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Abstract

This paper investigates the desirability of constitutional constraints on capital taxation in an environment without government debt and where benevolent governments have limited commitment. In our setup, governments can choose proportional capital and labor income taxes subject to the constitutional constraint but cannot commit to the actual path of taxes. First, we explore a form of constitutional constraint: a constant cap on capital tax rates. In our quantitative exercise, we show that a three per cent cap on capital taxes provides the highest welfare at the worst sustainable equilibrium. However, such cap decreases welfare at the best sustainable equilibrium (both because it constrains feasibility and tightens the incentive compatibility constraint). Second, we identify a form of constitutional constraint that can improve all sustainable equilibria. That constraint features a cap on capital taxes that increases with the level of capital.

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1 Introduction

The literature on optimal taxation provides one central conclusion: capital taxes should be very high in the short run and zero in the long run.[1] However, as time passes, governments are tempted to revise the previously chosen capital taxes.[2] Such temptation generates a well-known time-inconsistency problem that can lead to higher capital taxes, lower capital accumulation and welfare losses (as, for example, illustrated in Fischer (1980)). In order to alleviate this problem, several authors have suggested to impose constitutional constraints on capital taxation.[3]

This paper evaluates the desirability of constitutional constraints on capital taxation. We define a constitutional constraint as a cap on capital taxes, i.e. an exogenously imposed upper limit on the tax rates allowed to be set on capital income. We consider an economy with benevolent governments that have limited commitment. In order to finance public consumption, governments choose linear capital and labor tax rates subject to the constitutional constraint but cannot commit to the specific tax rates. Therefore, our environment is one with limited commitment in the sense that governments can commit to the cap but not to the actual sequence of tax rates.

In this setup, we provide a quantitative evaluation of different constitutional constraints on capital taxes by assessing their effects on the Best Sustainable Equilibrium (BSE) and the Worst Sustainable Equilibrium (WSE). We focus on both equilibria rather than only on the best because the coordination of beliefs required to sustain the best is not under the control of the government.[4]

First, we explore a form of constitutional constraint: a constant cap on capital tax rates. Our numerical results show that welfare at the BSE increases with the cap. Then the uncapped case delivers the highest and the zero cap the lowest welfare at the BSE. We find that the welfare at the WSE displays an inverted U-shape relative to the cap. For our baseline parameters, the welfare provided by the WSE is lowest at the uncapped case. Without caps, the WSE features

[1] See Chamley (1986), Judd (1985), and Chari and Kehoe (1998).

[2] Chari et al. (1994) find that 80% of the welfare gains of switching from the current tax to the optimal come from the high initial capital taxes.

[3] Some authors, such as Lucas (1990) and, more recently, Mankiw et al. (2009), have suggested an extreme form of constitutional constraint on capital taxation: the abolition of capital taxes.

[4] Rogoff (1987) provides an insightful discussion of the multiplicity of equilibria and the associated coordination problems in reputational models. He writes "the government can achieve some degree of coordination by placing external restraints on itself ... amounts to changing the structure of the game so that there are less equilibria". Our paper aims to find constitutional constraints that eliminate "bad" equilibria and lessen the coordination problem.

very high capital taxes and low capital accumulation. The highest welfare at the WSE is provided by a positive but low cap on capital taxes, more specifically at a three per cent cap.[5] We explain the results as follows. For the BSE, we find that long-run distortions are not present. At the BSE, a cap has a direct negative effect by reducing the short-run benefits of capital taxes and an indirect ambiguous effect through the sustainability via the welfare at the WSE. For our baseline parameters, the overall effect is negative and lowering the cap reduces the payoff at the BSE. For the WSE, the reduction of the cap lowers the long-run distortions at the WSE, but it also reduces the short-run benefits of less distortionary taxation. For moderate and large caps, the first effect dominates and the cap is welfare enhancing at the WSE. For very low caps, the second effect is larger and the cap is welfare reducing at the WSE.

Second, we investigate whether there exists a constitutional constraint on capital taxation that could benefit all equilibria. We find that the maximal tax rate on capital income prescribed by the Ramsey (with full commitment) for each level of capital stock provides a natural upper limit on capital taxation. For our balanced-budget policies, the incentives to tax capital are small (large) for low (high) levels of capital stock. Those incentives deliver an 'optimal' cap on capital taxes that increases with the level of capital. We show that such natural upper limit on capital taxation can improve the welfare provided by the WSE without worsening the welfare at the BSE.

Since Kydland and Prescott (1977)'s seminal work, several papers have studied optimal capital taxation without commitment. Chari and Kehoe (1990) use a model of intra-period capital accumulation to provide a game-theoretic formulation for the taxation problem. This formulation is extended to a dynamic setting by Phelan and Stacchetti (2001) and Sleet (1997). An alternative approach (that rules out reputational mechanisms) is used in Klein et al. (2008). They find a Markov-perfect equilibrium that is quantitatively close to our WSE in terms of capital taxes and capital stock. Following the same approach, Martin (2010) analyzes the effects of bounds on taxes.[6]

The earlier works of Kydland and Prescott (1977), Lucas (1986) and Chari (1988) suggest the use of institutional changes (implementation lags, monetary standards, budget balance, the elimination of capital taxes, etc.) to ameliorate time-inconsistency problems. A recent example

[5] We find that the specific level of the cap that maximizes welfare at the WSE depends on the relative need of distortionary taxation.

[6] Martin (2010) finds that, for some parameters, the Markov-perfect equilibrium may coincide with the WSE.

is that of Athey et al. (2005) who study the optimal degree of discretion in monetary policy and find that a cap on inflation can implement the best incentive-compatible equilibrium. Domínguez (2010) studies the effects of debt limits and deficit restrictions on the time-inconsistency problems of default and devaluation of government debt.

The rest of the paper follows the following structure. Section 2 presents the model. Section 3 describes the policy game and quantifies the effects of different constitutional constraints. Section 4 concludes. Proofs, Tables and Figures are relegated to the Appendix.

2 The Economy

Time is discrete and indexed by t. The economy is populated by a continuum of infinitely-lived identical households, a continuum of perfectly competitive firms and a benevolent government.

The representative household is characterized by the following life-time utility:

$$(1-\beta)\sum_{t=0}^{\infty}\beta^{t}\left[u(c_t,n_t)+G(g_t)\right], \tag{1}$$

with the discount factor $\beta \in (0,1)$.[7] The instantaneous utility $u(\cdot,\cdot)+G(\cdot)$ is a function of private consumption c_t, labor n_t and public consumption g_t, and takes the following form:

$$u(c_t,n_t)+G(g_t)=\frac{{c_t}^{1-\sigma}}{1-\sigma}-\gamma_n\frac{{n_t}^{1+\chi}}{1+\chi}+\gamma_g\frac{{g_t}^{1-\sigma}}{1-\sigma}, \tag{2}$$

where $\sigma \geq 0$ and $\chi \geq 0$ are respectively the inverses of the elasticity of intertemporal substitution of consumption and of labor. The parameters $\gamma_n \geq 0$ and $\gamma_g \geq 0$ represent the weights on labor disutility and on public consumption utility respectively.

Each individual is endowed with the initial capital k_0. Taking prices and the government policy as given, the representative household chooses consumption, labor and capital to maximize his welfare (1) subject to the budget constraint

$$R_t k_t+(1-\tau_t^n)w_t n_t \geq k_{t+1}+c_t, \tag{3}$$

[7] The instantaneous utility is normalized by $(1-\beta)$.

and the no-Ponzi-game condition

$$\lim_{t\to\infty} p_t k_{t+1} \geq 0. \tag{4}$$

Here p_t is the multiplier on the budget constraint (3), w_t the real wage, τ_t^n the tax rate on labor income, R_t the gross return on capital, after tax τ_t^k and depreciation δ rates, and r_t the net return on capital, i.e. $R_t = 1 + (1 - \tau_t^k)(r_t - \delta)$, at date t. The first-order conditions for this problem are

$$-u_{n,t} = (1 - \tau_t^n) w_t u_{c,t}, \tag{5}$$

$$u_{c,t} = \beta R_{t+1} u_{c,t+1}, \tag{6}$$

and the transversality condition $\lim_{t\to\infty} \beta^t u_{c,t} k_{t+1} = 0$, where u_c and u_n denote the marginal utility with respect to consumption and labor, respectively. Other derivatives follow similar notation.

A representative competitive firm produces the final good using the technology $y_t = f(k_t, n_t) = Ak_t^{\alpha} n_t^{1-\alpha}$, with $A > 0$ and $\alpha \in (0, 1)$. Taking factor prices as given, the firm chooses capital and labor to maximize profits, which implies

$$r_t = f_k(k_t, n_t) \text{ and } w_t = f_n(k_t, n_t). \tag{7}$$

We consider a benevolent government that must finance an endogenous public consumption g_t with taxes on labor income and on capital income. We restrict attention to balanced-budget policies with no initial government debt.[8] Then the per-period government's budget constraint is

$$\tau_t^n w_t n_t + \tau_t^k (r_t - \delta) k_t = g_t. \tag{8}$$

A key feature is that governments must comply with the constitution. In the absence of constitutional constraints, we assume that the rates, τ_t^n and τ_t^k, can take any value in the inter-

[8] Two reasons justify this assumption. First, even with commitment, a government would default on any initial positive public debt. Second, our tax problem is computationally demanding and allowing for government debt would substantially increase the dimensionality of the problem.

val $[0, \tau^{max}]$, with $0 < \tau^{max} \leq 1$.[9,10] We let $\tau_t^n \in T^n = [0, \tau^{max}]$. Then, a constitutional constraint on capital taxes takes the form of an upper limit on the capital tax rate, i.e. $\tau_t^k \in T^k = \left\{[0, \tau^{k,UL}] | 0 \leq \tau^{k,UL} \leq \tau^{max}\right\}$. For example, the abolition of capital taxes corresponds to $\tau^{k,UL} = 0$.

