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# Eureka! A Yurt!

## Integrating Mathematics, Cooperative Learning, and Community Service

by Jo Anna Allen

The significance of "Eureka!" in the title derives from the exhilaration of discovery that took place through a project in which a class of sixth graders built a Yurt.<sup>1</sup> There was a "Wow!" sense of awe and achievement akin to completing one's first rockclimb or a thousand-piece puzzle. In the words of many of the sixth graders who participated—"I can't believe we did it!" They built a scaled-down Yurt (see photo), a circular building having walls that slant upward and outward from a round base, with an overhanging, cone-like roof. The roof has a skylight in the middle, capping the top of the walls. After a fresh four-inch snowfall, the Yurt looks like a frosted cupcake. This wooden Yurt has a folded roof which resembles the folds in cupcake papers.



Jo Anna Allen

*The Yurt built by Class 6B of the Perry Hill Middle School in 1988.*

The main goal of the project, however, was not to build just a unique structure, but to teach mathematics experientially and to facilitate growth through service-learning. The Yurt became part of the playground facilities of a special education school for severely and profoundly handicapped students.

The project was full of adventure and learning because, first of all, it had never been done before by such young students. One of them even said in her evaluation "it was too difficult for sixth graders, but

not for us!" Secondly, in a large public school system there is always the suspense of whether someone will suddenly object to the project, or if red tape will undermine or throw off schedule even the best laid plans. Finally, the project was begun without knowing exactly how the money for materials would be obtained.

Because of the many positive outcomes that are possible through this project, it is being shared in the hopes that others may adopt it and report back their findings. Some of the more general outcomes were: direct application of mathematical principles, increased awareness of the benefits of doing community service, practical knowledge of carpentry, and leadership development. More specifically, over a span of three months during weekly class sessions,

twenty-five sixth grade math students at a middle school journeyed through these steps:

- 1) learned about the history and evolution of the modern Yurt,
- 2) designed eight different models of Yurts and learned the math needed to do it (three classes),
- 3) built Yurt models out of tagboard (three classes),
- 4) analyzed the structures and processed the design and building experiences (one class outdoors),
- 5) visited a special education school where they would build a wooden playground Yurt (field trip),
- 6) successfully solicited the building materials (\$200 in wood and galvanized nails) themselves from a local business (The math office picked up the

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remaining \$75 for costs of a skylight, hardware, cable, caulking, and engraved nameplates honoring the class and the donor.),<sup>2</sup>

- 7) sawed by hand all the pieces needed for the wooden Yurt to make a kit for the big day (one class and two Saturdays),
- 8) and erected the Yurt in about four hours with the help of parents, teachers, and administrators (one final Saturday).

The project required vertical and horizontal collaboration of teachers and administrators. Even the special education school's PTA became involved and informed as they provided lunch on the day the Yurt was built. The special education principal was present that entire day.

The teachers at the middle school who were involved included the class's regular math teacher (who understood experiential education), their English teacher (who directed letter-writing to the lumber company and the reflection essays), and the industrial arts teacher (who supplied tools and advice). Parents helped to measure and saw, and assisted on the Yurt-raising day. Teachers and parents were learners along with students because none had ever built a structure having only one right angle amidst circles and slants and yet still put the Pythagorean theorem into use so frequently.

### How Much Math?

Only very basic geometry is needed to design a Yurt. Trigonometry can also be used if the class is sufficiently advanced. The Pythagorean theorem ( $c^2 = a^2 + b^2$ ), the formula for the circumference of a circle ( $C = 2\pi r$  or  $\pi d$ ), and a basic calculator are the only mathematical tools needed for the design process. The starting dimensions given to each group were the diameter of the center roof skylight and the diameter of the base. The students were allowed to choose on their own, after group discussion and consensus, the number of wall pieces and the remaining dimensions such as the height of the walls and the height of the completed structure. On the data sheet (see Table 1), the dimensions needed to build the model are listed in the order in which they need to be calculated. An examination of the vertical cross-section (Figure 1) of the structure will show the relationships between the dimensions. The two given diameters are sufficient to get the students started and yet create a range of models. The letters (A, B, C, etc.) of each dimension in Table 1 are labelled to correspond with the sketches of the pieces in Figures 1 & 2.

