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Semiotics: Notes on English to Mathematical Language **Translation**

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SEMIOTICS: NOTES ON ENGLISH TO MATHEMATICAL LANGUAGE TRANSLATION

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Abstract: The manuscript at hand is a presentation relating the processes and notation of mathematics to the processes and notation of standard written English. Semiotics is introduced as a way to describe the similarities in the processes and goals of both systems as methods of communication. The discussion of semiotics falls within three categories: 1) examining the parallel structures of written language and mathematics, 2) defining the language based nature of mathematical symbols, notation, and processes, and 3) exploring misconceptions between mathematical versus standard written vocabulary and notation.

Introduction

Mathematics is the purest, most concise, and most comprehensive language system known to humankind. Mathematics not only subsumes the formal structure, notation, and use of all written and spoken language under its own paradigm, it is, itself, the most precise mechanism we have for describing the universe and all it contains. From the complex vocal sounds by which we transmit our individual thoughts and feelings, to the various permutations of the alphabetic symbols used to make words and sentences, to the geometric translations of sign language into meaning; all forms of language are, and must naturally be, mathematically structured. Thoughtful consideration on a much broader scale even suggests that if we ever receive a message from intelligent life outside our own solar system, it will likely be a mathematical one. Yet with all of its elegant communication potential, mathematics is typically only utilized as a language by mathematicians and scientists, despite the obvious parallels to how our written language is structured and used by non-mathematicians.

The unfortunate fact is that most of us experience only the most trivial facets of mathematics from our earliest exposure to the subject; and very little of this exposure involves recognizing the language aspects of mathematics. This is not to say that opportunities for more communication based applications are not available in the instruction of mathematics, but rather that the concept of mathematics as a language is overshadowed by the procedural fluency aspects of the discipline as we learn about it. This fact is ironic considering that procedural fluency is actually a component of mathematical communication but is rarely described as such.

The narrative provided hereafter will make the case that secondary school mathematics curriculum would benefit from a moderate transition away from a computational and procedural fluency focus and more toward defining mathematical procedures and notation as communication tools aimed at better leveraging concepts in the sciences, engineering, and technology. This argument will be made in the context of semiotics which is defined in various ways, but can be generally understood as the study of symbols and their interpretation. The appropriateness of studying mathematics as a communication protocol within the context of semiotics will be made evident through the following discussions: examining the parallel structures of written language and mathematics, defining the language based nature of mathematical symbols, notation, and processes, and finally, exploring the innate misconceptions of mathematical versus standard written vocabulary.

The Parallel Structures in Written Language and Mathematics

The statements from the previous section are not merely suggestions that if we look hard enough, we can find subtle connections between written language and mathematics, but rather that communication itself exists within the structure of mathematics and that mathematics is primarily a communication based discipline. To demonstrate this idea, let us examine some structural and operational similarities between written language and mathematics. We can do this by comparing a few basic postulates that are common in all written systems of

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language to the processes and symbols mathematicians use. In mathematics, a postulate (or axiom) is a statement that is assumed to be true without the burden of formal proof, usually because these assumptions are necessary for defining a starting point for the mathematical system, and also because they are often self-evident. For example, the ancient mathematician Euclid built an entire system of two-dimensional geometry, which is still studied and taught worldwide, based on five simple postulates (see Euclid's Elements). We might now generate some basic postulates to define our system of language, which can be directly compared to the numeric processes and notation with which we are already familiar. Note that the same mathematical structures could be applied to any syntactic system, but also that the structure does not automatically carry over to issues of semantics. There is, however, a unique semantic aspect of mathematics that applies to the development of mathematical proofs, which will be discussed later. The following postulates for language will be used to demonstrate that written language exists within a system that is very similar to how mathematical notation is structured:

- 1. Postulate 1: Written language uses permutations of a finite set of alphabetic symbols (letters), which exist in a hierarchical form to express meaning (words and sentences). Mathematics uses permutations of a finite set of numeric symbols (digits) that exist in a hierarchical form to express quantity (place value and exponential notation). The permutation (or rearrangement) of symbols is critical to the operation of both systems. For example, the letters in the word "cat" give us a completely different meaning when rearranged as "act" or no identifiable meaning when arranged as "tca" because a specific meaning has not been universally defined for the arrangement "tca." Likewise, the arrangement of the digits 123 give us a different value than if we rearrange them as 231 , or no identifiable value if rearranged as $1³2$ because exponential notation within a number has not been universally defined.
- 2. Postulate 2: The symbols of written language include a specific set of delimiters (punctuation) to organize thoughts into manageable subsections and provide nuance to ideas. The symbols of mathematics include a specific set of delimiters (mathematical operators) to organize expressions and provide nuance to quantities. For example, there is an old joke about how a comma can save a life: "Let's eat Grandma." versus "Let's eat, Grandma." One small delimiter changes the meaning of an otherwise identical permutation of letters. The mathematical system is a bit more obvious. For example, it is clear that 314 is different than 3.14. Again a small delimiter completely changes the value of an otherwise identical permutation of digits.
- 3. Postulate 3: Written language uses different symbol combinations (e.g. words or phrases) that exhibit a degree of congruence or equality to other words or phrases. These equivalent word and phrase combinations can be substituted for one another to simplify communication or clarify meaning. Mathematics uses different symbol combinations (called expressions) that may exhibit congruence or equality and can be substituted for one another to simplify communication or clarify meaning. For example, we might substitute the phrase, "Those shoes stink." for the phrase, "Those shoes smell bad." We may do this to make the phrase more efficient (four words to represent the idea versus three words for the same idea) or to emphasize an aspect of our idea (the intensity of the smell of the shoes). In mathematics, this idea is captured by a formal mathematical property called the transitive property of equality. This property states that if $A = B$ and $B = C$, then $A = C$, and allows the three variables to be used interchangeably. A common application of this property might be illustrated by substituting an improper fraction into an expression in place of a mixed number. The symbol representations look different but hold the same value and can, therefore, be used interchangeably. The ability to substitute equivalent values within a communication system is perhaps the most critical operational structure of both of these systems. For instance, expository writing is not simply a matter of choosing appropriate words, but choosing the best words to convey the writer's meaning. A mathematician does the same thing by simplifying a complex expression through a set of successively simplified equivalences. This will be demonstrated in more detail in the next section.
- 4. Postulate 4: Language and numeric systems are interdependent. We use numbers in written language and letters and words in numeric systems. Clearly no kind of quantity or ordinal relationship can be expressed in written language without the underlying mathematical concepts supporting them. Conversely, generalizable algebraic and geometric relationships could not be stated without the use of letters as variables, or without the description of conditions using words.

The Language of Mathematical Symbols, Notation, and Processes

There are approximately 500,000 words in the English language. Factor in the knowledge that written language allows for many words to hold multiple meanings and you have a system of immense power to transmit ideas; however, such complexity comes with a much greater potential for ambiguity in the interpretation of ideas. The mechanism by which the transmission of ideas can be made more efficient and more precise is mathematics. Applying a mathematical process to language whereby we simplify a written expression through the substitution of a more concise statement reduces the ambiguity. As an exercise in mathematical process applied to writing, let us interpret the following paragraph:

It would certainly be most auspicious if you would evince a design, for which I would be abundantly grateful, whereby it would be resolved that the canine, currently residing in the immediate vicinity, be escorted from the premises; this pursuant to the beast's immediate biological need to vacate copious amounts of extraneous fluid due to an exceedingly high internal pressure index.

Many readers may appreciate various aspects of word usage in the previous narrative, or perhaps interpret it as an example of ironic overstatement of the intended message, or maybe even as the sarcastic tirade of a pseudointellectual. In reality, any interpretation may constitute a deviation from the writer's true intention because this passage is subject to the nuances of individual perception as readers digest each component to determine the meaning. By applying mathematical thinking to a written process or notational system we can reduce or even eliminate the interpretive ambiguity of a passage. Let us try it with this passage. Politely stated, a mathematician might generate a final equivalence that looks something like, "Thank you for taking the dog outside. He needs to tinkle." This would be done through a series of substitutions using simplified equivalencies until a distilled message emerges. Let us dissect the passage by identifying important components and making substitutions of reasonably congruent but simplified phrases:

- 1. Component 1: "It would certainly be most auspicious if you would evince a design" = "it would be favorable to consider a way"
- 2. Component 2: "for which I would be abundantly grateful" = "and I would be very thankful."
- 3. Component 3: "whereby it would be resolved that the canine, currently residing in the immediate vicinity" $=$ "where we decide that the dog living here"
- 4. Component 4: "would be escorted from the premises" = "would be taken outside"
- 5. Component 5: "this pursuant to" $=$ "because of"
- 6. Component 6: "the beast's immediate biological need to vacate copious amounts of extraneous fluid" = "the dog's need to get rid of extra water"
- 7. Component 7: "due to an exceedingly high internal pressure index" = "because of uncomfortable bladder pressure"

We can now reassemble the passage with the substitutions and possibly have a more concise and efficient statement although, in this form, it may appear to be somewhat awkward:

It would be favorable to consider a way… and I would be very thankful… where we decide that the dog living here… would be taken outside… because of… the dog's need to get rid of extra water… because of uncomfortable bladder pressure.

Now, let us consider a few more mathematical procedures that could be applied to this revised passage. There is a property in mathematics called the Commutative Property which states that the order of some mathematical operations within an expression does not influence the value of the expression. An example would be the Commutative Property of Addition where $2 + 3 = 3 + 2$. The placement of the digits in the expression does not impact the value of the expression. We can apply this property to the phrase, "and I would be very thankful" because the placement of this phrase within the message does not critically impact the meaning of the message. Additionally, we could combine components 3 and 4 into a single new reasonably equivalent component. The same could be done with components 5, 6, and 7. If we simply started with the phrase, "Thank you" and continued to simplify the

revised expression with reasonably congruent substitutions, the next iteration of the passage may look something like the following:

This passage can be simplified even farther based on the need for precision or specific attention to various details. The most basic message may end up reading something like, "Thanks for taking the dog out. He needs to tinkle."

Let us now examine mathematical problem solving with an eye focused on the same procedural mechanism. As complex as mathematical communication appears to be, there are fewer than 100 symbols in all of mathematics. Factor in that mathematical symbols hold more consistently to a single function rather than having multiple meanings the way that words do, and the system appears comparatively simple. So how does mathematical language have the ability to be so concise and so robust with relatively little notation? The answer is fairly straight forward in that a unique permutation of mathematical symbols typically has a single, exact meaning; moreover, the nature of mathematical notation is ultimately intended to distill numbers, expressions and relationships down to their most elegant, readable, or otherwise useful form. Theoretical constructs such as irony or sarcasm, good or evil, do not exist in mathematical notation, though those constructs can still be defined mathematically within a system. By eliminating the ambiguity of the notation, logic prevails because there is no opportunity to interpret the meaning past what is specifically stated by the symbols. As an exercise, let us examine the following simple relationship:

 $A=B$

This is a statement equating the values associated with the variables *A* and *B*. Though we do not know the values, or even the nature of the values, we know they are the same. The relationship is absolute and does not allow for parameters, conditions, provisos or special circumstances. This notion of absolute equality tends to offend our natural intuition about the world because so much of how we interpret communication is dependent on other contextual factors. For example a ship approaching a rocky shoreline in foggy conditions sees the single pulse of a lighthouse beacon. This single flash of light, repeated at regular intervals as the light rotates, carries a message of warning. The environmental conditions create a context for the communication. Now put the same ship in clear conditions on still water. A single repeating pulse of light would simply inform the ship's crew that the lighthouse was working. Successive flashes would need to be emitted in some other kind of mathematical pattern for a message of danger to be successfully transmitted.