The resource constraint can be written as

$$f(k_t, n_t) + (1 - \delta)k_t = c_t + k_{t+1} + g_t. \tag{9}$$

Finally, we define a competitive equilibrium as follows:

Definition 1 *Given the tax rates $\left\{\tau_t^k, \tau_t^n\right\}_{t=0}^{\infty}$, and initial capital k_0, a competitive equilibrium allocation $\{c_t, n_t, k_{t+1}, g_t\}_{t=0}^{\infty}$ and prices $\{p_t, r_t, w_t\}_{t=0}^{\infty}$ are such that: (i) given prices, tax rates and k_0, the representative individual maximizes welfare (1) subject to the budget constraint (3) and the no-Ponzi game condition (4); (ii) factors are paid their marginal products (7); (iii) the government budget constraint (8) is satisfied; and (iv) all markets clear.*[11]

3 Time-Consistent Policy subject to Constitutional Constraints

In this Section, we present the policy game and compute the time-consistent optimal fiscal policy subject to different constitutional constraints on capital taxes. This is a form of limited commitment as we allow future governments to reconsider their policy but they commit to do so within the range allowed in the constitution.

First, we describe our policy game and provide a definition of equilibrium. Second, we formulate the game recursively. Finally, we calibrate the economy and present our quantitative results by showing the effect of different constitutional constraints on the set of sustainable equilibria. Among those constitutional constraints, we identify a form that can be desirable for all equilibria.

[9] We require tax rates to be bounded above to guarantee that the competitive equilibrium can be written recursively. This can be justified on the grounds that in a more general model there would be an endogenous upper bound on the tax rates. For example, whenever the capital tax rate is such that $R_t \leq 1 - \delta$, individuals would prefer to leave capital idle (see Chamley (1986)). Similarly, for very high labor taxes, individuals may prefer not to work.

[10] Additionally, as in Phelan and Stacchetti (2001), we require tax rates to be non-negative, i.e. $\tau_t^i \geq 0$, for $i = \{k, n\}$. Similarly, capital taxes can be imposed only when capital income is positive $(r_t - \delta) \geq 0$. With these assumptions, we rule out subsidies. We discuss the relaxation of this assumption in the numerical section.

[11] Given that (3) and (8) hold, the resource constraint (9) is also satisfied in a competitive equilibrium.

3.1 The Policy Game

Here we describe our dynamic game between the government and households. Before the game starts, a set of constitutional constraints on the governments' policy choices is exogenously imposed. All households and government understand and conform with those constitutional constraints.

Households are anonymous. Then the choices of a given household are not observed either by the government or by other households. In accordance, we restrict attention to choices that depend on public history. Public history is denoted by $\zeta^t = (\zeta_1, ..., \zeta_t)$, where $\zeta_t = (\tau_t^n, \tau_t^k, k_{t+1})$.[12] It should be noted that, as in Feng (2015), we do not require a public randomization device.

In our sequential equilibria, the government chooses first. A strategy for the government at date t, denoted $\sigma_{G,t}(\zeta^{t-1})$, is a choice of current taxes (subject to the constitutional constraints) as a function of the history ζ^{t-1}, i.e. $\left(\tau_t^n, \tau_t^k\right) = \sigma_{G,t}(\zeta^{t-1})$. Households choose second. A symmetric strategy for them at date t, denoted $\sigma_{H,t}(\zeta^t)$, is a choice of a current allocation as a function of the public history $(\zeta^{t-1}, \tau_t^n, \tau_t^k)$, i.e. $(c_t, n_t, k_{t+1}) = \sigma_{H,t}(\zeta^{t-1}, \tau_t^n, \tau_t^k)$.[13] After each history ζ^{t-1}, a strategy profile (σ_G, σ_H) induces a continuation strategy profile. A strategy profile induces an outcome, which produces a payoff for the government and a payoff for the households. Now we define the conditions under which a symmetric strategy profile (σ_G, σ_H) is a sustainable equilibrium.

Definition 2 *A symmetric strategy profile (σ_G, σ_H) is a sustainable equilibrium if it satisfies the following conditions for all $t \geq 0$:*
(i) given the symmetric strategy for households $\sigma_{H,t}$, the continuation payoff for the government is higher than the payoff from any deviation to a different strategy $\tilde{\sigma}_{G,t}$ for every history ζ^{t-1}; and
(ii) given the strategy for the government $\sigma_{G,t}$, the continuation payoff for the household is higher than the payoff from any deviation to a different strategy $\tilde{\sigma}_{H,t}$ for every history $(\zeta^{t-1}, \tau_t^n, \tau_t^k)$.

The above definition builds on two conditions that guarantee sequential rationality (as in Chari and Kehoe (1990) and Phelan and Stacchetti (2001)). The first requires the government not to

[12] As discussed by Phelan and Stacchetti (2001), there is no need to include the history of the distributions of all choices made by households (even if they are publicly observed), because all households choose the same actions along the equilibrium. In addition, there is no need to include the household's private history when they make choices, because the convexity of the household's problem ensures the optimality of on-the-path behavior.

[13] Market clearing determines the returns to capital and labor, r_t and w_t. After the households decide, the collected tax revenue determines the level of public consumption g_t.

have incentives to deviate and the second requires individuals to behave competitively.

3.2 A Recursive Formulation for our Policy Game

The APS method (Abreu et al. (1990)) shows that a repeated game can be written recursively by adding as a state variable a continuation value (that is an equilibrium payoff of the repeated game beginning next period). In an environment similar to ours, Phelan and Stacchetti (2001) extend the APS method to policy games with natural state variables (such as capital) between the government and a continuum of households. In addition to a continuation value for the government, they incorporate the marginal value of capital as a continuation value for the households. Both values are equilibrium payoffs of the game beginning next period, which depend on the next period capital stock, and summarize all relevant information about the future.

3.2.1 Competitive Equilibria in Recursive Form

The main idea is to think of our dynamic economy as a sequence of static economies (which are explained below) for which the state variables evolve endogenously and according to their appropriate laws of motion.

Let's denote the marginal value of capital as m_{t+1}, i.e.

$$m_{t+1} \equiv u_{c,t+1}[1 + (1 - \tau^k_{t+1})(r_{t+1} - \delta)], \tag{10}$$

which is as in Phelan and Stacchetti (2001), except for that capital depreciation is tax-deductible. For a given exogenous m_{t+1}, and given τ^k_t, τ^n_t, and k_t, the household's static problem is defined as follows:

$$\max_{\{c_t, n_t, k_{t+1}\}} u(c_t, n_t) + G(g_t) + \beta m_{t+1} k_{t+1}$$

subject to the budget constraint (3).

Given m_{t+1} as in (10), this recursive problem (the above static problem considered in each period) is equivalent to the sequential problem for the household stated in Section 2.

Proposition 1 *For our utility function (2) and production function* $y_t = Ak^\alpha_t n^{1-\alpha}_t$ *and provided*

m_{t+1} is defined as in (10), the recursive and the sequential problems for the household are equivalent.

Proof. See the Appendix. ■

For m_{t+1} as in (10), the optimality conditions of the above static problem are Equations (5)-(6) of the sequence problem. In addition, we show that the transversality condition holds.[14]

For a vector $(k_t, \tau_t^k, \tau_t^n, m_{t+1})$, this static economy consists of the household's static problem, the firm's problem (which is static) and the government's budget constraint (8). Denoting next period variables with subscript $+$, a competitive equilibrium for the static economy is as follows:

Definition 3 *The vector (c, n, k_+, g, w, r) constitutes a competitive equilibrium of the above static economy, denoted $(c, n, k_+, g, w, r) \in CE(k, \tau^k, \tau^n, m_+)$, if and only if*

$$u_c(c, n) = \beta m_+, \tag{CE-1}$$

$$-u_n = (1 - \tau^n) w u_c, \tag{CE-2}$$

$$k_+ = [1 + (1 - \tau^k)(r - \delta)]k + (1 - \tau^n) wn - c, \tag{CE-3}$$

$$g = \tau^n wn + \tau^k (r - \delta) k, \tag{CE-4}$$

$$w = f_n(k, n), \tag{CE-5}$$

$$r = f_k(k, n). \tag{CE-6}$$

Moreover, provided m_+ is defined as in Equation (10), it follows from Proposition 1 that for given $\left\{\tau_t^k, \tau_t^n\right\}_{t=0}^{\infty}$, and k_0, a sequence $\{c_t, n_t, k_{t+1}, g_t, w_t, r_t\}_{t=0}^{\infty}$ that is a competitive equilibrium of the static economy in each period, is also a competitive equilibrium of our dynamic economy.

