MATHEMATICS CAPITAL IN MATHEMATICS EDUCATION

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By

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A DISSERTATION

Presented to the Faculty of

The Graduate College at the University of Nebraska

In Partial Fulfillment of Requirements

For the Degree of Doctor of Education

Major: Educational Administration

Under the Supervision of Dr. C. Elliott Ostler

Omaha, Nebraska

February, 2023

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This dissertation explores the reimagination of mathematics education through a Bourdieusian perspective. The conceptual framework includes Bourdieu's well-defined terms: capital, field, habitus, and doxa. A thorough review of mathematics education documents finds that mathematics education has not yet considered, mathematics capital, from a Bourdieusian lens. More, examined documents reveal that key variables, mathematics teaching and learning experiences, mathematics self-efficacy, mathematics mindset, and cultural capital, interact. In this examination, cultural capital emerges thematically through all texts. Next, six mathematics teachers with dual degrees in mathematics and education are interviewed. Data analysis included a process of inductive and deductive coding. In these, a *je ne sais quoi* factor permeates the participants. Lastly, the framework for structural equation modeling is presented to verify the qualitative findings. The purpose of this quantitative model is to explicitly and succinctly describe the interactions that exist between the key variables and to measure their associations to mathematics capital. This chapter is also designed to disentangle and differentiate mathematics capital from any of the key variables. This dissertation closes by describing mathematics capital as a parallel to Bourdieu’s definition of cultural capital. Mathematics capital’s definition includes two components: the first is competence in school mathematics, and the next is the act of using, intent to use, or accidental use of this mathematical competence to achieve financial, cultural, social, or other gains that can be associated with an already described form of capital.
Keywords: Mathematics capital, capital, doxa, mathematics education, Bourdieu.
Dedication

To my children who, without knowing it, require me to relentlessly make the most of each minute without forgetting to notice the unprecedented twists of the trees, the unanticipated colors of the flowers, and the impassioned songs of the birds. Matthew, Elizabeth, Patrick, Violet, and John Theodore: You are the heart within my heart. No words can express how much I love you.
Acknowledgements

Dr. C. Elliott Ostler, chair of my committee, I tell you each day that without you this would not have been possible. Each time you answer, “I know Fran Anderson and you don’t need me.” Then I smile. What you don’t realize is that it is in this very response that I know that it is because of you. What you teach me is to know, innately, the strength that exists from within me. What a teacher who can teach a student to do for themselves and to increase belief in self. You have made writing a joy and strengthened me through every minute of conversation.

Dr. Tami Williams, member of my committee who endlessly supported me through the entire process. You always leave your door open, and I’m learning to walk in more often. I’m so grateful for your compassion and fierce leadership. I’m grateful to call you a mentor.

Dr. Kay Keiser, member of my committee and the first professor in my program. You welcomed me into the degree openly and were the first person to tell me that you look forward to when you’ll call me doctor. It seemed so far away at the time and yet in a blink, here we are. Your continual reminders to work reverently yet not without self-care will not be forgotten. Thank you.

Dr. Chris Wilcoxen, member of my committee and one of the first people to welcome me to UNO. Your warm mentorship and willingness to be a continued support has been priceless. Every time I see you, I know that you have so many things to do but you never have too many things on your plate to make time for me. I’m so grateful for the many conversation that we have had and for your support with and time spent on my dissertation.

Dr. Gregory Snow what can I say except that I wish you were here to read this. First, I know that you would read each word. Next, I want you to know that you are one of the reasons that research is my passion. You were the first professor that introduced me to research as a freshman in my undergraduate degree and taught me how exciting and fulfilling it can be to
have an idea and to work an unknown amount of time on this idea. I remember when you told me that I would complete a doctorate and that your only sadness was that it wasn’t in physics. Greg, never be sad about this because my promise to you is that I will bring more people to physics through mathematics that will do more amazing things for the field of physics. You are always in my heart, and I wish I could share this with you.

Jeremiah Anderson, my husband. Thank you for your unwavering support. I’m grateful to be able to call you family and know that without your consistent support, this dissertation would not be written. Life with you has been peaceful. Cheers to many more years.

