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Abstract

This paper investigates the time-inconsistency problem of labor taxes in an economy with balanced-budget policies and no capital taxes. With full commitment, we show that Ramsey labor taxes change with the cost of distortionary taxation and with the cost of not being able to tax capital. We numerically show that these make labor taxes increasing over time. With limited commitment, we find that this time-inconsistency problem leads to underprovision of public consumption. For our baseline parameter values, we find that imposing carefully chosen bounds on labor taxes as constitutional constraints can be optimal. While our proposed bounds sustain the Ramsey as the best sustainable equilibrium, our lower bounds alone or, in combination with some upper bounds, induce higher public consumption and higher welfare in the worst sustainable equilibrium.

JEL Codes: E61, E62, H21, H30.

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1 Introduction

Following the rules versus discretion debate pioneered by Kydland and Prescott (1997), many economists have suggested not to tax capital at all in order to alleviate the time-inconsistency problem of capital taxation.[1] If capital income were not taxed, what would be the properties of optimal labor taxes and of the associated time-inconsistency problem? Would there be any additional constitutional constraints that could further alleviate that problem?

This paper studies a dynamic policy game between governments and households. This paper contributes to this literature by providing an application of already existing theoretical and numerical methods to the study of the time-inconsistency problem of labor taxes and how this problem is affected by constitutional constraints. As suggested in Rogoff (1987), changes in the constitutional constraints can be interpreted as changes in the structure of the policy game. In the paper, we evaluate the effect of different constitutional constraints on the outcomes and welfare that can be sustained in equilibrium.

The environment in this paper is very close to that of Phelan and Stacchetti (2001). We consider a benevolent government that, in order to finance an endogenous public consumption, chooses the labor income tax rate that maximizes welfare. We focus on balanced-budget policies with no capital taxes. First, we study this optimal policy problem with full commitment, that is, under the assumption that in all future periods, the government is committed to the sequence of taxes chosen at the initial date. Analytically, we show that optimal labor taxes change with the cost of distortionary taxation and with the cost of not being able to tax capital. Numerically, we find that these factors make optimal labor taxes increasing over time for all initial levels of capital.[2] Thus, governments find optimal to set current labor taxes low and future labor taxes high. However, as the future becomes present, the temptation is to lower labor taxes again.

Second, we study the optimal policy problem with limited commitment, that is, under the assumption that in all future periods, the government is free to choose the labor tax rate but is

[1]For example, Lucas (1990) and, more recently, Mankiw et al. (2009).

[2]The finding that optimal labor taxes are increasing over time when capital cannot be taxed is connected to the comments of Atkenson et al. (1999). They observe that, a positive tax on capital is comparable to an increasing tax rate on consumption and the latter corresponds to an increasing tax rate on labor income. These policies are comparable but not equivalent. In our economy, we don't count with consumption taxes and not having capital taxes eliminates an instrument that is necessary for decentralization and therefore changes the Ramsey problem.

committed to comply with some exogenously imposed constitutional constraints. As in Domínguez and Feng (2014) for capital taxes, we evaluate the Ramsey plans starting from any initial conditions and propose the highest and lowest Ramsey labor taxes for each level of capital as the natural upper and lower bounds on labor taxes. Then we quantify the effects of these natural bounds (as exogenously imposed constitutional constraints) on the whole set of sustainable equilibria.

Our findings are as follows. Our proposed natural bounds on labor taxes rates have no effect, or minimal effect, on the Best Sustainable Equilibrium (BSE). For our baseline parameterization, the Ramsey can be sustained as the best and this is not changed (or just slightly) by these bounds. However, our proposed natural bounds affect the Worst Sustainable Equilibrium (WSE). The upper bounds reduce the welfare provided by the WSE as they rule out optimal reaction choices by the government when facing worst expectations by the public. The lower bounds, on the other hand, increase the welfare of the WSE, as they prevent worst expectations that lead to underprovision of public consumption. Interestingly, considering both upper and lower bounds together, leads to WSE with even higher welfare. We suggest that tighter bounds induce smooth higher taxes in the worst that lead to higher and smooth public consumption and higher welfare gains.

This paper builds on the literature on optimal taxation.[3] This literature finds that labor taxes should be roughly constant.[4] In contrast, in our setup, we find that optimal labor tax rates are increasing over time during the transition. Thus, we argue that the time-inconsistency problem is now relocated towards labor taxes. This result is related to those of Correia (1996) and Rogoff (1985). Correia (1996) studies capital taxation when another factor of production cannot be taxed and finds that the optimal steady state capital tax is not longer zero. In our model, the constraints that capital cannot be taxed and bonds cannot be issued change the properties of optimal labor taxes. A similar result is also found in McCallum (1995)'s analysis of delegation in monetary policy as in Rogoff (1985). Delegation relocates the time-inconsistency problem to that of the choice of the conservative central banker.

This paper provides an application of the recent developments in the literature on time-

[3] See Lucas and Stokey (1983), Chari and Kehoe (1998), Chari et al. (1994), Stockman (2001), etc.

[4] More precisely, in Ramsey problems with full-commitment, access to bonds, capital taxes, and with homothetic preferences, optimal labor taxes are typically low (even negative) in the initial period and then positive and roughly constant from then on. In contrast, in our paper, labor taxes are increasing over the transitional phase.

inconsistency of optimal plans, as originated by Kydland and Prescott (1977). This type of problems were formalized as a policy game by Chari and Kehoe (1990) and extended to dynamic games with natural state variables by Phelan and Stacchetti (2001) and Sleet (1997).[5] Judd et al. (2003) and, more recently, Feng et al. (2014) and Sleet and Yeltekin (2015) provide algorithms to numerically approximate the equilibrium value correspondence in dynamic games.

The rest of the paper follows the following structure. Section 2 describes the economy. Section 3 characterizes optimal labor taxes with full commitment. Section 4 discusses the solution method and characterizes optimal labor taxes with limited commitment. Section 5 concludes. Details of the calibration, Tables and Figures can be found in the Appendix.

2 The Economy

The economy is populated by a continuum of identical households, a continuum of identical firms, and a benevolent government. Time is infinite and indexed by subscript $t = 0, 1, 2, ...$

The representative household lives forever and values private consumption c_t, labor n_t and public consumption g_t over its life-time, and according to

$$(1-\beta)\sum_{t=0}^{\infty}\beta^t u(c_t, n_t, g_t), \tag{1}$$

with the discount factor $\beta \in (0,1)$, and the (normalized by $(1-\beta)$) instantaneous utility

$$u(c_t, n_t, g_t) = \frac{{c_t}^{1-\sigma}}{1-\sigma} - \gamma_n \frac{{n_t}^{1+\chi}}{1+\chi} + \gamma_g \frac{{g_t}^{1-\sigma}}{1-\sigma}, \tag{2}$$

where $\sigma \geq 0$ and $\chi \geq 0$ are the inverses of the elasticity of intertemporal substitution of consumption and of labor respectively, and $\gamma_n \geq 0$ and $\gamma_g \geq 0$ are the weights on labor disutility and on public consumption utility respectively.