We can now define the equilibrium value correspondence of our dynamic economy. Let's denote the value for the government as $h = u(c, n) + G(g) + \beta h_+$. Our recursive formulation requires a continuation value for households, denoted m_+, and a continuation value for the government, denoted h_+, to define the set of values (m, h) that can be attained in a sustainable equilibrium. For a given initial capital k, this set of values is called the equilibrium value correspondence $\mathbf{V}(k)$.

[14] Our proof extends those of Phelan and Stacchetti (2001) and Feng (2015) by allowing for a utility function that does not satisfy $\lim_{n \to 1} u(., n) = -\infty$.

3.2.2 Self-Generation

Exploiting the recursivity of the problem and imposing the conditions for a sustainable equilibrium, the equilibrium value correspondence can be found as a fixed point of an arbitrary value correspondence that contains $\mathbf{V}(k)$. We first define an arbitrary value correspondence $\mathbf{W}$ as any mapping from k into sets of payoffs (m, h). Then we define consistency:

Definition 4 *The vector $\psi = (\tau^k, \tau^n, c, n, k_+, g, w, r, m_+, h_+)$ is said to be consistent with respect to the value correspondence $\mathbf{W}$ at k if $(c, n, k_+, g, w, r) \in CE(k, \tau^k, \tau^n, m_+)$, $\tau^i \in T^i$, for $i = \{k, n\}$, $(m, h) \in \mathbf{W}(k)$, and $(m_+, h_+) \in \mathbf{W}(k_+)$, where the payoffs for the households and for the government are given as*

$$m(k, \psi) := u_c(c, n)[1 + (1 - \tau^k)(r - \delta)], \tag{11}$$

$$h(k, \psi) := u\,(c, n) + G(g) + \beta h_+. \tag{12}$$

Next, we define admissibility as follows:

Definition 5 *The vector $\psi = (\tau^k, \tau^n, c, n, k_+, g, w, r, m_+, h_+)$ is said to be admissible with respect to the value correspondence $\mathbf{W}$ at k if it is consistent and*

$$h(k, \psi) \geq \tilde{h}(k, \tilde{\psi}), \tag{13}$$

with $\tilde{\psi} = (\tilde{\tau}^k, \tilde{\tau}^n, \tilde{c}, \tilde{n}, \tilde{k}_+, \tilde{g}, \tilde{w}, \tilde{r}, \tilde{m}_+, \tilde{h}_+)$, and where $\tilde{h}(k, \tilde{\psi})$ is the worst possible payoff for the government when it deviates, that is,

$$\tilde{h}(k, \tilde{\psi}) = \max_{\tilde{\tau}^k, \tilde{\tau}^n} \left\{ \min_{\tilde{c}, \tilde{n}, \tilde{k}_+, \left(\tilde{m}_+, \tilde{h}_+\right) \in \mathbf{W}(\tilde{h}_+)} \left[u(\tilde{c}, \tilde{n}) + G(\tilde{g}) + \beta \tilde{h}_+ \right] \right\},$$

such that $(\tilde{c}, \tilde{n}, \tilde{k}_+, \tilde{g}, \tilde{w}, \tilde{r}) \in CE\left(k, \tilde{\tau}^k, \tilde{\tau}^n, \tilde{m}_+\right)$ and $\tilde{\tau}^i \in T^i$ for $i = \{k, n\}$.[15]

Thus, admissibility captures the two conditions required in the definition of a sustainable equilibrium. Through consistency, it satisfies that individuals behave competitively. Through the

[15] Note that $\tilde{\psi} = (\tilde{\tau}^k, \tilde{\tau}^n, \tilde{c}, \tilde{n}, \tilde{k}_+, \tilde{g}, \tilde{w}, \tilde{r}, \tilde{m}_+, \tilde{h}_+)$ is then also consistent with respect to $\mathbf{W}$ at k.

incentive compatibility constraint (13), it implies that the government does not want to deviate. As in Abreu et al. (1990), the government does not need to evaluate the consequences of all possible actions, it suffices to consider the payoff associated with the best deviation.

It is worth noticing that the constitutional constraints on taxation affect admissibility directly and indirectly. The direct effect is by reducing the number of competitive equilibria that are consistent and the taxes that can be imposed in the best deviation (the worst). This direct effect induces an indirect effect by restricting the expectations that households can hold.

We then define an operator $\mathbb{B}$, $\mathbb{B} : \mathbf{A} \to \mathbf{A}$, where $\mathbf{A}$ is the space of all value correspondences. The operator $\mathbb{B}$ is the convex hull of all sets (m, h) that satisfy admissibility (and therefore consistency). That is, the payoffs (m, h) that form part of a sustainable equilibrium. Computing the mapping $\mathbb{B}$ amounts to find a set $\mathbb{B}(\mathbf{W})$, that is the set of (m, h) that can be enforced today

$$\mathbb{B}(\mathbf{W})(k) = \left\{(m, h) | \exists (\tau^k, \tau^n, c, n, k_+, g, w, r, m_+, h_+) \text{ that are admissible w.r.t. } \mathbf{W} \text{at } k\right\}.$$

Then, Phelan and Stacchetti (2001) show that an arbitrary value correspondence containing $\mathbf{V}$ converges to the equilibrium value correspondence. Their results (which apply to our setup) can be summarized as:

1. If $\mathbf{W} \subseteq \mathbb{B}(\mathbf{W})$, then $\mathbb{B}(\mathbf{W}) \subseteq \mathbf{W}$.

2. $\mathbf{V}$ is compact and the largest set of $\mathbf{W}$ such that $\mathbf{W} = \mathbb{B}(\mathbf{W})$.

3. $\mathbb{B}(.)$ is monotone and preserves compactness.

4. If we define $\mathbf{W}_{r+1} = \mathbb{B}(\mathbf{W}_r)$ for all $r \geq 0$, and the equilibrium value correspondence is such that $\mathbf{V} \subset \mathbf{W}_0$, then $\lim\limits_{r \to \infty} \mathbf{W}_r = \mathbf{V}$.

Our quantitative results rely on a numerical implementation of the above iterative method and deliver an outer approximation of the equilibrium value correspondence.

To facilitate the computation of $\mathbb{B}(\mathbf{W})$, Phelan and Stacchetti (2001) introduce a public randomization device to convexify the equilibrium set in order to apply the approximation technique developed by Judd et al. (2003). Instead, in line with Feng (2015), we assume that $\mathbf{W}$ is convex-valued at given (k, m) and use the method developed in that paper to approximate equilibrium sets. As argued in Feng (2015), the assumption that $\mathbf{W}$ is convex-valued is weaker than assuming that the value correspondence is convex. This assumption implies continuity in h inside $[\underline{h}, \bar{h}]$. More specifically, for given (k, m), there exist strategies that support any $h \in [\underline{h}, \bar{h}]$, where $\underline{h}$ and $\bar{h}$ are the lower and upper boundaries of $\mathbf{W}(k)$, that is

$$\bar{h}(k, m) := \max_{h} \{h | (m, h) \in \mathbf{W}(k)\}, \tag{14}$$

$$\underline{h}(k, m) := \min_{h} \{h | (m, h) \in \mathbf{W}(k)\}. \tag{15}$$

For a given k, the lowest value in $\mathbf{W}(k)$ yields the worst value for the government $\tilde{h}(k) = \min_m \underline{h}(k, m)$, which corresponds to the Worst Sustainable Equilibrium. Similarly, the highest value $\max_m \bar{h}(k, m)$ in $\mathbf{W}(k)$ corresponds to the Best Sustainable Equilibrium.

We refer to Feng et al. (2014) for details in the approximation of convex-valued sets and for Feng (2015) for an application of the algorithm to a similar game (and for recovering the strategies).

3.3 Quantitative Analysis

3.3.1 Calibration

We start with a benchmark economy: a calibration of an initial steady state that corresponds to an economy with similar policy and statistics to those of the US. This initial steady state provides initial capital and parameters for our quantitative exercise. We also use this benchmark economy as a reference against which to compute the welfare gains/losses of a particular equilibrium.

Our calibration relies substantially on that of Chari et al. (1994). More specifically, parameter values are chosen such that certain moments in the initial steady state allocation are consistent with the U.S. data. We consider the utility function (2) and a Cobb-Douglas production function $y_t = Ak_t^{\alpha} n_t^{1-\alpha}$. In our simulations one period corresponds to one year. We assume a capital share in production of 0.34 and a depreciation rate of 0.08. The discount factor is chosen to obtain a capital to output ratio of 2.71 in the initial steady state. In the utility function the degree of relative risk aversion σ is set equal to unity and the labor-supply elasticity is set so that $\chi = 0.32$.[16] The weight on labor is chosen so that hours worked is 0.23 in the initial steady state, which is in the range found by time allocation studies based on microeconomic evidence [c.f. Juster and Stafford (1991)]. The weight on public consumption γ_g is chosen so that the government spending to output ratio in the social planner's solution coincides with the one of our initial steady state, which is close to 19 per cent. This number represents the non-Social Security government spending in the U.S.. We later perform sensitivity analysis with respect to γ_g to consider higher needs of distortionary taxation. For the initial steady state, the tax rates on capital and labor income are set equal to 27.1 and 23.7 per cent respectively, which are in the range of the average U.S. tax rates estimated by Mendoza et al. (1994). Table 1 shows our calibration targets and Table 2 summarizes the parameter values used in the initial steady state and for our baseline economy.

[Insert Tables 1 and 2 about here.]

