)).

\[
y(t) = [x(t), x(t + T), x(t + 2T), \ldots x(t + (d_E-1) T)] \quad \text{Equation (3).}
\]

where \(d_E\) is number of embedding dimensions, \(y(t)\) is the \(d_E\)-dimensional state vector, \(x(t)\) is the original joint angle data, and \(T\) is the time delay. A global false nearest neighbors algorithm with the time delay determined from the local minimum of the average mutual information was used to determine the number of necessary embedding dimensions to reconstruct the joint angle time series \(^{(23)}\). The calculated embedding dimension indicates the number of governing equations that are necessary to appropriately reconstruct the dynamics of the system \(^{(23)}\). Custom MATLAB (Mathworks Inc, MA) software was used to calculate the embedding dimension.

After calculating the appropriate time delay and embedding dimension and reconstructing the joint angle time series, the largest Lyapunov Exponent was calculated using the *Chaos Data Analyzer* (professional version, American Institute of Physics \(^{(21)}\)). The *Chaos Data Analyzer* calculates the rate of divergence between two vectors \(^{(23)}\) (Eq. (4)).
\[
\lambda = \frac{1}{t_M - t_0} \sum_{k=1}^{(d_E - 1)M} \ln \frac{L'(t_k)}{L(t_{k-1})}
\]  
Equation (4).

where \(y(t)\) is the \(d_E\)-dimensional state vector and serves as the reference trajectory. \(L(t_0)\) is the distance between \(y(t)\) and its nearest neighbor. \(L'(t_1)\) is the distance between the \(y(t)\) and its nearest neighbor after moving forward \(n\) steps (we used \(n = 3\)). Then a new state vector replaces the evolved neighboring state vector if it meets the following two conditions:

1) The distance of a replacing vector from the evolved state vector on the reference trajectory denoted as \(L(t_1)\) is small.

2) The angular separation between the evolved reference state vector and replacing vector is small.

New vectors are repeatedly generated \(M = N - (d_E - 1)\) times where \(N\) is the length of the original time series. Then, the largest Lyapunov Exponent is defined by Equation (4), where \(k = 1, 2, \ldots, M\) and \(n = t_{k+1} - t_k\).