In period 0, each household owns the same initial level of capital k_0. In each period, households receive capital income (the principal plus a return r_t net of the depreciation rate δ on their capital holdings, $(1 + r_t - \delta)\, k_t$) and wage income (the real wage w_t net of taxes τ_t on their supplied labor,

[5] An alternative approach is to consider Markov-perfect equilibria as in Klein et al. (2008).

$(1-\tau_t)\, w_t n_t)$. This income is used to consume in that period and accumulate capital for the next period. Then the budget constraint of the representative household is

$$k_{t+1} + c_t = (1 + r_t - \delta)\, k_t + (1-\tau_t)\, w_t n_t. \tag{3}$$

The representative household chooses $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize his welfare (1) subject to the budget constraint (3), and the no-Ponzi-game (NPG) condition $\lim_{t\to\infty} p_t k_{t+1} \geq 0$, where p_t is the multiplier on (3). The resulting first-order conditions are

$$-u_{2,t} = (1-\tau_t) w_t u_{1,t}, \tag{4}$$

$$u_{1,t} = \beta\, (1 + r_{t+1} - \delta)\, u_{1,t+1}, \tag{5}$$

and the transversality (TV) condition $\lim_{t\to\infty} \beta^t u_{1,t} k_{t+1} = 0$, where u_i denotes the partial derivative of u with respect to its ith argument. Other derivatives follow a similar notation.

The representative firm produces the final good using the technology $y_t = f(k_t, n_t) = k_t^{\alpha} n_t^{1-\alpha}$, with $\alpha \in (0,1)$. We assume firms operate in perfectly competitive markets, then r_t and w_t equal the marginal products of capital and labor respectively

$$r_t = f_1(k_t, n_t) \text{ and } w_t = f_2(k_t, n_t). \tag{6}$$

In each period, the benevolent government finances an endogenous public consumption g_t with taxes on labor income

$$g_t = \tau_t w_t n_t. \tag{7}$$

In the absence of constitutional constraints on labor taxes, the tax rate τ_t can take any value in the interval $[0, \tau^{max}]$, with $0 < \tau^{max} \leq 1$.[6] Then, a constitutional constraint on labor taxes takes the form of lower and/or upper limits on the labor tax rates, i.e. $\tau_t \in T = \left\{[\tau^{LB}, \tau^{UB}] | \tau^{LB} \leq \tau_t \leq \tau^{UB}\right\}$.

[6] In this model, with only one source of government income, the non-negativity of public consumption requires a non-negative labor tax. Moreover, Laffer curve effects generate a 'natural' upper bound on labor tax rates.

The resource constraint of the economy is

$$f(k_t, n_t) + (1-\delta)k_t = c_t + k_{t+1} + g_t. \tag{8}$$

A competitive equilibrium is defined as follows:

Definition 1 *Given the tax rates $\{\tau_t\}_{t=0}^{\infty}$, and initial capital k_0, a competitive equilibrium allocation $\{c_t, n_t, k_{t+1}, g_t\}_{t=0}^{\infty}$ and prices $\{p_t, r_t, w_t\}_{t=0}^{\infty}$ are such that: (i) given prices, tax rates and k_0, the representative household maximizes welfare (1) subject to the budget constraint (3) and the NPG condition; (ii) factors are paid their marginal products (6); (iii) the government budget constraint (7) is satisfied; and (iv) all markets clear.*

3 Optimal Labor Taxes with Full Commitment

In this Section we study optimal labor taxes with full commitment. We present the Ramsey problem, solve it and characterize the optimal taxes with full commitment analitically and numerically.

We follow the primal approach. We substitute the first-order conditions (4)-(6) into the budget constraint (3) to obtain for each period the implementability condition

$$u_{1,t}c_t + u_{2,t}n_t + u_{1,t}k_{t+1} = \frac{1}{\beta}u_{1,t-1}k_t, \tag{9}$$

whose right-hand side is replaced by $u_{1,0}\left(1 + f_1(k_0, n_0) - \delta\right)k_0$ in period 0. In addition, as capital cannot be taxed, governments lack an instrument to decentralize the Euler condition (5). Then, in each period $t \geq 1$, governments must satisfy the zero capital tax constraint

$$u_{1,t-1} = \beta(1 + f_1(k_t, n_t) - \delta)u_{1,t}. \tag{10}$$

Furthermore, any bounds on labor tax rates $\tau_t \in T$ are assumed not to bind with full commitment.

The government's optimization problem is defined as follows. The government at date 0 chooses the sequences $\{c_t, n_t, g_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize the welfare of the representative household (1) subject to the resource constraint (8), the implementability condition (9), and the zero capital tax constraint (10), given the initial condition k_0, and the TV condition. All future governments are committed to follow the sequence of taxes chosen by the government at date 0.

The Lagragian for this optimization problem is

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t \{u(c_t, n_t, g_t) + \lambda_t [u_{1,t}c_t + u_{2,t}n_t] + (\lambda_t - \lambda_{t+1}) u_{1,t}k_{t+1} \\
&+ \mu_t [f(k_t, n_t) + (1-\delta)k_t - c_t - k_{t+1} - g_t] \\
&+ \theta_{t+1} [u_{1,t} - \beta(1 + f_1(k_{t+1}, n_{t+1}) - \delta)u_{1,t+1}]\} - \lambda_0 u_{1,0}[1 + f_1(k_0, n_0) - \delta]k_0. \qquad (11)
\end{aligned}$$

Note that $\lambda_{-1} = 0$ and $\theta_0 = 0$. The solution to this problem satisfies constraints (8)-(10), and the next first order conditions for consumption, labor, public consumption and capital for all $t \geq 1$:[7]

$$c_t^{-\sigma} \left\{ 1 + \lambda_t(1-\sigma) - \sigma (\lambda_t - \lambda_{t+1}) \frac{k_{t+1}}{c_t} + \sigma \left(\frac{\theta_t}{c_t}(1 + f_{1,t} - \delta) - \frac{\theta_{t+1}}{c_t} \right) \right\} = \mu_t, \qquad (12)$$

$$\gamma_n n_t^{\chi} (1 + \lambda_t(1+\chi)) + c_t^{-\sigma}\theta_t f_{12,t} = f_{2,t}\mu_t, \qquad (13)$$

$$\gamma_g g_t^{-\sigma} = \mu_t, \qquad (14)$$

$$\mu_t = \beta\mu_{t+1} (1 + f_{1,t+1} - \delta) + c_{t+1}^{-\sigma} (\lambda_t - \lambda_{t+1}) + \beta c_{t+1}^{-\sigma}\theta_{t+1} f_{22,t}, \qquad (15)$$

where μ_t, λ_t, and θ_t are the Lagrange multipliers on (8), (9), and (10), respectively.[8]

Let's define $\Psi_t \equiv \sigma \left(\frac{\theta_t}{c_t}(1 + f_{1,t} - \delta) - \frac{\theta_{t+1}}{c_t} \right) - \theta_t \frac{f_{12,t}}{f_{2,t}}$, and $\Gamma_t \equiv \sigma \frac{k_{t+1}}{c_t}$, combining (12)-(13) together with (4) and (6), we find:

Proposition 1 *The optimal labor tax rates with full commitment for all periods $t \geq 1$ are*

$$\tau_t = \frac{\lambda_t(\sigma + \chi)}{1 + \lambda_t(1+\chi)} + \frac{(\lambda_t - \lambda_{t+1})\Gamma_t}{1 + \lambda_t(1+\chi)} - \frac{\Psi_t}{1 + \lambda_t(1+\chi)}. \qquad (16)$$

Proof. See the Appendix. ■

[7]The first order conditions in period 0 are different due to the initial wealth.
[8]As usual, an optimal interior solution is assumed exist.