Given the above parameter values and the initial condition for the capital stock provided by the initial steady state, we use an algorithm that implements the iterative method described before to solve for the optimal time-consistent fiscal policy subject to different constitutional constraints.[17] In what follows, we present our quantitative results.

3.3.2 A Constant Cap on Capital Taxes

In this Section, we explore a form of constitutional constraint: a constant cap on capital taxes. For our baseline parameters, our results are illustrated in Figures 1, 2, 3a, 3b, 4 and Table 3.

[Insert Figures 1, 2, 3a, 3b, 4 and Table 3 about here.]

Figure 1 presents the set of sustainable equilibria for different constant caps on capital taxes for a given initial capital stock k_0. In particular, we consider $\tau^k \in [0, \tau^{k,UL}]$ and let $\tau^{k,UL}$ vary between 0 and 0.90.[18] We find that all value correspondences are flat at the bottom, with a large number

[16] For the given capital share, this elasticity of labor supply allows us to solve for labor analytically in one of the steps of the computation. This reduces the time required for computation substantially and has allowed us to improve the accuracy in the computation.

[17] We discretize the state space with 400 equally spaced points for $k \in [0.01, 1.5]$, 400 points for m, and 100 points for $\tau \in [0, 0.9]$. We use linear interpolation for variable values falling outside of the grid. We ran our *C++ MPI* code using an IBM iDataPlex cluster, with 50 Intel Sandy Bridge 2.6GHZ processors.

[18] As mentioned, we proxy the absence of constitutional constraints as $\tau^k, \tau^n \in [0.0, 0.90]$ and constant caps on capital taxes as $\tau^k \in T^k = [0, \tau^{k,UL}]$ while $\tau^n \in T^n = [0, 0.90]$.

of marginal values of capital m supporting the WSE, and characterized by an inverted U-shape at the top, with a smaller number of marginal values of capital m sustaining better equilibria and just one m sustaining the BSE.

The largest set of sustainable equilibria corresponds to the uncapped case, which nearly contains the sets for all caps. The imposition of a 0.50 constant cap on capital taxes increases slightly the payoff at the WSE, reduces the payoff at the BSE and eliminates a large set of low marginal values of capital. Relative to the 0.50 cap, the 0.30 cap has a similar effect. As the cap is further lowered, high marginal values of capital are also eliminated and the effect on welfare at the WSE becomes more pronounced. The smallest set is the one with the cap $\tau^{k,UL} = 0.03$. Reducing the cap from 0.03 to 0 expands the set. Then, in general but not in all specific cases, the imposition of caps moves the set of sustainable equilibria towards the top center of the set for the uncapped case.

Figure 2 displays the payoffs provided by the BSE and the WSE for different $\tau^{k,UL}$. The payoff of the BSE increases monotonically with $\tau^{k,UL}$. Thus, the highest payoff at the BSE is provided by the uncapped case and the lowest payoff by the zero cap. Compared to the BSE, a cap on capital taxes has a larger level effect on the WSE. Moreover, the payoff at the WSE displays an inverted U-shape relative to the cap $\tau^{k,UL}$. For our benchmark economy, we find that the payoff at the WSE increases relatively little as the cap is reduced from 0.90 to 0.20, increases sharply as the cap goes from 0.20 to 0.03 and then decreases markedly as the cap is reduced from 0.03 to 0.

Our computation recovers the strategies that can implement the above equilibria. These strategies (allocations and policies) are not unique.[19] Some of those allocations and policies are illustrated in Table 3, Figures 3a and 3b and Figure 4.

Table 3 shows the steady state allocations and policies and the resulting steady state and overall welfare. We find that the BSE does not coincide with the Ramsey equilibrium for high caps, but it does for caps below 0.10. The BSE for all caps delivers welfare gains relative to the initial steady state. For all caps, we find that the BSE displays long-run capital taxes, labor taxes, capital to output ratios and government to output ratios very similar to those in the Ramsey

[19] There are two reasons for that. First, as mentioned before, there is a continuum of marginal values of capital m that can sustain the WSE. Second, for a given m, there are multiple combinations of tax instruments that can deliver the same payoff for the government. This multiplicity is illustrated in Figure 4.

equilibrium (which do not vary with the cap).[20,21] Accordingly, a lower cap has little effect on the long-run welfare but reduces the overall welfare provided by the BSE. We find that the WSE delivers welfare losses relative to the initial steady state and that those losses vary substantially with the cap. Those losses are negligible at $\tau^{k,UL} = 0.03$, and larger as the cap moves away from that level. In the uncapped case, the WSE is very bad and displays long-run capital taxes that are as high as 80 per cent and capital stocks that are 78 per cent lower than in the Ramsey allocation. As the cap is reduced, the long-run burden of taxation shifts from capital to labor income. This reduces the long-run distortions at the WSE. For very low caps, this is illustrated by the relatively high steady-state welfare and by capital to output ratios very similar to those in the Ramsey. For caps between zero per cent and 10 per cent (30 per cent and 90 per cent), the WSE displays some degree of underprovision (overprovision) of government spending.

The paths for the simulated capital stock, capital taxes and labor taxes during the transition for the different caps on capital taxes at the BSE (WSE) are depicted in Figure 3a (3b). As the cap is reduced, the burden of taxation at the WSE is permanently shifted from capital to labor income. However, at the BSE, such a shift is short-lived and within few periods all BSE present a similar level of labor taxation. For the WSE, the cap on capital taxes binds for many periods (even for high caps). This is not the case at the BSE. For the BSE, we see that as the cap decreases, the number of periods in which it binds increases. Therefore, it seems that for low caps, a commitment to a cap is almost identical to a commitment to an actual path of capital taxes.

Figure 3b shows the strategies that supports the WSE payoff for a given m. If we pick a different m within the equilibrium set, the allocations and policies are different. This is illustrated in Figure 4. The same payoff may be sustained with high capital taxes and low labor taxes or with low and volatile capital taxes and high labor taxes. This may explain why a reduction of the cap from 0.90 to 0.20 eliminates more 'good' equilibria than 'bad' equilibria, as very different policies may be able to sustain the payoff at the WSE.

In summary, we find that lowering the constant cap on capital taxes $\tau^{k,UL}$ decreases the BSE

[20] Albanesi and Armenter (2012) provide a sufficient condition for ruling out permanent intertemporal distortions in the second best. Our environment does not satisfy such a condition, but still our third best policy (the BSE with no constitutional constraints) does not feature significant long-run intertemporal distortions.

[21] Stockman (2001) provides a quantitative analysis of Ramsey taxation under balanced-budget rules.

but has a non-monotonic effect on the WSE. For the WSE, the reduction of the cap lowers the long-run distortions at the WSE, but it also reduces the short-run benefits of less distortionary taxation.[22] For large caps, the first effect dominates and the cap is welfare enhancing at the WSE. For low caps, the second effect is larger and a further reduction of the cap is welfare reducing at the WSE. For the BSE, we find that long-run distortions are not present. At the BSE, a cap reduces the short-run benefits of capital taxation and indirectly affects the incentive compatibility constraint (ICC) through the payoff at the WSE. As explained above, the second effect may be reinforcing (if it tightens the ICC) or weakening (if it loosens the ICC). For our baseline parameters, the overall effect is negative and lowering the cap reduces the payoff at the BSE.

Figures 5 and 6 provide sensitivity analysis with respect to the weight on government consumption and the risk aversion parameter.[23] Figure 5 depicts the payoff at the WSE and BSE for different caps for $\gamma_g = 0.49$. This higher weight on government consumption implies larger needs of distortionary taxation.[24] We find that, as before, the BSE increases with the cap and the WSE displays an inverted U-shape relative to the cap. But now the fifteen per cent cap is the one that provides the highest welfare at the WSE. Then, for higher needs of distortionary taxation, the 'best' constant cap on capital taxes at the WSE is higher. Moreover, starting with the uncapped case, declines in the cap have a larger positive effect on the payoff at the WSE.

[Insert Figures 5 and 6.]

Figure 6 shows the payoff at the WSE and at the BSE for different constant caps on capital taxes for a higher risk aversion parameter ($\sigma = 1.50$). Figure 6 shows a pattern very similar to that of Figure 2. The payoff at the BSE increases with the cap. The payoff at the WSE displays an inverted U-shape and 3 per cent is again the cap that provides the highest payoff. The main difference is that, starting with the uncapped case, lowering the cap improves the payoff at the WSE relatively more than under the benchmark parameterization. This may be explained as

[22] We thank a Referee for this insight.

[23] We do not provide sensitivity analysis with respect to the labor elasticity. We have chosen the labor elasticity so that a particular step in the computation can be solved analytically. This has allowed us to increase substantially the accuracy of our approximations. Our intuition is that when labor supply is less elastic, the benefits of raising capital taxes (so that the government can reduce labor taxes and the distortions against labor) would decrease; then, the government's incentives to set high capital taxes in the WSE would be lower. Then, for a less elastic labor supply, we expect that an optimal cap on capital taxes would not affect as much the set of sustainable equilibria.

[24] For this parameterization, the welfare gains of a Ramsey tax reform are larger than for our baseline parameters and equal to 3.4 per cent of permanent increase in initial steady state private consumption.

follows. With a lower elasticity of intertemporal substitution, individuals are less responsive to higher capital taxes and the government at the WSE may face higher incentives to increase capital taxes. There a cap in capital taxes should bind more and in turn have a larger impact on welfare.