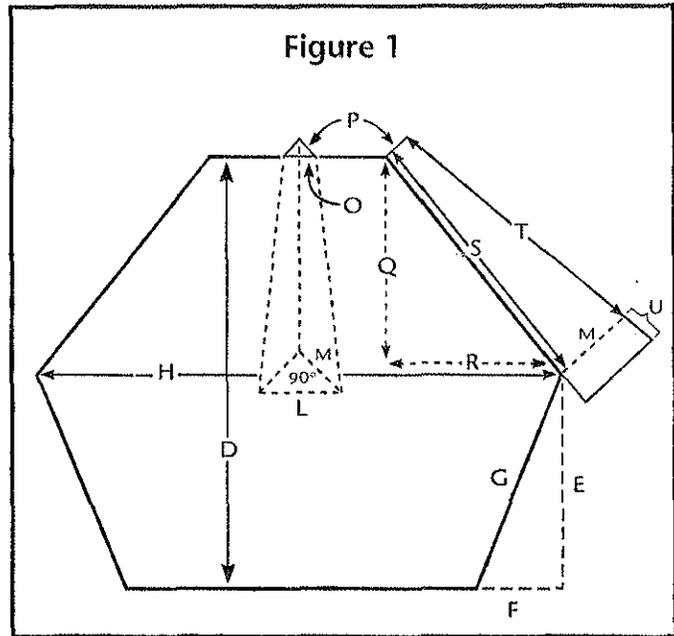


Figure 1  
Cross-section of the dimensions of the Yurt with the letters corresponding to the dimensions labeled in Tables 1 & 2.

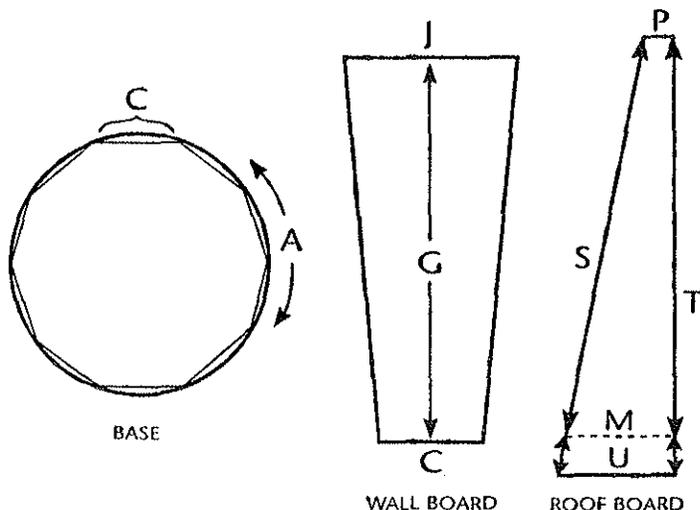
Table 1 gives a description of how to calculate each dimension for a model posterboard Yurt. In designing a wooden Yurt, the width of the lumber must be taken into account and the sawing of the roof boards must be exact. In our Yurt, zeal in planing edges resulted in an error that multiplied to leave a gap in the roof. After a brief moment of disappointment, this was treated as a repairable error and part of the learning process. The sample calculations in the table are for a model Yurt having a base diameter of eight inches and a skylight diameter of three inches. Table 2 provides the step-by-step derivation of the sample numbers in Table 1 as a means to illustrate the calculation process for those who want to put it into practice.

Teaching the derivation of the structural dimensions from the two given numbers, plus the calculations for their own designs, required three class sessions for the group. Older students would take less time.

### Using Cooperative Learning and An Experiential Approach

It does help to have somewhat of a spatial sense to facilitate the design process. Student groups can be formed in accord with this ability or their computational skills. The cooperative learning technique of putting low, medium, and high ability students together in groups of three or four is very effective. Designated duties may be assigned such as materials manager, recorder, and calculator operator. The highest ability student should not work the calculator or the likelihood is that s/he will come up with the figures without the others understanding their origins.

Figure 2



Dimensional diagrams of the base, wall board and roof board of the Yurt.

Table 1

This table lists the dimensions required for the calculations to design a Yurt and corresponds to the lettered dimensions in Figures 1 & 2. The process typically starts with the designation of the base diameter and the skylight diameter. All of the other dimensions can either be decided upon by the students (noted with a \* below) or calculated. Actual numbers in inches are filled in as a sample and as a means to demonstrate the calculation process.