Proposition 1 characterizes optimal labor taxes with full commitment. There we see that Ramsey labor taxes move with the multiplier λ_t and the term Ψ_t. The multiplier λ_t on the implementability condition (9) measures the distortionary cost of taxation. If governments could issue bonds, there would be one life-time implementability condition and the multiplier λ_t would be constant. However, as they cannot, there is one implementability condition per period and the multiplier changes over time. The term Ψ_t changes with the multiplier θ_t, which measures the cost of not being able to tax capital. If governments could tax capital, θ_t and Ψ_t would be zero. Then, for our homothetic utility function (2), when governments can issue bonds and tax capital, optimal labor taxes with full commitment are constant over time for all $t \geq 1$. However, when they cannot issue bonds and cannot tax capital, what is the pattern of optimal labor tax rates?

For the calibration detailed in the Appendix, Figure 1 displays the optimal labor taxes with full commitment and other terms for different initial levels of capital. We see that Ramsey labor taxes are increasing over time towards the optimal steady state level for all initial levels of capital. The multiplier λ_t is increasing (decreasing) over time for low (high) levels of initial capital. This suggests that, with endogenous public consumption and no bonds, a Ramsey planner shifts the distortionary cost of taxation towards periods with a higher capital stock. The multiplier θ_t is positive and converges to zero, which is consistent with the optimality of no permanent intertemporal distortions (see Albanesi and Armenter (2012)). For our specification, $\Psi_t = \left[(1-\alpha)(1-\delta) + \alpha \frac{k_{t+1}}{k_t} - \frac{\theta_{t+1}}{\theta_t}\right] \frac{\theta_t}{c_t}$. During the transition, Ψ_t is negative (positive) whenever the ratio $\frac{\theta_{t+1}}{\theta_t}$ is sufficiently large (small). As θ_t increases with capital, this happens for low (high) levels of initial capital.

Plotting the first two terms in (16), we find that without the third term, Ramsey taxes would follow the pattern of the cost of distortionary taxation and would be increasing (decreasing) over time for low (high) levels of initial capital. With the third term, the pattern changes. At low levels of initial capital, the multiplier λ_t is increasing but the term Ψ_t is negative, overall the first effect dominates and labor taxes are increasing. At high levels of capital, the multiplier λ_t is decreasing but the term Ψ_t is large and positive, overall the second effect dominates and labor taxes are again increasing. Then, for all initial levels of capital, Ramsey labor taxes are increasing over time.

[Insert Figure 1 about here]

This pattern for Ramsey labor taxes has the following implications for time-inconsistency. The benevolent government lowers labor taxes today and promises higher labor taxes for the future. However, as the future becomes present, the government is tempted to lower labor taxes again. This form of time-inconsistency problem in labor taxes seems quite different from the one in capital taxes (where governments increase capital taxes today and promise zero capital taxes for the future). However, the temptation behind both time-inconsistency problems is in essence the same. Looking at the implementability condition (9), a Ramsey planner would like to reduce the RHS of that constraint (and make the overall cost of distortionary taxation lower) by increasing capital taxes. In the absence of a capital tax, a Ramsey planner can reduce the RHS by inducing a boom in consumption that depresses the marginal utility of consumption. Such a boom in consumption can be achieved by lowering labor taxes today and increasing labor taxes in the future.

4 Optimal Labor Taxes with Limited Commitment

This Section departs from the assumption of full commitment. Here we consider limited commitment in the sense that future governments can reconsider the labor taxes but they commit to comply with the constitutional constraints.

This Section follows closely Domínguez and Feng (2014), but without capital taxes, and extends the analysis to the case of constitutional constraints on labor taxes. We next describe the game.

4.1 The Policy Game

Our game is a dynamic policy game between a strategic government and atomistic households. Before the game starts, a set of constitutional constraints is exogenously imposed. We assume that government and households understand these constraints and governments conform with them.

In our setup, households are anonymous and their choices cannot be observed by the government or by other households. On the other hand, aggregate choices are observable. Then, households' and government's choices depend only on the public history, denoted by $\zeta^t = (\zeta_1, ..., \zeta_t)$, where $\zeta_t = (\tau_t, k_{t+1})$. As in Feng (2014), we do not make use of public randomization devices.

In any given period t, the timing of choices is as follows. First, the government chooses a labor tax rate for that period. The government's strategy, denoted $\sigma_{G,t}(\zeta^{t-1})$, is a choice of labor tax, within the range allowed in the constitution, as a function of ζ^{t-1}, i.e. $\tau_t = \sigma_{G,t}(\zeta^{t-1})$. Second, households choose consumption, labor and savings for that period. The household's strategy, denoted $\sigma_{H,t}(\zeta^t)$, is a choice of a current allocation as a function of the public history (ζ^{t-1}, τ_t), i.e. $(c_t, n_t, k_{t+1}) = \sigma_{H,t}(\zeta^{t-1}, \tau_t)$. We assume a symmetric strategy equilibrium, where all households make the same choices along the equilibrium path. Payments to capital and labor, r_t and w_t are determined by market clearing. Public consumption g_t is pinned down by the collected tax revenue. After each history ζ^{t-1}, a strategy profile (σ_G, σ_H) induces an outcome and a continuation strategy profile, which produces a payoff for the government and a payoff for the households.

As in Chari and Kehoe (1990) and Phelan and Stacchetti (2001), we define a sustainable equilibrium as follows:

Definition 2 *A symmetric strategy profile (σ_G, σ_H) is a sustainable equilibrium if, for all $t \geq 0$, the following two conditions are satisfied:*
(i) given the symmetric strategy for households $\sigma_{H,t}$, the continuation payoff for the government is higher than the payoff from any deviation to a different strategy $\tilde{\sigma}_{G,t}$ for every history ζ^{t-1}; and
(ii) given the strategy for the government $\sigma_{G,t}$, the continuation payoff for the household is higher than the payoff from any deviation to a different strategy $\tilde{\sigma}_{H,t}$ for every history (ζ^{t-1}, τ_t).

The above two conditions guarantee sequential rationality. The first condition implies that governments have no incentive to deviate. The second implies that households behave competitively.