My parents Mark and Jacqueline Keating. Without you, there is no me. Thank you for giving me life and for, during my early years, introducing me to key academic and intellectual development that informed my disposition and capacity for my entire life. I’m grateful for the gift of learning that you bestowed on me from birth. Lastly, thank you for each of you as you supported me through this work. Mum, you were there at the beginning, telling me not to worry and that you are there to support me. And Dad, you were there at the end when listened while I read this entire document to you, as you were weathering the effects of chemo. I love you both so much.

Patrick Keating, my older brother, and lifelong friend, I’m so grateful to have you in my daily life. You are often the light in my days. As you look to help each person catch the fish of their lifetime, I want you to know that it is in this dissertation that you have allowed me to (metaphorically) catch the biggest fish of my lifetime. I know it is not on the water and in theory this is not a fish. And yet, I had the fight of my life “reeling” this in, but I was never alone. You stood beside me, told me when to increase tension and when to let the fish be in control. I love you big bro. Thank you for always being there for me.
Ariel Root, my sister, and nonetheless, I love you for every moment you spent on zoom with me while you worked on your dissertation as well. No matter how tight your deadline was, you would selflessly give time to read mine, whenever I asked. Even if I didn’t ask, at times you would just look at me and say, “What are you working on? Stuck?” I’m so glad to have had a partner in you.

The Siermacheskys, my adopted family, you have been a part of my life for more years than I can count. Thank you for showing me endless love, compassion, and support through this process. Thank you for continuing to include me as a family member; the richness that this gives me is often left unsaid. Let it be known that I never take it for granted and look forward to a time when we can all be together again.

Teresa Rivera Avelar, my “daycare lady”, my guardian angel on earth, none of this would be possible without your support. I tell you that I need to work, and you tell me, “Well Frances, we do what we need to do.” which means, you’ll support me no matter what. Teresa you have so selflessly been by my side since August 2014. I cannot imagine life without you and I know with certainty that none of this would exist without your support.

Dr. Kelly Gomez Johnson, the person who guided me down this path. I remember the day you told me, “Good things happen to good people. This is real.” I sat in disbelief as I watched you show me that all my dreams would come true. Without you none of this would exist. I’m grateful for your mentorship, friendship and cherished sisterhood. Thank you for always caring about me.

Dr. Jon Santo who has so freely taken me under his wing to teach me so many things. You provided key insight and guidance for this work. I fearfully wonder how many times I’ve said to you, “Okay, this is my last question.” Not only have you been a substantial portion of my learning for this work, but you’ve also guided me on a personal level. Specifically, you’ve shown
me that there will be weight lifted once this process is completed. Alongside this, you’ve also
played a critical role in assuring me that I will find success with this work and in future work.

Dr. Paula Jakopovic, thank you for being my friend through this process. For providing
space to arrive exactly as myself. You offered me evenings, weekends, time during the workday,
and cherished “Tommy photos”.

Dr. Yvonne Lai who selflessly gave me time your time to teach me so many skills that
went into this product. What a relief it was when I got to certain places in my dissertation to
have had the experience of working with you. I am deeply indebted to you and hopefully look
forward to many more years of collaboration. Cheers to us Canadians working in Nebraska!

My WAGS group Jennifer Lemke, Sara Churchill, and Christina Wilcoxen. Thank you for
the hours that you spent reading this work. I assure you that each moment has been cherished
and that each piece of feedback was valued through deep contemplation. Being able to bring
work, as it is and have eyes on it was a huge part of forward momentum during this process.
Thank you for being partners with me.

Brenda Peters who takes endless phone calls from me helping me stay focused on what
is now, what is critical to accomplish when my thoughtful mind wants to take dark turns. I’m
grateful for everything you’ve taught me. Most especially that you’ve taught me to manage
work and to do it with relative ease.

Dr. Grant Tietjen, a long-time friend. I remember how I felt reading my name in your
dissertation. I can say that I never, in that moment, dreamed that I’d be able to produce the
reciprocal gesture. Instead, I sit staring in disbelief. Thank you for always taking my calls, for
walking me through this wonderful opportunity that I was given and providing me with
invaluable insight in critical moments.
Dr. Kathy Danielson who has shown me endless compassion and insisted on me to always put one thing first, myself and my family. I could ask for no better leader.

Dr. Greg Sand who always appears out of nowhere in exact moments when needed. You are a friend beyond friends; I’m so grateful that we remain in each other's lives.