Base Diameter (BD) = 8

Skylight Diameter (SD) = 3

A. Circumference of the base	A = 25.10
B. Number of wall boards *	B = 24.0
C. Bottom width of the wall boards	C = 1.05
D. Height from floor to skylight *	D = 8.00
E. Vertical height from floor to top of wall *	E = 5.00
F. Smaller leg of wall triangle	F = 1.92
G. Length of the wall board	G = 5.36
H. Diameter of the top circle of the wall	H = 11.84
I. Circumference of top circle of the wall	I = 37.20
J. Top width of the wall board	J = 1.55
K. Number of windows *	K = 8.00
L. Base of triangular window	L = 4.65
M. Leg of triangular window	M = 3.28
N. Skylight circumference	N = 9.42
O. Base of triangle at skylight	O = 1.18
P. Leg of triangle at skylight	P = 0.83
Q. Height from top of wall to skylight	Q = 3.00
R. Segment of diameter of top circle of the wall	R = 4.42
S. Length of the valley edge of the roof piece	S = 5.34
T. Length of the peak edge of the roof piece	T = 6.27
U. Overhang of the roof piece	U = 0.50
V. Valley edge of the roof piece plus the overhang	V = 5.84
W. Peak edge of the roof piece plus the overhang	W = 6.77

This is analogous to an orienteering adventure in which the youngest child or the slowest-moving one is designated the compass-carrier. "Numbered heads together" (another cooperative learning structure) can be used to report on group progress so that the normally dominant student does not continually take the lead (Kagan, 1989). "Pairs check" can be used to double check the figures within the group or even between groups.<sup>3</sup>

If the teacher happens to be unfamiliar with Yurt design, s/he can compensate by being one who likes to learn with the students and values the mistakes that will undoubtedly happen. The teacher should consider whether to allow the model-building speak for the accuracy of the calculations, or to play it safe and check each and every number. My preference is to let the consequences teach the students, and use reflection and group debugging to decide how models can be improved or rescued. A wrong-sized roof can be given a new base and vice versa.

The teacher can also dive right into the project without making a test structure beforehand. I recommend starting from scratch with the students, and making your own Yurt model alongside the class. It is important to keep in mind that it is the process that teaches and not the product. The students in my group saw slides of various Yurts, but never saw a completed tagboard model except for their own. We did not want them to have preconceived ideas of a "correct" answer. This is not to say the teacher does not have to work out the mathematics ahead of time to understand where the pitfalls are for the students. One group capped off their wall base with the roof, only to have it fall inside, which resulted in peals of laughter and light-hearted jokes rather than tears or frustration.

The "correctness" of the results is a matter of aesthetics. Following the completion of the model-building, we arranged for the class to meet outdoors in a grassy setting with the

## Table 2

This table describes the step-by-step process for designing a Yurt. The lettered dimensions correspond to the dimensions designated in Table 1 and Figures 1 & 2.

1. The circumference of the base (A) is found by using the formula for the circumference of a circle,  $c = \pi d$ , in which d is the diameter of the circle and  $\pi$  is 3.14. In this case,  $A = \pi(\text{Base Diameter})$ .
2. Decide by consensus how many wall boards (B) the Yurt is to have. The sample shows twenty-four. Students might sketch a circle and divide it with chords representing wall boards to help decide (see the sketch of the base in Figure 2).
3. The bottom width of the wall boards (C) is found by dividing the circumference (A) by the number of wall boards (B). Given a sufficient number of wall boards, the chord length is close enough to the actual arc length.
4. Decide on the height from the floor to the skylight (D). The sample shows 8 inches.
5. Also decide how high the top of the boards will be from the floor (E), i.e., the vertical height. Notice in Figure 1 that E is the longer leg of a right triangle and the wall board is the hypotenuse of the right triangle. In the sample, E was chosen as 5 inches.
6. Calculate the smaller leg of the wall triangle (F) using proportions so that E divided by F equals 13 divided by 5. With 5 substituted for E, the bottom leg of the wall triangle becomes 1.92 inches. This slope can be changed to whatever is aesthetically pleasing or practical. If the wall is to be a back rest for benches, a less steep slant might be desired such as 13/4 or 14/5.
7. The length of the wall board (G) is calculated using the Pythagorean theorem. In this project's playground Yurt, a standard four-foot board was used to determine the vertical height rather than vice versa.
8. The diameter of the top circle (H) is found by adding the base diameter (BD) to two times the F dimension ( $H = BD + 2F$ ).
9. The circumference of the top circle of the wall (I) is calculated using the circumference formula (see step 1) with H being the diameter.
10. To get the width of the top of the wall board (J), divide the circumference of the top of the wall (I) by the number of wall boards (B), which is 24 in the example.
11. Decide how many windows (K) the Yurt will have. The windows are the triangular openings formed where the roof boards meet the top of the wall. In the sample, 8 windows were decided upon based on using three wall boards per triangular window.
12. The width of the base of the window (L) is determined by dividing dimension I (the circumference of the top wall) by K (the number of windows).
13. The legs of the window (M) have a 90 degree angle between them and are equal. Use the Pythagorean theorem once again, which in this case is,  $L^2 = M^2 + M^2$ . Solve for M.
14. To get the skylight circumference (N), use the skylight diameter (SD) given at the outset (3 inches) and the formula for circumference (see step 1).
15. Divide answer N by K (the number of windows) to obtain the base of the triangle at the skylight (O).
16. The leg of the triangle at the skylight (P) is calculated the same way M was in step thirteen, only using O as the hypotenuse rather than L ( $O^2 = P^2 + P^2$ ).
17. The length of the valley edge of the roof piece (S) is the hypotenuse of a right triangle in which one leg (Q) is the vertical height from the top of the wall to the skylight and the other leg (R) is one segment of H, the diameter of the top circle of the wall. Q is calculated by subtracting E from D (the height from the floor to the skylight minus the vertical height from the floor to the top of the wall), or 8 minus 5 equals 3 in the example. R is calculated according to  $R = (H - SD)/2$  (the diameter of the top of the wall minus the skylight diameter, all divided by 2). S is now calculated using the Pythagorean theorem as follows:  $S^2 = Q^2 + R^2$ , or 5.34 inches in the sample. Note that in a wooden Yurt, the grain of the wood should run parallel to this edge for maximum strength.
18. Ignore the P length and assume a right triangle in order to calculate the length of the peak edge of the roof piece (T), based on the lengths S and M. Use the Pythagorean theorem:  $T^2 = S^2 + M^2$ .
19. For overhang (U), .5 inch is sufficient, although a 1 inch overhang might insure that the roof doesn't fall in.
20. Add the overhang (U) to S to get the actual valley length of the roof board (V).
21. Add the overhang (U) to T to get the actual peak length of the roof board (W). Eureka! A Yurt!