In Domínguez and Feng (2014), we follow Phelan and Stacchetti (2001) (who extend the APS method, Abreu et al. (1990), to dynamic policy games) to show that the above game has a recursive structure once added as state variables a continuation value for the government and a continuation value for the households. The first is the next period's equilibrium payoff (welfare) of the game and the second is the next period's marginal value of capital, i.e.

$$m_{t+1} \equiv (1 + r_{t+1} - \delta)\, u_{1,t+1}. \tag{17}$$

As in Phelan and Stacchetti (2001), our dynamic economy can be thought of a sequence of static economies with endogenously changing state variables. In such static economy, for given m_{t+1}, τ_t, and k_t, the household's problem is that of choosing c_t, n_t, k_{t+1} to maximize $u(c_t, n_t, g_t) + \beta m_{t+1} k_{t+1}$ subject to the budget constraint (3). In Domínguez and Feng (2014), we show that, for the specific utility and production functions considered in this paper, and given m_{t+1} as in (17), the solution to this recursive problem (the static household's problem considered in each period) is equivalent to that of the sequence problem as presented in Section 2.

It is useful to consider the competitive equilibrium of the static economy (composed of the static household's problem, the firm's static problem and the government's budget constraint). Denoting next period variables with subscript $+$, that competitive equilibrium is defined as follows:

Definition 3 *The vector (c, n, k_+, g, w, r) constitutes a competitive equilibrium of the above static economy, denoted $(c, n, k_+, g, w, r) \in CE(k, \tau, m_+)$, if and only if*

$$u_1 = \beta m_+, \tag{CE-1}$$

$$-u_2 = (1-\tau) w u_1, \tag{CE-2}$$

$$k_+ = (1 + r - \delta)k + (1-\tau)wn - c, \tag{CE-3}$$

$$g = \tau^n wn, \tag{CE-4}$$

$$r = f_1(k, n), \tag{CE-5}$$

$$w = f_2(k, n). \tag{CE-6}$$

Given m_+ as in (17), the household's static problem is equivalent to the sequential problem. Then, it follows that for given $\{\tau_t\}_{t=0}^{\infty}$, and k_0, a sequence $\{c_t, n_t, k_{t+1}, g_t, w_t, r_t\}_{t=0}^{\infty}$ that is a competitive equilibrium of the static economy in each period, it is also a competitive equilibrium of the dynamic economy.

In a competitive equilibrium, for given continuation values h_+ and m_+, the values (payoffs) for the government and for the household are defined as $h = u(c, n, g) + \beta h_+$ and $m = (1 + r - \delta)\, u_1$ respectively. For a given initial capital k, the set of values (m, h) that can be attained in a sustainable equilibrum is called the equilibrium value correspondence $\mathbf{V}(k)$.

Let's define an arbitrary value correspondence $\mathbf{W}$ as any mapping from k into (m, h) and assume that contains $\mathbf{V}(k)$. The equilibrium value correspondence $\mathbf{V}(k)$ can be found and numerically approximated as a fixed point of $\mathbf{W}(k)$ by imposing the conditions for a sustainable equilibrium. These conditions are summarized in consistency and admissibility.

Definition 4 *The vector $\psi = (\tau, c, n, k_+, g, w, r, m_+, h_+)$ is consistent with respect to $\mathbf{W}$ at k if $(c, n, k_+, g, w, r) \in CE(k, \tau, m_+)$, $\tau \in T$, $(m, h) \in \mathbf{W}(k)$, and $(m_+, h_+) \in \mathbf{W}(k_+)$, where*

$$m(k, \psi) := (1 + r - \delta)\, u_1(c, n, g), \tag{18}$$

$$h(k, \psi) := u\,(c, n, g) + \beta h_+. \tag{19}$$

Definition 5 *The vector $\psi = (\tau, c, n, k_+, g, w, r, m_+, h_+)$ is admissible with respect to $\mathbf{W}$ at k if it is consistent and*

$$h(k, \psi) \geq \tilde{h}(k, \tilde{\psi}), \tag{20}$$

with $\tilde{\psi} = (\tilde{\tau}, \tilde{c}, \tilde{n}, \tilde{k}_+, \tilde{g}, \tilde{w}, \tilde{r}, \tilde{m}_+, \tilde{h}_+)$, where $(\tilde{c}, \tilde{n}, \tilde{k}_+, \tilde{g}, \tilde{w}, \tilde{r}) \in CE\,(k, \tilde{\tau}, \tilde{m}_+)$, $\tilde{\tau} \in T$, and

$$\tilde{h}(k, \tilde{\psi}) = \max_{\tilde{\tau}} \left\{ \min_{\tilde{c}, \tilde{n}, \tilde{k}_+, \left(\tilde{m}_+, \tilde{h}_+\right) \in \mathbf{W}(\tilde{k}_+)} \left[u(\tilde{c}, \tilde{n}, \tilde{g}) + \beta \tilde{h}_+ \right] \right\}.$$

Here $\tilde{h}(k, \tilde{\psi})$ is the worst possible payoff for the government (the best deviation). For a vector ψ that satisfies consistency, households behave competitively. If the vector ψ satisfies also admissibility, the incentive compatibility constraint (20) guarantees that the government does not want to deviate. The constitutional constraints on labor taxes change the structure of the game by affecting both conditions. They reduce the set of vectors ψ that are consistent, the set of continuation values that can be expected and the taxes that can be imposed in the best deviation.

4.2 Computation of the Equilibrium

In order to compute the set of equilibrium values, we define an operator $\mathbb{B}$ as follows.

Definition 6 *For a given set of equilibrium values* $\mathbf{W}$*, operator* $\mathbb{B}$ *is defined as*

$$\mathbb{B}(\mathbf{W})(k) = \{(m, h) \mid \exists \ \ \psi \textit{ is admissible with respect to } \ \mathbf{W} \textit{ at } \ k\}.$$

Phelan and Stacchetti (2001) prove that this operator has the following properties:

1. If $\mathbf{W} \subseteq \mathbb{B}(\mathbf{W})$, then $\mathbb{B}(\mathbf{W}) \subseteq \mathbf{V}$.

2. $\mathbf{V}$ is compact and the largest set of equilibrium values $\mathbf{W}$ such that $\mathbf{W} = \mathbb{B}(\mathbf{W})$.

3. $\mathbb{B}(\cdot)$ is monotone and preserves compactness.

4. If we define $\mathbf{W}_{n+1} = \mathbb{B}(\mathbf{W}_n)$ for all $n \geq 0$, and the equilibrium value correspondence$\mathbf{V} \subset \mathbf{W}_0$, then $\lim_{n\to\infty} \mathbf{W}_n = \mathbf{V}$.

Result 1 has been called self-generation. From the definition of the set of equilibrium values, it is straightforward to see that $\mathbf{V} \subseteq \mathbb{B}(\mathbf{V})$. Together with result 1, it is fairly easy to reach the second result. Results 3 and 4 will be used to approximate the set of equilibrium values.