In our setup, we have ruled out subsidies. However, Martin (2010) finds that subsidies on labor income are optimal when there is a generous upper bound on capital taxes and non-existent when capital cannot be taxed. Consistent with that, we find that labor taxes are strictly positive at the WSE for caps below 0.50. Looking at Figures 2, 5 and 6, our conjecture is that allowing for subsidies would not affect the best constant cap at the WSE, but it would affect the steepness of both the WSE and the BSE for high caps.

3.3.3 The Natural Upper Limit on Capital Taxation

In the previous Section, we have examined the effects of constant caps on capital taxation. Overall, we have learned that a positive but low cap on capital taxation maximizes the welfare at the WSE. However, such cap has obvious costs at the BSE. In this Section, we explore whether there exists a constitutional constraint on capital taxes that could improve all equilibria. That is, a rule that could improve the WSE without worsening the BSE.

The answer to the above question is yes. For balanced-budget policies, the Ramsey provides us with a natural upper limit on capital taxation. Consider the Ramsey without constitutional constraints, where $\tau^n, \tau^k \in [0, \tau^{max}]$. While for high levels of capital stock, Ramsey capital taxes are high in the short-run. For low levels of capital, Ramsey capital taxes may be moderate and optimally below τ^{max} even in the initial period. Then, let us define the natural upper limit on capital taxation (NULKT) as the highest level of capital taxes for a given level of capital stock that a Ramsey planner would prescribe (starting from any initial conditions).[25] Figure 7 illustrates the NULKT for our baseline parameters and depicts a maximum tax on capital income that is not longer constant but increases with the level of capital.

[Insert Figure 7 about here.]

Consider a situation in which the BSE coincides with the Ramsey. That is, the worst is so bad

[25] In our numerical exercise, the highest optimal level of capital taxes for a given level of capital stock coincides with the Ramsey capital tax in the initial period for that level of capital stock as initial condition.

that the Ramsey can be sustained as the best. Let us now impose the NULKT as the constitutional constraint. First, the NULKT is likely to improve the WSE, as it rules out worst expectations that induce high capital taxes when the stock of capital is low. Second, the NULKT is likely not to affect the feasibility of the BSE, as the NULKT does not bind at the Ramsey. However, as the welfare of the WSE increases, the incentive compatibility constraint (13) tightens and it may decrease the BSE (if it binds) or may not affect the BSE (if it does not bind). Therefore, the NULKT has the potential of improving all equilibria.

For our baseline parameters and without caps, the BSE does not coincide with the Ramsey. Nevertheless, Figure 8 shows that the NULKT improves dramatically the payoff at the WSE, but has a very small negative effect on the payoff at the BSE. This is illustrated in Table 4 and Figures 9a and 9b. Table 4 shows that the BSE displays no long-run distortions and provides a welfare that is close to the uncapped case (displayed in Table 3). This Table also shows that long-run taxes and capital to output ratios are larger at the WSE with the NULKT. Figures 9a and 9b present the simulated stock of capital, capital tax rates and labor tax rates during the transition for the BSE and the WSE for the uncapped case and when the NULKT is imposed. Both environments deliver very similar transitions at the BSE, but very different ones at the WSE. At the WSE, the NULKT displays lower levels of capital taxes and induces a larger accumulation of capital.

[Insert Table 4 and Figures 8, 9a and 9b about here.]

Next, we consider a larger weight on public consumption ($\gamma_g = 0.49$). Figures 10 and 11 respectively depict the NULKT and the value correspondences without caps and with the NULKT. Comparing Figure 10 to Figure 7, the larger weight on public consumption induces higher incentives to tax capital for each level of capital. For this parameterization, the BSE coincides with the Ramsey, and Figure 11 shows that the NULKT improves the WSE without worsening the BSE. This constitutional constraint proves optimal as it eliminates only 'bad' equilibria.[26]

[Insert Figures 10 and 11 about here.]

[26]In this paper, we have not computed the expected social welfare of the implementation of a constitutional constraint. We do not do so because it is unclear how to assign probabilities to each of the multiple sustainable equilibria. However, if all of the continuum of equilibria were assumed to be equiprobable, then we could certainly claim that this optimal constitutional constraint increases expected social welfare, as it benefits the entire set of sustainable equilibria.

4 Conclusions

We have investigated the desirability of constitutional constraints on capital taxation in economies where governments lack full commitment and have no access to government debt.

We have studied constant caps on capital tax rates. In our quantitative exercise, we found that the welfare provided by the worst sustainable equilibrium displays an inverted U-shape relative to the cap and that a three per cent cap on capital taxes provides the highest welfare at the worst. However, such cap decreases welfare at the best sustainable equilibrium.

We have identified a form of constitutional constraint that could benefit all equilibria. We have found that the maximal tax rate prescribed by the Ramsey planner for each level of capital provides a natural upper limit on capital taxation. Such natural upper limit on capital taxation (which increases with the level of capital) can improve the worst without worsening the best.

There are several interesting extension to our analysis. One important extension would be to allow for government debt. As shown by Domínguez (2007) and Reis (2013), this assumption can affect the properties of capital taxation without commitment. Another interesting extension would be to consider environments where capital (or capital income) can contemporaneously respond to changes in capital taxes. Two examples are the following. Conesa and Domínguez (2013) consider an economy where firms can react to current capital taxes through intangible investment (which can be expensed or sweat). Gervais and Mennunni (2015) allow for investment to become productive within the period. In both frameworks, the incentives to tax capital heavily in the short run are very limited and upper bounds on capital taxes never bind. The properties and the effects of optimal constitutional constraints on capital taxation could be quite different in those environments.

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5 Appendix

5.1 Proof of Proposition 1

The main difference between our environment and those of Phelan and Stacchetti (2001) and Feng (2015) is that our utility function (2) does not satisfy $\lim_{n \to 1} u(., n) = -\infty$. However, it is easy to see that as long as labor is bounded above, the results developed in Lemmas 1-3 of Phelan and Stacchetti (2001) and extended in Feng (2015) also hold in our environment.

Let's proceed by contradiction and suppose there could be a competitive equilibrium for which labor n tends to infinity. Then there are three possibilities: $\frac{k}{n}$ could approach (i) zero, (ii) a constant value, or (iii) infinity. First, it is easy to see that (i) would violate optimality. For the given utility and production functions, the consumption-leisure decision (CE-2) can be written as $\gamma_n n^{\chi} c^{\sigma} = (1-\tau^n)(1-\alpha)A\left(\frac{k}{n}\right)^{\alpha}$. As $\frac{k}{n}$ approaches zero, this equation would require consumption c to go to zero. Then, given the Inada conditions, the individual would be better off by working less. Second, one can check that (ii) can never occur. As $\frac{k}{n}$ approaches a constant value, condition (CE-2) requires $n^{\chi} c^{\sigma}$ to approach a constant value. However, from (CE-5)-(CE-6) a constant $\frac{k}{n}$ implies constant w and r. Moreover, dividing the government budget constraint (CE-4) by labor, a constant $\frac{k}{n}$ implies a constant $\frac{g}{n}$. Likewise, dividing the household's budget constraint (CE-3) by labor, constant $\frac{k}{n}$, w, r and $\frac{g}{n}$ require a constant $\frac{c}{n}$. This last would imply that consumption also tends to infinity, which contradicts $n^{\chi} c^{\sigma}$ approaching a constant value. Third, let's consider (iii), where $\frac{k}{n}$ approaches infinity. For our production function, this implies that the returns to capital approach zero. In the spirit of Phelan and Stacchetti (2001)'s Lemma 1, as the returns to capital are very low, a household would prefer to save a little bit less and that would further increase consumption. Then, for all possibilities, we reach a contradiction. Therefore, labor is bounded above $n \leq \bar{n}$.

For $n \leq \bar{n}$, one can define an upper bound for the capital stock as $\bar{k} = f(\bar{k}, \bar{n}) + (1-\delta)\bar{k}$. Then it is straight-forward to reproduce Lemmas 1-3 of Phelan and Stacchetti (2001) and extended in Feng (2015). ■

5.2 Tables and Figures

Table 1: Targets in the Initial Steady State

β	Target: $\frac{k}{y} = 2.71$
γ_n	Target: $\bar{n} = 0.23$
γ_g	Target: $\frac{g_{planner}}{y_{planner}} = \frac{\bar{g}}{y}$

Table 2: Parameter Values for the Baseline Economy

Preference	$\beta = 0.968$	$\sigma = 1.0$	$\gamma_n = 7.694$	$\chi = 0.32$	$\gamma_g = 0.333$
Technology	$A = 1.0$	$\alpha = 0.34$	$\delta = 0.08$		
Policy	$\tau_0^n = 0.237$	$\tau_0^k = 0.271$			

Table 3: Welfare Gains and Final Steady State Allocation and Policy for Different Constant Caps on Capital Taxes