various Yurt models positioned in the center of the circle of students. (It is best not to do this on a windy day.) The sun, shining on the unique shapes with curved shadows, nicely set off the structures.

It became very clear to the students that some of the Yurts looked better than others. Their words included "too squatty," "too tall" or "has a better shape." Their observations were a perfect lead-in to a discussion of proportions and aesthetics, one of the goals of our school system's math program, but one which is difficult to address within the curriculum.

From this discussion, it was decided by the class that one particular model looked the best, and that became the basis for the wooden playground Yurt. To save time, I calculated those dimensions myself, taking into account the 3/4-inch thickness of the board lumber we would use. For the model-building, this was a factor that could be ignored. Although we used tagboard, I now recommend using poster board which is thicker and can be bought in an assortment of colors. The "winning" Yurt was the largest and had to be strengthened with shoe strings woven into the walls because of the thinness of the tagboard, much like the

## How Big a Project?

There are all sorts of ramifications and extensions that can be tacked on to this initial phase. However, there is no need to go beyond the model building in order to have a complete project or to learn mathematics experientially. But the growth and self-esteem component of the project was greatly enhanced by its extension into building a playground Yurt as a service project. So many of the essays that culminated the endeavor spoke of the feelings the students experienced from having contributed to the special education population. Repeatedly they expressed the disbelief they had felt on being told of the upcoming project, skeptical that as sixth graders they would build a structure that they "could really walk into!" When they finished, the most repeated phrase was "I can't believe we did it."

A drafting class, or an art class, or any group could be encouraged to be creative with the design. I chose the folded roof simply because of its beauty and uniqueness. There are many other possibilities for both the wall and roof which have been tried out in actual structures all over the world. If more time had been spent on scale drawings, it would have expanded the students' experience. We found the time needed to develop this skill was too great to fit into the original project timetable. Scale drawing is also a somewhat difficult task for sixth-graders. I am not sure at what point scale drawings can be used with ease for a two-dimensional preview of the aesthetics of the chosen designs.

A science class could delve into the heat efficiency aspects of insulating the structure, using their mathematical skills to calculate surface area versus volume. Because the space under the folded roof consists of diminishing equilateral triangles rising at an angle, the best way to calculate the volume is to use calculus. The volume of the base is most simply derived by using the formula for the volume of a cone,  $V = (\pi r^2 h) / 3$ .

One extra variation of the project would be to have students use a computer spreadsheet to derive the figures for a multitude of Yurt sizes, not all of which may be aesthetically pleasing, but which would result in a data table that could be used to check student calculations. However, once devised, such a spreadsheet should not be allowed to take away from the sheer discovery and pleasure of calculating for oneself, and finding that the numbers one derives by hand can be translated into concrete results.