Fernï¿œndez-Villaverde and Tsyvinski (2001) and Domínguez (2010) provide a numerical implementation for $\mathbb{B}$ by adapting the approximation technique developed by Judd, Yeltekin and Conklin (2003) and Sleet and Yeltekin (2015). This method requires the convexity of the set of equilibrium values $\mathbf{V}$, which can be guaranteed by incorporating a publicly observed random variable as in Phelan and Stacchetti (2001). This approximation scheme uses a polar coordinate system to represent the position of an arbitrary point on a manifold. Thus, to approximate a convex set, one only needs to keep track of the supporting hyperplanes at each angle around the polar. It reduces the computational costs substantially. However, randomization and convexification will arbitrarily enlarge the equilibrium value correspondence.

Instead, we apply the methodology developed by Feng et al. (2014).[9] Their method partitions the state space into a finite set of simplices. Compatible with this partitioning, they then consider

[9]We refer to Feng et al. (2014) for details in the approximation of convex-valued sets and for Feng (2014) and Domínguez and Feng (2014) for applications of the algorithm to similar games (and for recovering the strategies).

a sequence of step correspondences, which take constant set-values on each simplex. The main advantage of this method is that it does not require convexity of the equilibrium set, and, thus, it is not necessary to introduce a randomization device as in Phelan and Stacchetti (2001). However, this method faces some computational challenges. To facilitate the computation, we assume that $\mathbf{W}$ is convex-valued at given (k,m). This assumption implies that, for given (k,m), there exist strategies that support any $h \in [\bar{h}, \underline{h}]$, where

$$\bar{h}(k,m) := \max_{h} \{h|(m,h) \in \mathbf{W}(k)\}, \tag{21}$$

$$\underline{h}(k,m) := \min_{h} \{h|(m,h) \in \mathbf{W}(k)\}. \tag{22}$$

For a given k, the worst sustainable equilibrium yields the lowest value for the government $\tilde{h}(k) = \min_m \underline{h}(k,m)$, and the best sustainable equilibrium yields the highest value $\max_m \bar{h}(k,m)$. Feng (2014) proves that the boundaries of the set of equilibrium values for sustainable equilibria can be characterized by themselves. The repeated application of operator $\mathbb{F}$ as described below generates a sequence of sets that converges to the equilibrium value correspondence $\mathbf{V}$. We refer the reader to Feng (2014) for the proof of the theorems and the numerical implementation of operator $\mathbb{F}$.

Definition 7 *For any convex-valued correspondence $\hat{\mathbf{W}} = \left\{(m,h)|h \in \left[\underline{h}^0(k,m), \bar{h}^0(k,m)\right]\right\}$, define operator $\mathbb{F}$ as follows:*

$$\mathbb{F}(\hat{\mathbf{W}})(k) = \left\{(m,h)|h \in \left[\underline{h}^1(k,m), \bar{h}^1(k,m)\right]\right\},$$

where

$$\bar{h}^1(k,m) = \max_{\tau} u(c,n,g) + \beta \bar{h}^0(k_+, m_+) \tag{23}$$

$$\underline{h}^1(k,m) = \max\left\{\max_{\tau} u(c,n,g) + \beta \underline{h}^0(k_+, m_+), \tilde{h}^0(k)\right\} \tag{24}$$

$$\tilde{h}^0(k) = \max_{\tau}\left\{\min_{c,n,k_+,m_+} u(c,n,g) + \beta \underline{h}^0(k_+, m_+)\right\} \tag{25}$$

such that vector $(\tau, c, n, k_+, g, w, r, m_+, h_+)$ is admissible with respect to $\hat{\mathbf{W}}$ at k. Define $\underline{h}(k,m) = -\infty, \bar{h}(k,m) = +\infty$ if no such vector exists.

4.3 Quantitative Analysis

For the calibration detailed in the Appendix, we first propose some constitutional constraints on labor taxes and then approximate the set of sustainable equilibria of our policy game under those constraints.[10]

4.3.1 Natural Bounds on Labor Tax Rates

Following Domínguez and Feng (2014), we explore whether there exist constitutional constraints on labor taxes that could improve all equilibria. In order to construct these 'optimal' constraints, in that paper we propose to examine the prescriptions of a Ramsey planner with full commitment and without constitutional constraints. For a given level of capital, we define the natural upper and lower bounds on labor taxation as the highest and lowest levels of labor tax rates that a Ramsey planner (starting from any initial conditions) would prescribe for that level of capital.

Figure 2 depicts these natural bounds on labor taxes for our benchmark parameter values and for other selected parametrizations. For balanced-budget policies and no capital taxes, these bounds can be described as follows. The natural lower bound on labor taxes increases with the level of capital. Let's define a threshold level of capital $\hat{k}$. The natural upper on labor taxes increases (decreases) with the level of capital for all $k \leq \hat{k}$ ($\geq \hat{k}$). Interestingly, this threshold level of capital coincides numerically with the steady state level of capital in the Ramsey allocation.

[Insert Figure 2 about here.]

Jointly, both bounds define a range of allowed taxes that is larger for higher (lower) levels of capital for all $k \leq \hat{k}$ ($k \geq \hat{k}$). That is, for low or very high levels of capital, the Ramsey delivers very tight bounds on labor taxes. However, around the steady state level of capital, the natural bounds on labor taxes are wider. While the shape of these bounds is robust across different parametrization, the level and width of the bounds is affected by the parameters. For a lower elasticity of labor supply, the natural bounds are tighter. For a higher weight on public consumption, both bounds shift upwards and define a wider set of allowed taxes.

[10] We discretize the state space with 400 equally spaced points for $k \in [0.01, 1.5]$, 400 points for m, and 100 points for $\tau \in [0, 0.9]$. We use linear interpolation for values falling outside of the grid. We ran our *C++ MPI* code using an IBM iDataPlex cluster, with 50 Intel Sandy Bridge 2.6GHZ processors.

4.3.2 The Set of Sustainable Equilibria for Different Constitutional Constraints

The above natural bounds on labor taxes have the potential to improve all sustainable equilibria. Suppose the economy faces a situation in which, without constitutional constraints, the BSE coincides with the Ramsey. Let us now impose the natural bounds as the constitutional constraint on labor taxes. First, these bounds have the potential to eliminate worst outcomes (if those are sustained by taxes outside of the bounds) and then may improve the WSE. Second, the bounds do not bind at the Ramsey. Then, if the increase in the WSE is not enough to make the incentive compatibility constraint (20) bind, these natural bounds do not worsen the BSE and may improve all equilibria.

Figure 3 shows the set of sustainable equilibria under different constitutional constraints for our benchmark parameter values and for other parametrizations. In our experiment, we consider no constraints and three other distinct constitutional environments: the natural upper bound on labor taxes; the natural lower bound on labor taxes; and both natural bounds on labor taxes. Figure 4 depicts the allocation and taxes that support the WSE for our benchmark parameters.

[Insert Figures 3 and 4 about here.]

Across all selected parameterizations and relative to the equilibria with no constitutional constraints, we find the following. First, the three constitutional environments have none or little effect on the welfare provided by the BSE. However, they do affect the WSE.

The natural upper bound on labor taxes actually decreases the welfare provided by the WSE. The intuition is that while the upper bound does not affect the optimal government's choices at the best it does restrict the government's optimal choices when facing worst expectations. With no constraints, Figure 4 shows that taxes in the WSE are in some periods higher than without the upper bounds. Moreover, at the WSE, the taxes with the upper bound display an irregular pattern that delivers uneven sequences of (public and private) consumption and labor.