	Uncapped			0.50 cap			0.30 cap			0.10 Cap			0.03 Cap			Zero Cap		
	R	BSE	WSE	R	BSE	WSE	R	BSE	WSE	R	BSE	WSE	R	BSE	WSE	R	BSE	WSE
Δh_0	0.80	0.78	-4.32	0.62	0.48	-4.13	0.48	0.38	-3.94	0.28	0.28	-2.88	0.18	0.18	-0.02	0.15	0.15	-1.02
$\Delta \bar{h}$	2.02	2.02	-22.1	2.02	1.45	-7.23	2.02	1.69	-5.97	2.02	1.91	-4.03	2.02	1.80	1.75	2.02	1.90	0.58
$\bar{\tau}^k$	0.00	0.01	0.80	0.00	0.02	0.45	0.00	0.02	0.14	0.00	0.02	0.10	0.00	0.02	0.03	0.00	0.00	0.00
$\bar{\tau}^n$	0.29	0.28	0.04	0.29	0.32	0.11	0.29	0.29	0.12	0.29	0.28	0.20	0.29	0.28	0.25	0.29	0.29	0.27
$\bar{k}$	1.19	1.19	0.42	1.19	1.17	0.86	1.19	1.19	0.89	1.19	1.19	0.89	1.19	1.18	1.18	1.19	1.19	1.17
$\frac{\bar{k}}{y}$	3.00	2.99	1.42	3.00	3.09	2.43	3.00	2.99	2.42	3.00	2.99	2.30	3.00	2.98	2.98	3.00	3.00	2.97
$\frac{\bar{g}}{y}$	0.19	0.19	0.21	0.19	0.19	0.21	0.19	0.19	0.24	0.19	0.19	0.14	0.19	0.19	0.17	0.19	0.19	0.18

Table 4: Welfare Gains and Final Steady State Allocation and Policy for the NULKT

	NULKT		
	R	BSE	WSE
Δh_0	0.80	0.68	-1.21
$\Delta \bar{h}$	2.02	1.90	-3.12
$\bar{\tau}^k$	0.00	0.00	0.68
$\bar{\tau}^n$	0.29	0.29	0.09
$\bar{k}$	1.19	1.19	0.62
$\frac{\bar{k}}{y}$	3.00	3.00	1.86
$\frac{\bar{g}}{y}$	0.19	0.19	0.19

Note: R stands for Ramsey. Δh_0 ($\Delta \bar{h}$) represents overall (steady state) welfare gains measured in terms of % change in initial steady state private consumption.

Figure 1: The Set of Sustainable Equilibria for Different Constant Caps on Capital Taxes

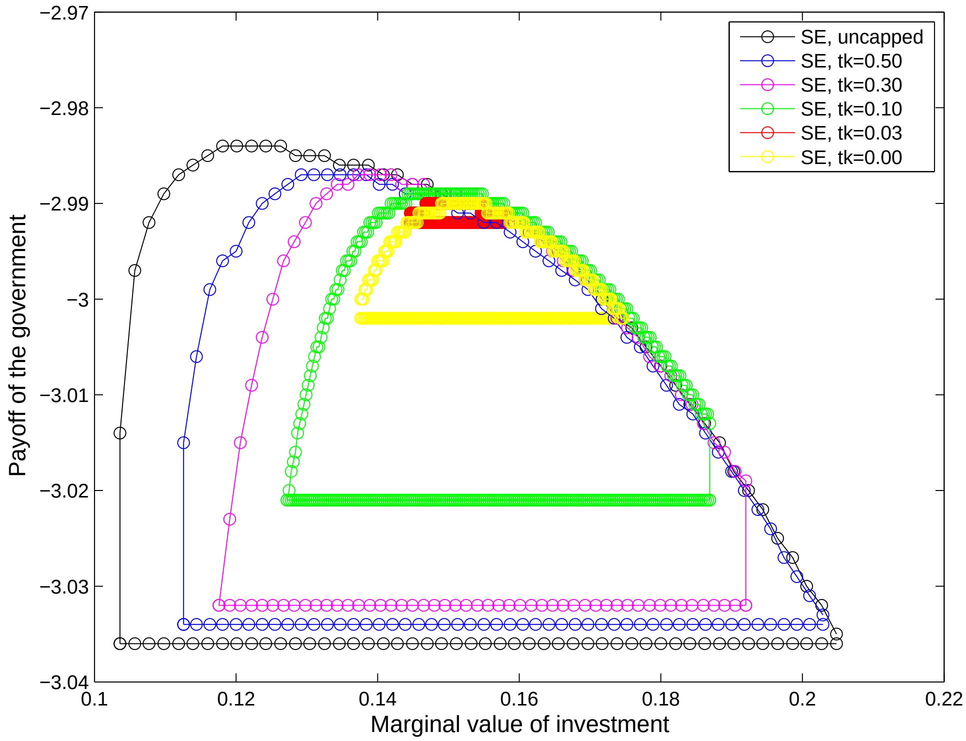


Figure 2: Welfare at the WSE and BSE for Different Constant Caps on Capital Tax Rates

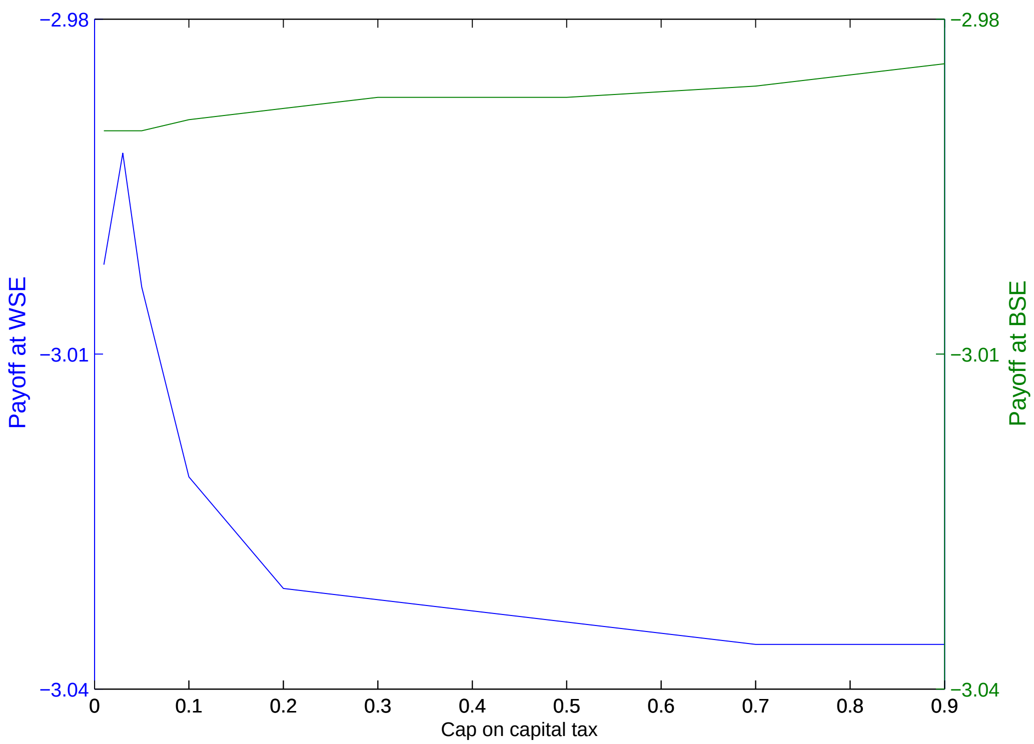
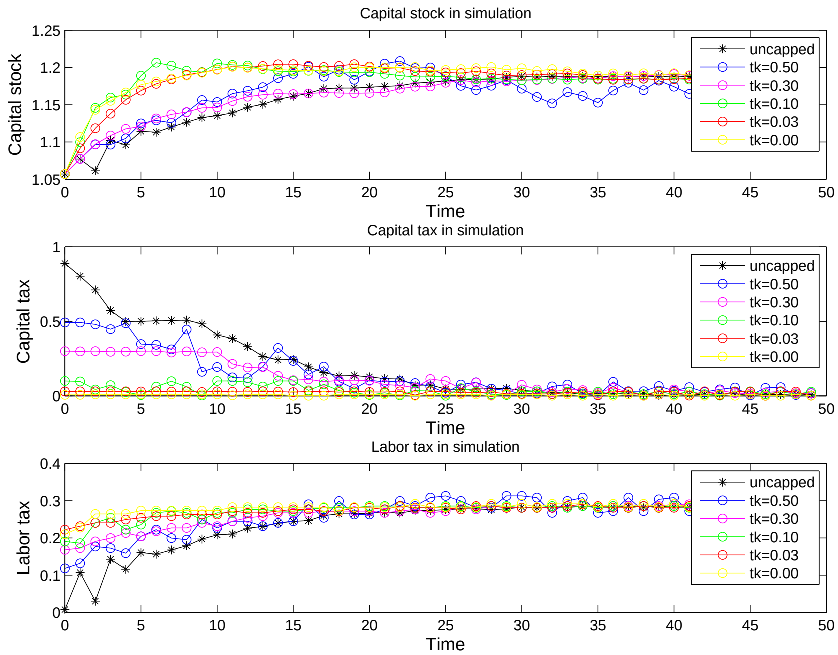


Figure 3: Capital Stock, Capital Taxes and Labor Taxes during the Transition for Different Caps on Capital Taxes