Context is important in mathematics as well but it is generally supplied by additional notation. The absence of this additional notation creates ambiguity. For example, at some point we have all learned that the sum of the interior angles of a triangle is 180 degrees. However, without more contextual communication, this equivalency may not be true. If the triangle is constructed on the surface of a convex manifold, say, the surface of the earth, then the sum of the angles would be different. Try it. Place one vertex of a triangle at the North Pole of a globe. Draw a straight line due South until you intersect the equator. Make a right angle turn (90 degrees) to define the second vertex and follow the equator due East one fourth of the way around the globe. Define the third vertex by turning back due North and draw your last side as a straight line that ends at the North Pole. You have made three right angles in constructing this triangle, the sum of which is 270 degrees, not the 180 that you have been taught. Is the fabric of our mathematical system breaking down because of this contradiction? Of course not. We have simply failed to communicate the conditions under which our 180 degree relationship holds true, and this is one of the great oversights of mathematics instruction at the secondary level. This example is evidence of why looking at the notation of mathematics as a language is so important. Critical parts of messages cannot be omitted while maintaining an expectation that the intended meaning will be accurately relayed. Let us suppose that we want to describe a *rational* number. Most math teachers adopt a shorthand approach to this kind of description by suggesting that a rational number is basically a constant or variable in fractional form where we have a number over a number. The description seems fairly complete, but consider the precision of the following notation:

> N defines the Natural Numbers: 1, 2, 3… Z defines the Integers: $0, \pm 1, \pm 2, \pm 3...$ Q then defines the rational numbers thusly: \int_{b}^{a} $\frac{a}{b}$ |∀a ∈ Z,∃b ∈ N $}$

The notation of Q can be translated to English as follows: *Rational Numbers* are the *set* of numbers organized in the form *a* over *b* such that *for every* value of *a* that is an *element* of the *Integers*, *there exists* a value *b* that is an *element* of the *Natural Numbers*.

Admittedly, the formal notation defining a rational number looks intimidating, and maybe even seems excessive, but the message being conveyed requires an explicit notation to eliminate ambiguity. If we were to simply say that a rational number is a number over a number or an integer over an integer, we would have to consider zero to be a possible character in the denominator of a fraction, which in computational mathematics defies definition as a constant value.

The message here is that equivalence relationships are only factual if we are meticulous about how we communicate them. For this reason, mathematical notation is not a language system intended to describe truth, but rather, to define fact, irrefutable and absolute within a given context. This, of course, begs the question how are facts determined?

If we can get past the notion that fact is only as pure as the postulates on which we base our system, we can begin to focus on the goals of our communication. The language of computational mathematics has only four broad procedural goals in the determination of fact, and the goals occur in a hierarchy:

- 1. to define known values as contextualized constants
- 2. to define unknown values as mathematical expressions in terms of identified and contextualized variables
- 3. to establish relationships between and among different combinations of constants and expressions in the form of equations
- 4. to create concrete notions of equality from abstract situations that occur numerically, algebraically, and geometrically.

 If you were to examine the contents of an Algebra textbook, you would discover that every section of every chapter involves procedures focused on one or more of the goals stated above. Mathematical instruction often conceptualizes "doing math" as procedural applications, or simply the steps to reaching a solution to a problem. This is a misrepresentation of mathematical process. These so called "steps" of a problem are merely restatements of equivalent expressions. Successive substitutions (what appears to be steps of a problem) of these equivalent expressions allow us to remove the complexity and ambiguity of the original expression. Each *step*, in reality, is a simpler restatement of the initial expression using the exact same protocols as we did when we simplified the written passage in the last section. Though the written versus mathematical problems look different, the processes needed to simplify or interpret them are nearly identical. Suppose, for example, that we want to derive a popular relationship stating that the area of a circle is equivalent to the product of the constant Pi and the numeric square of the measurement of the circle's radius. Many of us know this formula by rote memorization $(A = \pi r^2)$. This familiar formula is an elegant, factual statement connecting two expressions from a complex relationship, but how was the relationship established? There are many different notational approaches to establishing this equivalence relationship, but the purest notation can be found in calculus. Note that each "step" of this process is a notational restatement or translation of a previous expression, beginning with equivalence relation for the numeric representation of a circle.

$$
x2 + y2 = r2
$$

$$
y = \sqrt{r2 - x2}
$$

$$
\int_{0}^{r} \sqrt{r2 - x2} dx
$$

$$
\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a}
$$

$$
\frac{x}{2}\sqrt{r^2 - x^2} + \frac{r^2}{2}\sin^{-1}\frac{x}{r}\Big|_0^r
$$

$$
\left[\frac{r}{2}\sqrt{r^2 - r^2} + \frac{r^2}{2}\sin^{-1}\frac{r}{r}\right] - \left[\frac{0}{2}\sqrt{r^2 - 0^2} + \frac{r^2}{2}\sin^{-1}\frac{0}{r}\right]
$$