In contrast, the natural lower bound on labor taxes improves the WSE. With no constraints and with lower bounds, both WSE induce similar patterns of capital, labor and output. Relative to no constraints, the lower bounds induce higher taxation and higher (lower) levels public (private) consumption. This mix of private/public consumption delivers higher welfare.

Interestingly, both natural bounds together deliver the WSE with the highest welfare. As Figure 3 shows, the set of sustainable equilibria shrinks dramatically when both bounds are considered. From Figure 4, we see that these tighter bounds induce smooth taxes in the worst and a smooth and higher public consumption that induces higher welfare.

The constitutional environment affects the equilibrium marginal values of capital that are supported. The natural upper (lower) bound on labor taxes supports lower (higher) marginal values of capital. With both bounds together, the set of equilibrium marginal values of capital is very small. This may explain the smoothness of the allocation at the WSE seen in Figure 4.

5 Conclusions

This paper has investigated the time-inconsistency problem of labor taxes when governments cannot issue bonds and cannot tax capital. We have learned that, in this environment, governments find optimal to set low labor taxes for today and high labor taxes for the future. However, in the future, governments are tempted to lower labor taxes again. With limited commitment, we have found that this time-inconsistency problem leads to underprovision of public consumption. In dealing with this problem, we have learned that imposing carefully chosen lower bounds on labor taxes (alone or in combination with upper bounds) as constitutional constraints can be optimal. Such bounds induce higher public consumption and higher welfare in worst equilibria.

An important extension to our analysis would be to allow for government bonds. As shown in Domínguez (2007), government debt affects the properties of optimal taxes without commitment.

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6 Appendix

6.1 Calibration

In order to calibrate the parameters for our quantitative exercise, we consider an initial steady state that corresponds to an economy with similar statistics to those of the US. In our simulations one period corresponds to one year.

Our calibration follows that of Chari et al. (1994), which is consistent with U.S. data. In the initial steady state, the tax rates on capital and labor income are set equal to 27.1 and 23.7 per cent respectively. We choose a public consumption to output ratio equal to 19 per cent and consider no public debt.

We assume the utility function (2) and a Cobb-Douglas production function $y_t = k_t^{\alpha} n_t^{1-\alpha}$. We choose a capital share of 0.34 and a depreciation rate of 0.08. The discount factor is set to deliver a capital to output ratio of 2.71 in the initial steady state. In (2), the coefficient of risk aversion σ is set equal to unity and the labor-supply elasticity is set so that χ= 0.32. The weight on labor disutility γ_n is chosen so that hours worked are 0.23 in the initial steady state. The weight on public consumption γ_g is chosen so that social planner's solution delivers a public consumption to output ratio equal to 19 per cent.

Table 1 summarizes the parameter values used in our baseline economy.

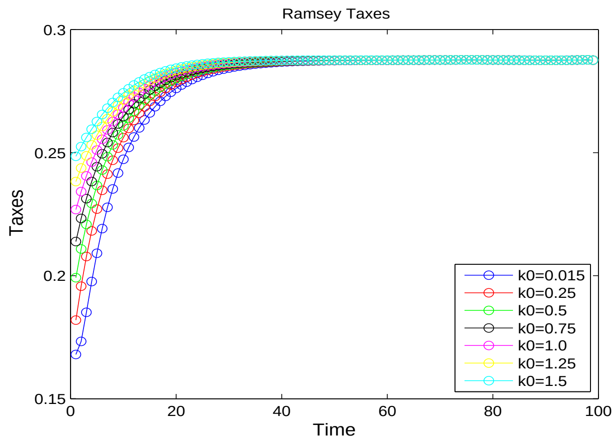
Table 1: Parameter Values for the Baseline Economy

Preference	$\beta = 0.968$	$\sigma = 1.0$	$\gamma_n = 7.694$	$\chi = 0.32$	$\gamma_g = 0.333$
Technology	$\alpha = 0.34$	$\delta = 0.08$			

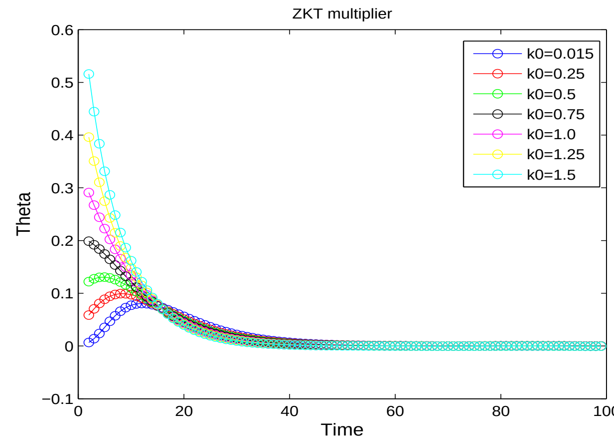
6.2 Figures

Figure 1: Ramsey Labor Tax and some Terms for Different Initial Levels of Capital

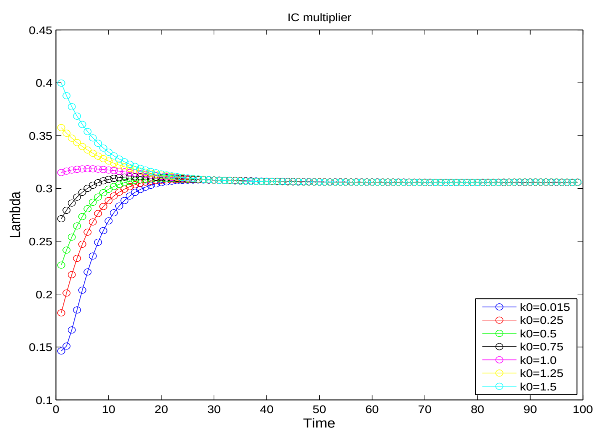
(a) Ramsey Labor Taxes



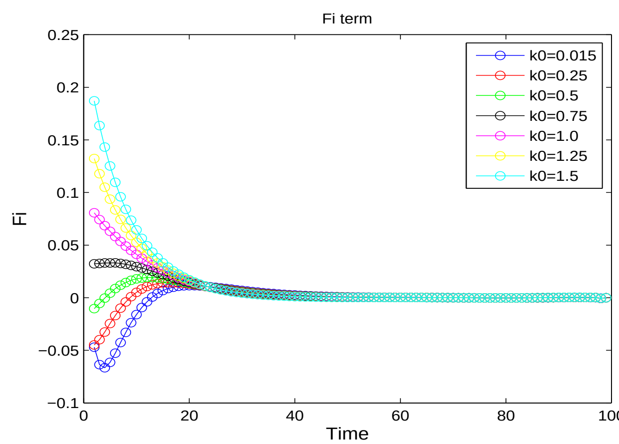
(b) Zero Capital Tax Multiplier



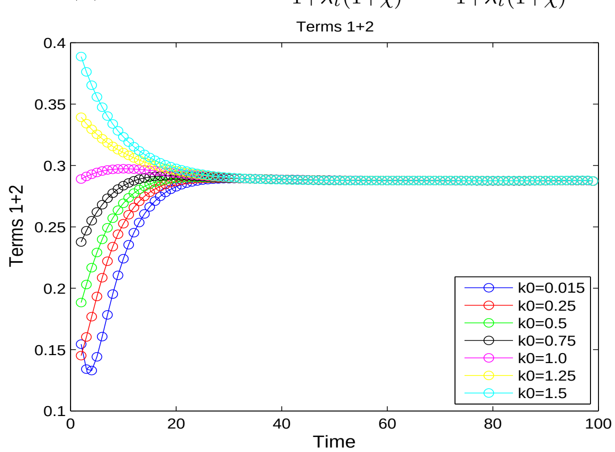
(c) Implementability Condition Multiplier