(a) BSE



(b) WSE

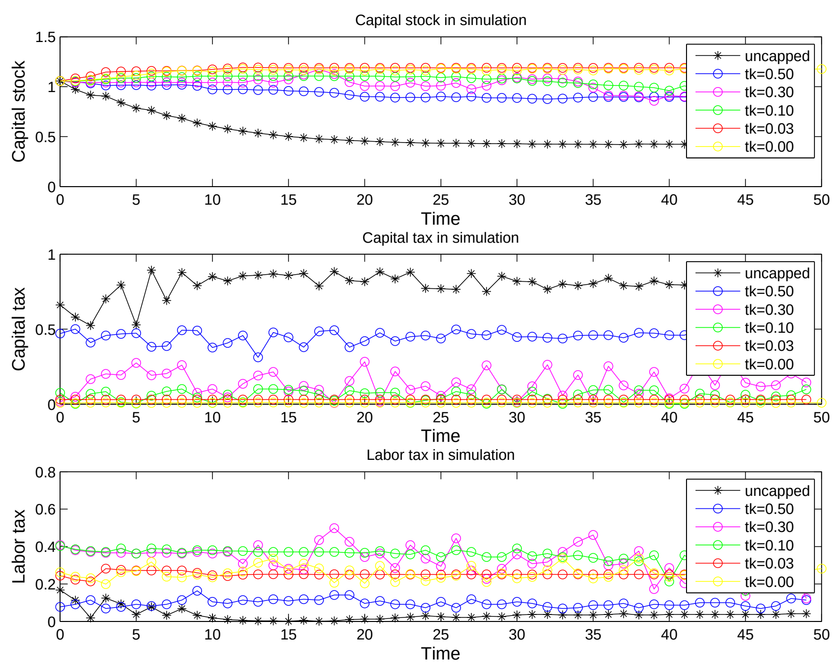


Figure 4: Simulated Paths for Different m for the WSE with $\tau^{k,UL} = 0.50$

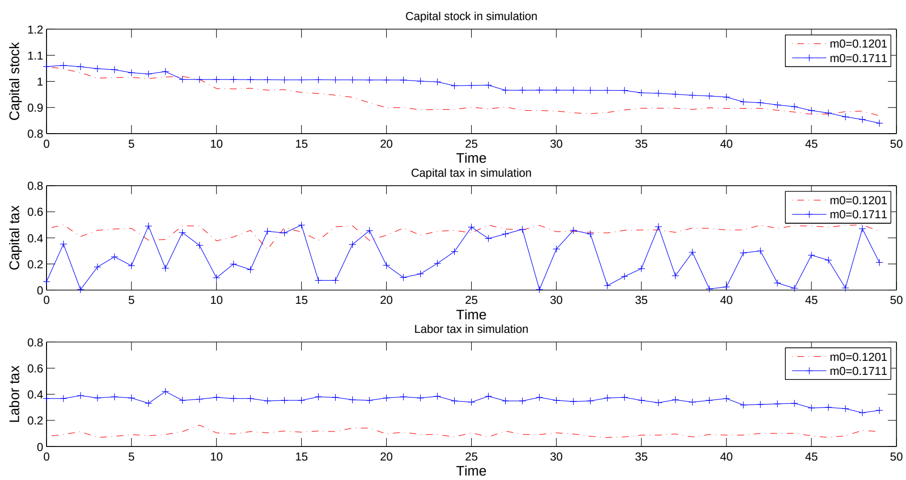


Figure 5: Welfare at the WSE and BSE for Different $\tau^{k,UL}$, $\gamma_g = 0.49$

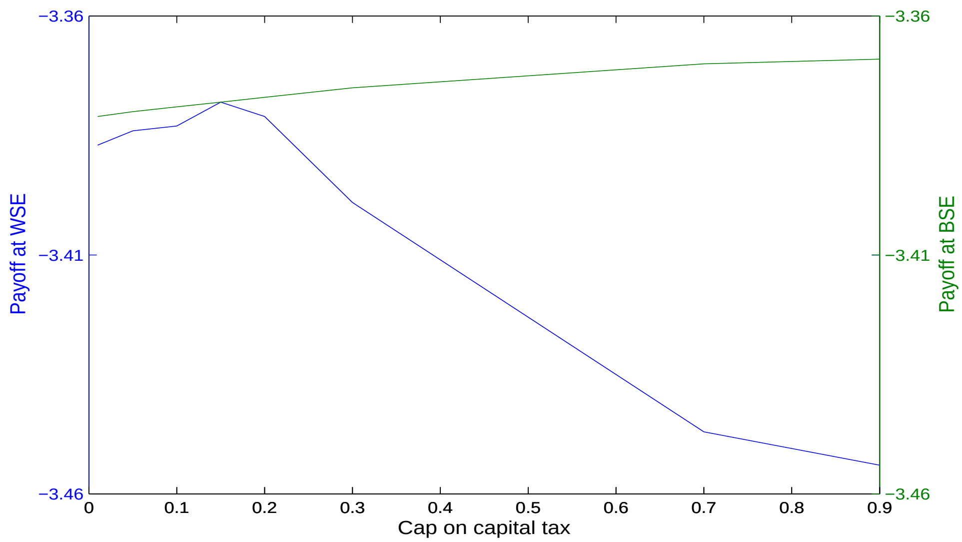


Figure 6: Welfare at the WSE and BSE for Different $\tau^{k,UL}$, $\sigma = 1.50$