$$
\left[\frac{r}{2}(0) + \frac{r^2}{2}\sin^{-1}1\right] - \left[\frac{0}{2}(r) + \frac{r^2}{2}\sin^{-1}0\right]
$$

$$
\left[0 + \frac{r^2}{2} \cdot \frac{\pi}{2}\right] - \left[0 + \frac{r^2}{2}(0)\right]
$$

$$
\frac{\pi r^2}{4}
$$

Just as the goal of a written document somewhat determines the language style and composition appropriate for representing the idea, mathematical processes are determined by the nature of the solution being sought. In the area formula derivation illustrated above, the final expression allows us to determine the relationship between a circle's area and its radius. The purpose is to determine a complex measured value (the area of the circle) by relating it to a value that is easier to obtain, that being the radius. In this example, a non-mathematician only needs to know that each statement provides an equivalent representation in successively less complex terms under given conditions, exactly as we did when simplifying the written passage. In considering the outcome, which is a simple formula involving only two variables, it is easy to overlook the precision of the communication protocol that allows the formula to exist.

 Using the four procedural goals stated above, we can define a notational system of mathematical rules which seem complex relative to the written English rules we use almost every day, but in reality, are just an application of the same logical principles under two different communication paradigms. The mathematical procedures that we have used for the both the written passage and formula derivation also apply to sign language and voice patterns. In fact, they are also so simple that even a computer can understand them! Our instincts may tell us that if we need a computer, our task must be difficult. Consider, however, that a computer can only function by being told *exactly* what to do. The communication protocol that a computer uses must be simple enough to be distilled down into a finite number of combinations using only ones and zeros. Let us conclude this section by exploring a modern application of computers and language that helps illustrate this phenomenon.

There are approximately 44 sounds (phonemes) in the English language depending on the linguistic source we choose to consider. Each sound combination in a spoken word is analogous to the letter combinations used to make written words. Our brains recognize and interpret sounds in spoken language similar to how they recognize and interpret words in written language. Amazingly enough, a computer can be *taught* to do the same thing. By analyzing a range of values for the variables of pitch, duration, frequency, volume, pacing, etcetera, the computer can begin to *recognize* the vocalization of different words and phrases by comparing strings of binary digits. Of course this is a bit of an oversimplification. The actual programming model must be significantly more complex because an English word may be spoken a number of different ways by the same individual, not to mention, many different ways by the English speaking world. Mathematics helps the computer estimate what the sounds of words and phrases should digitally look like by using a mathematical model called a *Markov Chain*. When we speak, the sounds our voices make are converted to binary code (numbers consisting of only ones and zeros). A mathematical analysis can then compare the numeric code to a database of existing codes for possible written word substitutions. The Markov model is a Stochastic modeling process that relies on the idiosyncrasies of a given language to help

determine the intended order of words based on how the language is commonly structured. For example, the words "happy birthday" have a higher probability of being spoken in sequence than "aptly earth pay," which might sound similar to the computer depending on who is speaking. The mathematical model would help the computer select the most likely translation based on the context of the surrounding phrase combinations.

Misconceptions in Mathematical versus Written Vocabulary

Although we know that English words can potentially have several meanings, many of us may not fully appreciate how the meaning of a word is established by the context of the surrounding language. For example, when we use the word "root" in English, we might mean the underground part of a plant or tree, the fundamental cause of a problem, or even a *root*-word extraction. In each option, the surrounding words provide context for how we interpret "root" but note that each of the interpretive contexts fall under an organizational system for written words. In short, with words having multiple meanings, we must use the context of surrounding language to accurately interpret which possible meaning is most appropriate, but consider what might occur if there were a crossover of English words into mathematical language contexts.