Anyone wishing to build a wooden Yurt would benefit from experience working with wood and doing carpentry.<sup>4</sup> Our Yurt's dimensions included a forty-four inch diameter base (from a cable reel) and a height of approximately six feet (see photo). The skylight was eighteen inches in diameter with a twenty-four inch diameter plexiglass skylight bolted on top. There is no

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cables that support a full-sized Yurt. This can be avoided by using heavier stock.

One of the comments by one group whose Yurt had many problems was, "I guess we could have worked more carefully." Calculating carefully and measuring, marking, and cutting exactly makes a tremendous difference. With twenty-four wall pieces, an error is multiplied by twenty-four.

Other mathematical concepts that were picked up experientially were the need to round off answers on the calculator, and the meaning of .45 feet which needed to be translated into inches, pointing out the advantage of the metric system. The idea of limits is actually inherent in the design because the more wall pieces there are, the closer the chords approximate the circle. Without prompting, students will probably come up with their own techniques for constructing the models. The fastest group actually chose the most labor-intensive method, cutting and taping individual wall and roof sections. Others found they could score and fold the tagboard and do less cutting. We tried to give no hints as to which way to proceed. Once the decision was made, groups worked oblivious to what other groups had chosen, convinced that their way would work just as well. Everyone was right.

door closing the Yurt but benches were added a week after the Yurt was built. The custodial staff of the special education school painted the structure orange and blue, inside and out, to match the other playground structures.

A standard-sized Yurt, seventeen feet in diameter at the eaves, makes an excellent classroom-in-the-round with benches built at a comfortable height against the wall (yes, only one wall!). It also can be built as housing for one or two persons.<sup>5</sup> Much larger, concentrically-designed Yurts have been built as homes.

The possibilities are endless, starting with an understanding of the mathematics, or even better, not knowing the math. Where the class goes depends on the energy and creativity of both the teacher and students working together. It is a unique way of combining mathematics and hands-on experience with the lessons of service learning.

### Notes

1. The word Yurt is being capitalized to distinguish it from, and out of respect for, the Mongolian yurt, a portable shelter whose building principles inspired the modern version.

2. Class 6B of the Perry Hill Middle School built the Yurt for Ridge School in the Spring of 1988. Both schools are in Baltimore County, Maryland.

3. *Numbered Heads Together* originated with Spencer Kagan (see Kagan, 1989). It is useful for reviewing material and responding or reporting to the class. The students in each team are numbered (a team of four would have 1, 2, 3, 4). They do not know who is going to be called on. Once the teacher is ready for reports from groups or answers to a question, (s)he calls out a number, all of whom answer taking turns, or simultaneously using hand signals, showing response cards, or even in chorus. In a project such as this, this reporting process allows the quieter student to be the spokesperson for the group. The student must also collaborate with the team to get a consensus on the report.

*Pairs Check* is another Kagan structure. Within each team of four, students can be paired to calculate the Yurt dimensions. One member of each pair solves the problem while being coached by the partner. They can then check with the other pair in the team to see whether that pair also obtained the same answer for the dimension. Both pairs are working with the same starting numbers for the base and skylight diameters. For this project, we used eight teams of three persons each in order to get eight different models. Groups of four would facilitate *pairs check* but would result in fewer models to study.

4. There are a variety of additional considerations when building a wooden yurt. Attaching the wall boards to the floor can be tricky, but trial and error will probably suffice. It helps to see a building firsthand to understand the overlap of the wall boards to produce the slope. Yurts with larger diameters are built with a steel cable around the top of the wall, cinched up tightly and fastened in brackets or clinched nails, in order to help support the weight of the roof pressing out on the wall. The wall boards overlap more at the bottom than at the top, and are nailed the length of the overlapped area. In a small-sized Yurt such as this one, the cable is actually not necessary.

The roof boards converge around the skylight and in large Yurts, the skylight opening is strengthened through the use of a steel or wooden compression ring which maintains the shape of the round skylight and bears the weight of the roof forcing in. The small Yurt done by the students did not need a compression ring in the skylight, although we did put a steel cable around the top of the wall boards.

5. For more information on Yurts and Yurt workshops, contact Dr. Wm. S. Coperthwaite, Director, Yurt Foundation, Bucks Harbor, Maine 04618.

### References

- Kagan, Spencer (1989). *Cooperative learning: Resources for teachers*. San Juan Capistrano, CA: Spencer Kagan.