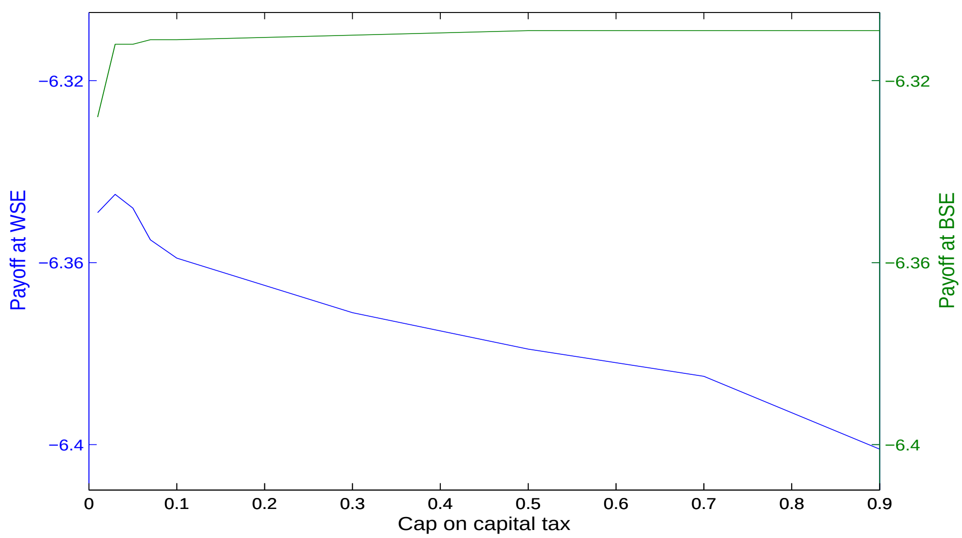


Figure 7: The Natural Upper Limit on Capital Tax Rates

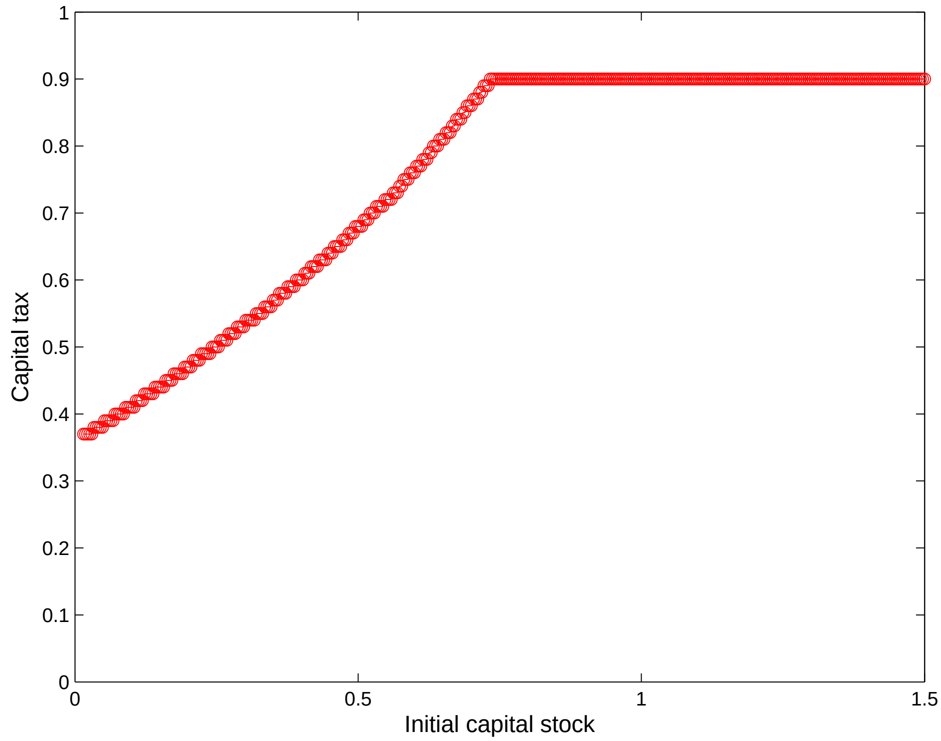


Figure 8: The Set of Sustainable Equilibria: Uncapped vs the NULKT

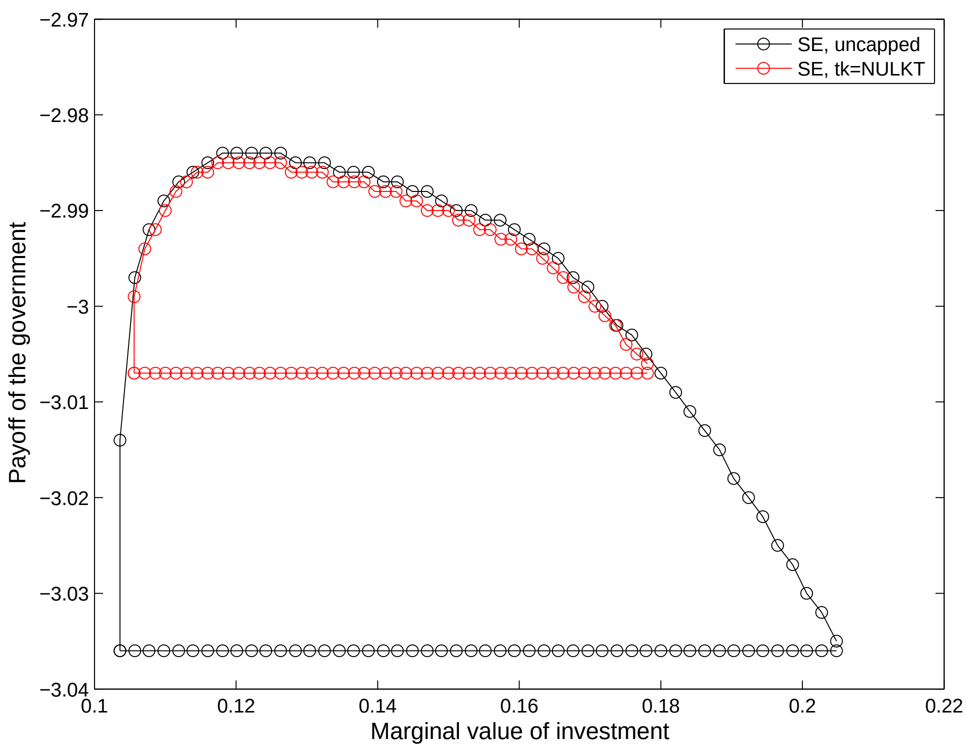
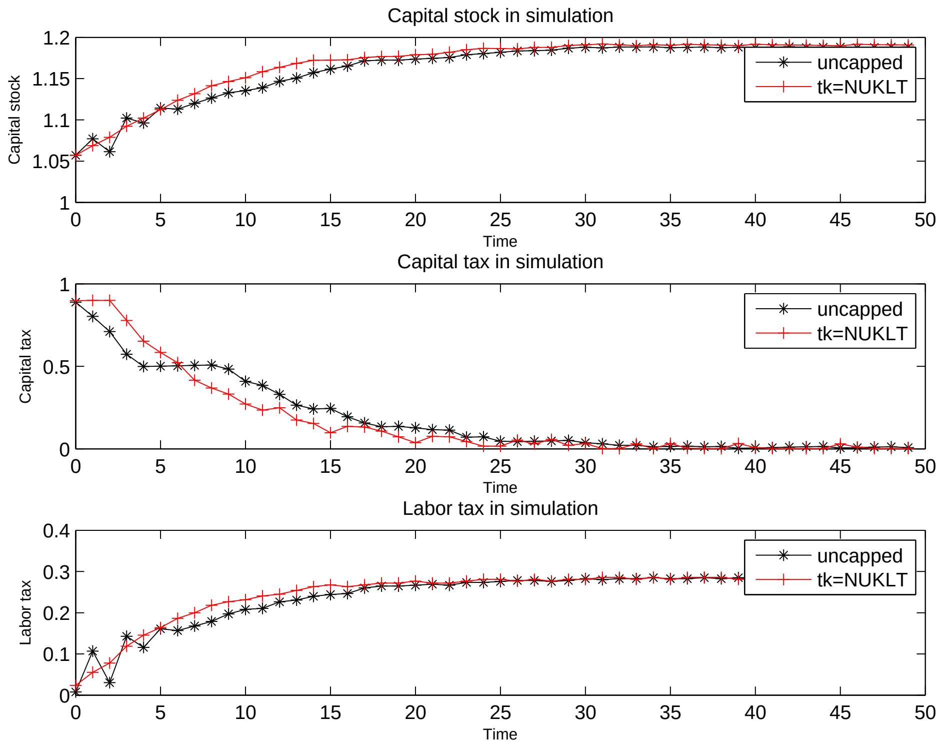


Figure 9: Capital Stock, Capital Taxes and Labor Taxes during the Transition: Uncapped vs. the NULKT

(a) BSE



(b) WSE

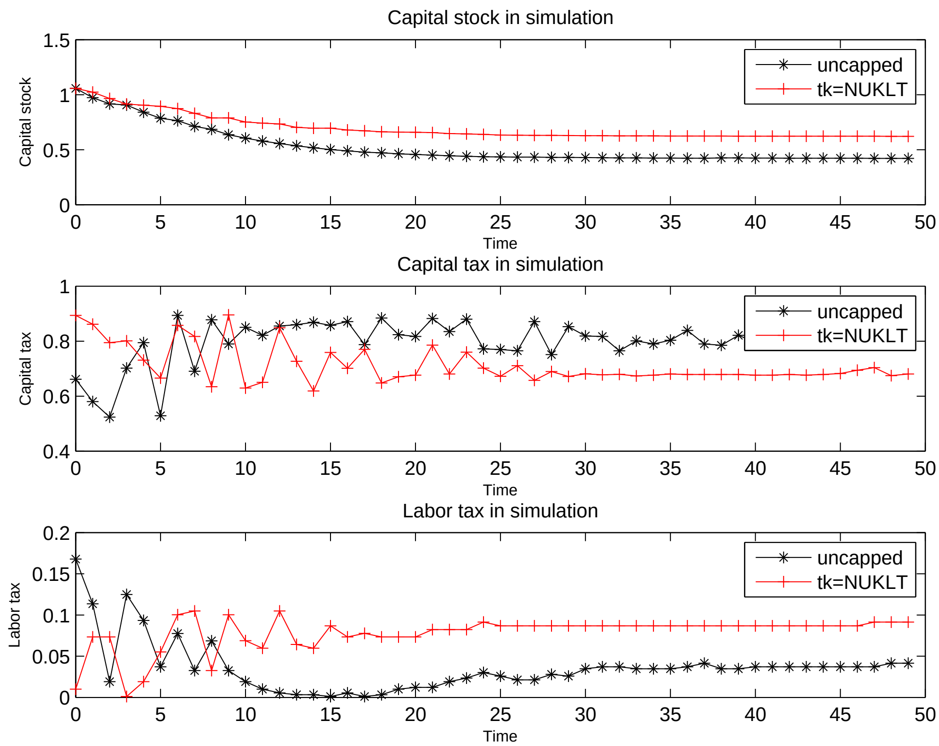


Figure 10: The Natural Upper Limit on Capital Tax Rates, $\gamma_g = 0.49$

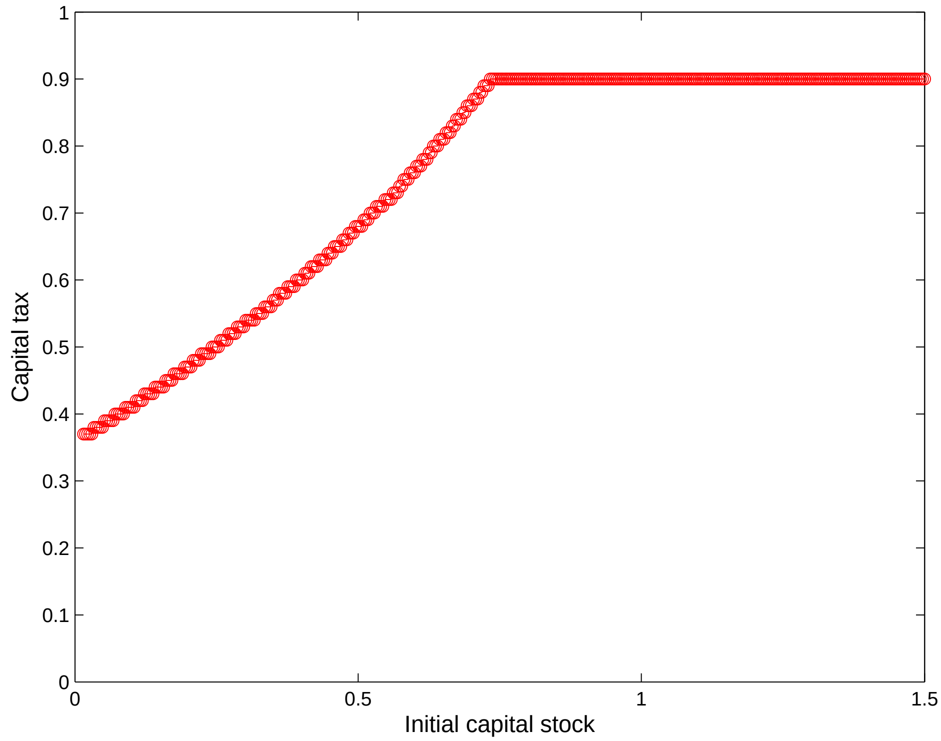


Figure 11: The Set of Sustainable Equilibria: Uncapped vs the NULKT, $\gamma_g = 0.49$