(d) The term Ψ_t



(e) Terms 1+2: $\frac{\lambda_t(\sigma+\chi)}{1+\lambda_t(1+\chi)} + \frac{(\lambda_t-\lambda_{t+1})\Gamma_t}{1+\lambda_t(1+\chi)}$



(f) Term 3: $\frac{\Psi_t}{1+\lambda_t(1+\chi)}$

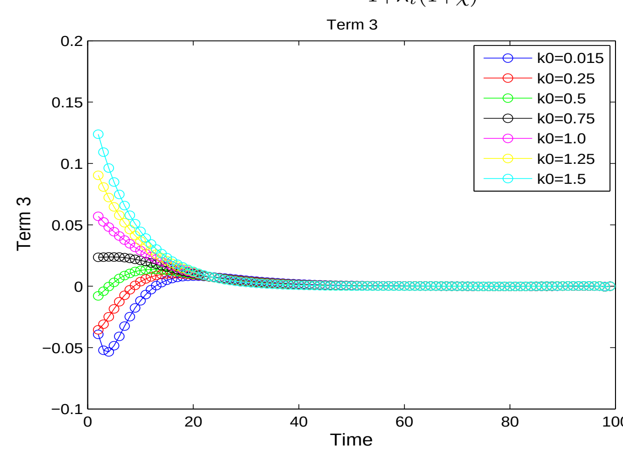
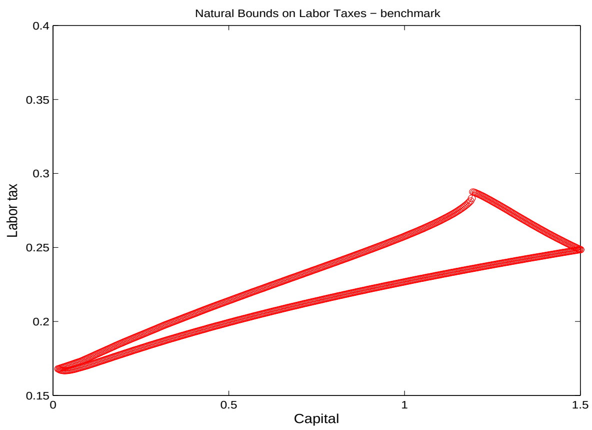
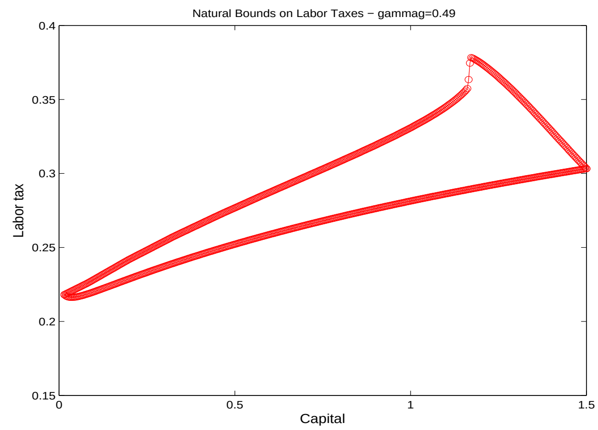


Figure 2: Natural Bounds on Labor Tax Rates

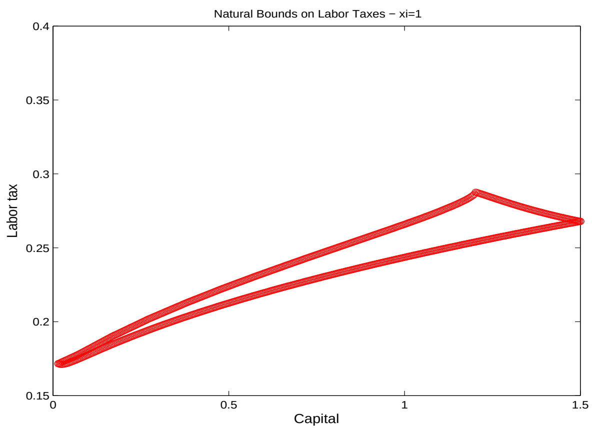
(a) Baseline Parameters



(b) Higher Weight $\gamma_g = 0.49$



(c) Lower Elasticity, $\chi = 1.00$



(d) Elasticity $\chi = 1.00$, Weight $\gamma_g = 0.49$

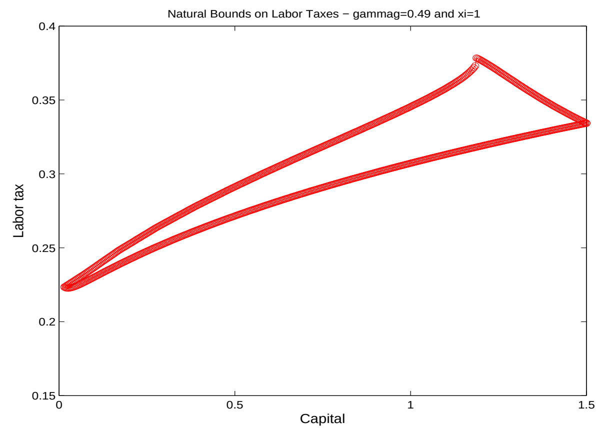
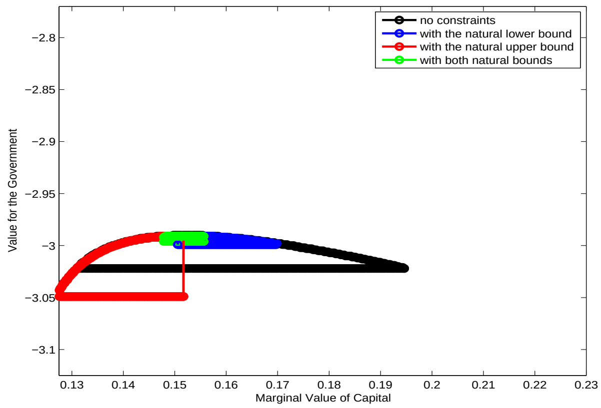
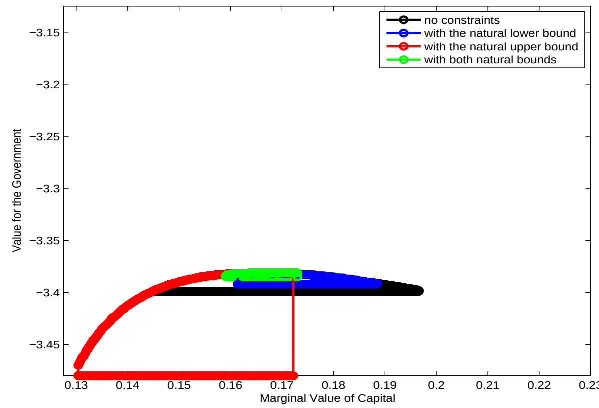