Friends and family at Omaha Central. You have to know where you came from to know where you are heading. So much of what I believe is rooted in what I lived with you at Central. Thank you for living with me through these moments in my life. The years spent at Central remain highlights in my heart and mind. I miss you each day and hope that this is the start of giving back to every staff, faculty member and student at Central.

My family members and friends from each walk of life. I resist naming so that no person is left out. You know me exactly as I am and love me all the same. You are the people who hold me when I cry, who listen when I do not stop talking, and who are always there when I want to believe that I’m alone. You always support me, without question, and remind me that I am chronically capable.
Grant Acknowledgement Information

This study was supported by the National Science Foundation Grant No. 1852908. This study represents the opinions of the author and does not reflect the opinion of the National Science Foundation. A special thank you to the six participants, graduates of the NOYCE scholarship program: your invaluable perspectives allowed for integral, informative insight. Thank you for participating in this study.
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Chapter 1: An Introduction to Mathematics Capital in Mathematics Education

Seminal research in social, cultural, financial, and science capital defines capital by explaining that capital positively influences a person’s outcomes in the context in which it is being described (Archer et al., 2014; Archer et al., 2015; Bourdieu 1977, 1998; Coleman 1988). One factor with limited representation in research is the concept of mathematics capital (Williams & Choudry, 2016). Recent educational research has sought to level the playing field for all students, creating justice for our comprehensive population (Safir & Dugan. 2021).

Mathematics education research has successfully identified factors that influence student success in mathematics without identifying *mathematics capital* as an underlying factor of a student’s outcome in mathematics (NCTM, 2014). Capital experts report that capital exists as an overarching power in any definable context (Bourdieu, 1977; Coleman 1988). Mathematics education is an established, well-defined context, suggesting that *mathematics capital* is an operationalizable phenomenon.

Bourdieu explains that researchers must consider whether they are subjects within their research field. If the researchers are a part of the subjects, the research will not remain without influence (Bourdieu, 1990). Operationalizing *mathematics capital* offers the opportunity to reimagine the mathematics education field holistically. This view will provide a clearer vision of mathematics education, and individual student needs; any stakeholder in the field of mathematics education, be it mathematics learners, teachers, researchers, or other will know that mathematics outcomes lack the integral descriptor, *mathematics capital*, and this descriptor influences mathematics success.

Social Capital and Coleman
Physicians are learning more about fascia and its interconnectedness in the body. Fascia is a full-body communication network, operating to connect us from the inside out; everything done with our bodies is connected through fascia (Schleip et al., 2012). And yet, in previous years, fascia was misunderstood, undervalued, or ignored (Adstruma et al., 2017; Chen1, May 21, 2019, 0-42mins). As we searched for similar, misunderstood facets of our social world, we discovered something equally undervalued and interconnected: social capital (Bourdieu, 1986). Refined by Coleman, social capital began to describe the human social potential and is inherent connectedness to financial and human capital, impacting a person’s potentiality and opportunities in their lifetime (1988). As a society, the phrase, “It’s not what you know but who you know” is well-known and socially acceptable. When something so powerful and interconnected controls our relational sphere and potential life outcomes, mirroring how the fascia network connects our body, it is reasonable to wonder if this same interconnected “fascia network” is hidden in other spaces, impacting outcomes.

Similarly, social capital can be measured by the number of potential exchanges within the relationships of the actors in a community, how many communities each actor is connected to, and the diversity of actors’ connected communities. Communities must be built so that the parties are closely intertwined, offering each actor opportunities to draw from the network independently. Social capital withdrawals are only possible if the giver chooses to release to the receiver, which is why social capital is vastly underutilized in comparison to the other forms of capital (Bourdieu, 1977, 1986; Coleman 1988, 1990; Hoenig, 2003; Putnam, 1995). Social capital has been studied in educational contexts, in the home, and the workplace (Furstenberg & Hughes, 1995; Driskell et al., 2012; Archer et al., 2015; Teachman et al., 1996). In essence, social capital is the concept that “it takes a village” to raise a child, having a village to rely on, and choosing to rely on the village (Hanifan, 1916).
Coleman (1988) introduces capital as a web-like structure where he finds all forms of capital are intertwined and passed down through families. Human capital cannot make its way to children if the home’s social capital is nonexistent. Parents' relationship with their children is the determining factor of whether their human capital is passed down. In line with the funds of knowledge concepts from Esteban-Guitart and Moll (2014) who state that a person’s capacity to value and identity with specific things is oftentimes in close relationship to how their parents value and identified it. Thus, this interdependent nature of constructs is not secular to capital, but instead crosses over in a multidisciplinary fashion.