The word *root* has a unique mathematical meaning that is written the same way as all other English language notation, but has a very different semiotic notation in the language of mathematics. Suppose we wanted to evaluate the root of an expression to find the hypotenuse of a right triangle. In English, we would simply use the phrase, "evaluate the *root* of the expression to determine the length of the hypotenuse." Translated into semiotic notation for mathematical language, it would read as follows: $h = \sqrt{a^2 + b^2}$. The notation is relatively simple in this example, but with a more complex expression the notation not only assumes we know what the *root* symbol means in the language of mathematics, but also how the surrounding contextual nuances affect the process of simplifying it. As an exercise in English to mathematics language, translate into words the following mathematical notation: $\sqrt[3]{x}$. Syntactically, this expression is simple translate into English, "... the third root of x." Semantically, it is more difficult to capture, which is exactly why the study of mathematics needs to be taught beyond the aspects of procedural fluency. Semantically, the expression means that the *product* of three identical *factors* of a number results in the value *x*. Given this semantic description, we must also understand the definition and context of the terms *product* and *factors*, as well as understanding that the index value of the root determines the number of times we must multiply our identical factors of *x*.

Let us now turn our attention to some inconsistencies in translating from English to mathematics language using a few of the previous ideas. Some words are shared between English description and mathematical notation and can be translated directly from one system to the other without ambiguity. Or, as we have seen using the word "root," the translations can be more difficult and require some contextual information. In a third scenario, there are some delimiters that are shared between the systems but have completely different meanings. In the first case, we might use the example, "five is greater than three." The phrase can easily be translated into mathematical notation, $5 > 3$. The notation $>$ can be directly substituted for the phrase, "is greater than." As a counterpoint, now consider the use of delimiters. We could make an exclamatory statement in English, "… the shoes are red!" Clearly, the writer is expressing excitement about the shoes being red. On the other hand, we could structure a similar phrase in mathematics, "… the answer is 5!" This is not an exclamatory statement, but rather, a substitution of a numeric answer of 120 using *factorial* notation. In mathematics, an exclamation point has its own meaning, which is very different than how it is used in a standard written system. A *factorial* translates into written language as the product of all of the numbers counting down from a given number *n* to 1 and including *n* and 1 (i.e. *n* x (*n*-1) x (*n*-2) ... x 4 x 3 x 2 x 1). The mathematical notation describing a specific factorial such as 5! can be expressed as follows:

$$
5! = \prod_{n=1}^{5} n
$$

Finally, let us briefly look at an English translation of ideas that appear to be self-evident but are actually translated appropriately while interpreted incorrectly. Take the word "twenty" for example. This is given to equate to the numeric value of 20. In reality the number 20 is a notational representation of a quantity that only exists because of place value. That is to say the digits zero through nine are each represented by a single word. The number 20 is also represented by the word "twenty" but the quantity of 20 is actually not a single number in the way that the

digits zero through nine are. The number 20, in essence, a quantity represented by two digits (mathematical letters if you please) and place value notation to create the equivalent of a mathematical *word* meaning 20. The place value notation of the number 20 says we have two sets of a place value worth 10 and zero sets of the place value worth 1. Understanding how numbers follow this rule within an algebraic system is critical to understanding how polynomial operations in algebra can be used to create useful expression and equations.

Conclusion

Capturing the idea of how language is used in everyday communication is tremendously difficult. It is hard to imagine that the 26 alphabetic symbols we use can be permuted into half a million words, which, in turn, can be arranged to make sentences and stories with more possible symbolic permutations than there are atoms in the known universe. But within all of this complexity, there are some very simple underlying mathematical structures that define the use of all languages including the communication aspects mathematics itself. Recall the four postulates that govern the syntax of written and mathematical communication. Letters and digits create the symbols we understand as words, numbers, and abstract expressions. Punctuation and mathematical notation help organize and provide nuance. Equivalent expressions can be substituted for one another in both systems to clarify meaning or model efficiency. Written language and mathematical language are, and will remain, interdependent. Recall also that the mathematical processes of substitution and simplification are designed to reduce ambiguity and misinterpretation in the transmission and presentation of ideas. Finally, consider that vocabulary and symbolic notation is not universally defined between these two interdependent communication systems. These concepts are important to remember in the instruction of both subjects. As mathematics teachers strive to define mathematical processes, it may be effective to consider drawing parallels to how students communicate in written English. As language teachers try to reinforce the idea of clarity in their students' writing, they can draw parallels to simple mathematical processes. At any rate, we should all reinforce what we all know about transmitting messages to solve problems, particularly in the modern age of texting, tweeting, and emailing where abbreviations, shorthand, and colloquial symbolism reign supreme.

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