Figure 3: Set of Sustainable Equilibria

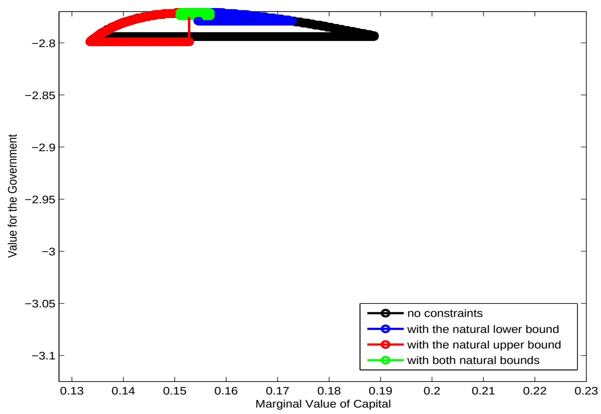
(a) Baseline Parameters



(b) Higher Weight $\gamma_g = 0.49$



(c) Lower Elasticity, $\chi = 1.00$



(d) Elasticity $\chi = 1.00$, Weight $\gamma_g = 0.49$

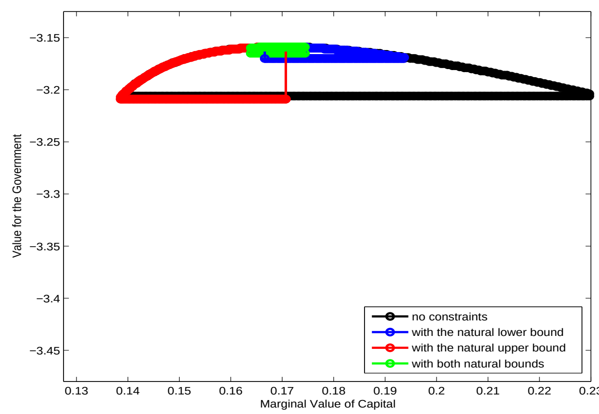
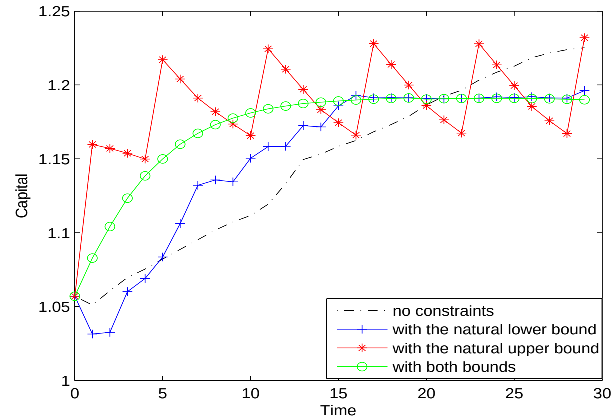
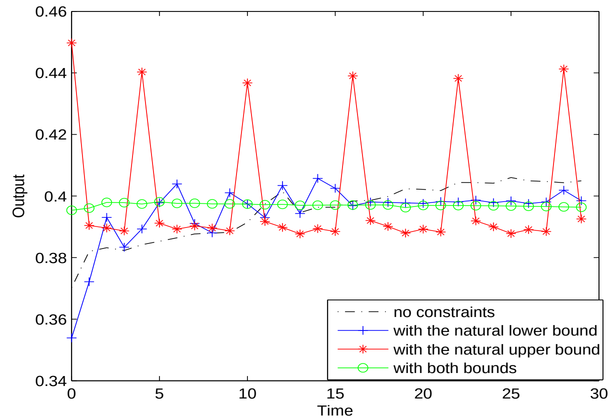


Figure 4: Simulations for the WSE during the Transition

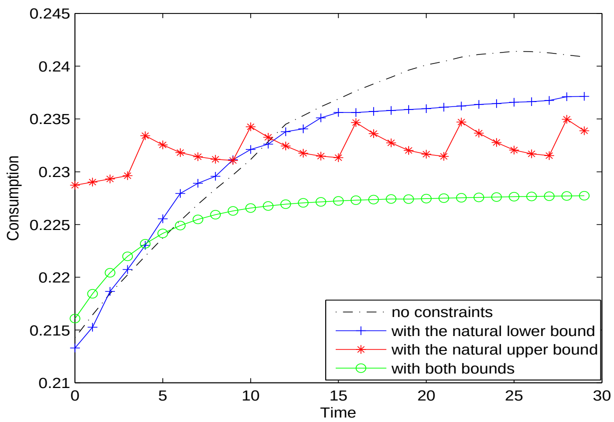
(a) Capital



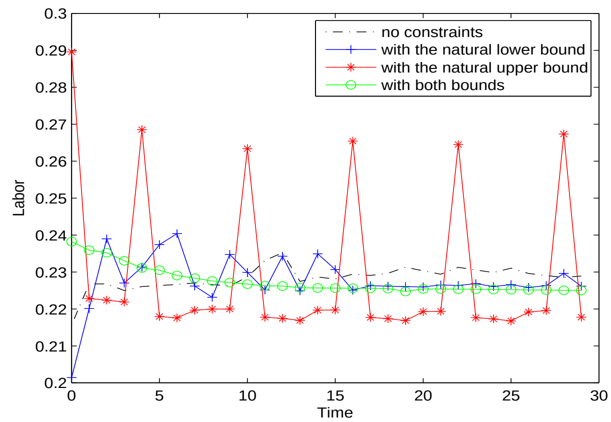
(b) Output



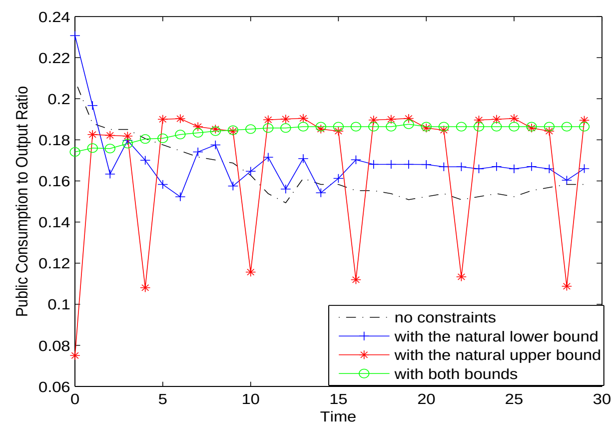
(c) Consumption



(d) Labor



(e) Public Consumption



(f) Labor Tax Rates