Bourdieu agrees with capital’s interdependency by finding economic capital as the root of all forms of capital and that “the convertibility of the different types of capital is the basis of the strategies aimed at ensuring the reproduction of capital” (1986, p. 25). Capital presents as a web of intertwined opportunities, fighting to protect those in it and holding firm to keep others out, whether this is intentionally, or unintentionally, is still negotiable (Nolan, 2012).

**Social and Ancillary Capital**

Extensive capital research has been done in educational contexts (Teachman et al., 1996; Turnball et al., 2019). Thus, social capital and its impact on education remain highly prevalent in research today while being described as early as 1916. Hanifan (1916) explains that when the school, families, and the community work together to build relationships, student outcomes improve. Specifically, the products are better attendance, and more family and student engagement. Paralleling these products are successful improvements within the community. Researchers conclude with certainty that their work resulted in positive outcomes.

Dewey (1899) describes the ideal home for a student and how this yields optimal educational results. “When children are invited into household tasks, the children have more
opportunities to gain knowledge” (1899, p. 17). He continues by discussing the differences between youth and how these differences arm each child uniquely. These differences are highly predictive of the student’s success and skill development which foreshadows the description of cultural and social capital.

Intentional development of skill, access to financial power and flexibility, access to machinery, or strong connections with a community or communities creates advantages (Bourdieu, 1977, 1986; Coleman, 1988, 1990). In literature, these advantages are now referred to as capital (e.g., human, social, physical). And yet, advantages have been alluded to in the way of skill development, financial power, machinery, or community since 1776 and possibly before. Smith (1776) alludes to what is now known as human capital via an array of positions that labor men have by contrasting their ability to perform each skill based on their prior experience with the noted skill. His discussion of nails notes that one untrained man could make two to three hundred nails of poor quality in a day. Smith notes, “I have seen several boys, under twenty years of age, who had never exercised any other trade but that of making nails, and who, when they exerted themselves, could make, each of them, upwards of two thousand three hundred nails in a day” (p. 4).

Coleman, in 1988, explained that social capital was no longer theoretical but instead conceptual. Although still used in a “theoretical enterprise,” social capital has been operationalized and validated as a measure of function and opportunity that exists within social spheres. Coleman’s work continues to reframe and expand social capital under the umbrella of education. This reach is not unreasonable, instead quite fitting when mirrored with Dewey’s work in 1899, where social capital, although not operationally defined, was first introduced and theorized as described above. With an overarching goal to present a new perspective for
consideration in mathematics education research, it becomes relevant to consider describing mathematics capital similarly to social, cultural, and human capital.

**Conceptual Framework**

Through a Bourdieusian lens, it can be argued that researchers can consider mathematics education as a field within which *mathematics capital* can be termed (Bourdieu, 1977, 1986, 1990; Bourdieu & Wacquant, 1992). Within such a lens, subjective and objective are retired, and instead, finite concepts are defined: *capital, field, habitus, and doxa*. An abbreviated explanation of each follows, yet an operational definition is included in the following chapter.

*Capital* is an overarching power that exists within a described field. This power influences the way people find success in the field. *Field* is any closed circuit that can be captured and described. Within this description, we can make clear what are considered gains and what are considered losses. Further, we know people’s lives are impacted within the described field. *Habitus* is what is understood as acceptable actions within the field. Said differently, habitus could be considered acceptable status quo. *Doxa* is the acceptable actions that are found in the unconscious mind, as it reigns over all people both willingly and unwillingly (Bourdieu, 1977). Capital can also inform people about acceptable habitus, which again only more positively influences persons within the field. When capital is described in the field, it is delineated from the word capital and instead is captured as a short sentence, two words where the precursor describes the field. For example, if one wanted to discuss the capital that exists within social structures, the phrase “social capital” would be used. Likewise, if one wanted to describe how capital exists in the world in how it relates specifically to the exchange of money and the strength of wealth, “financial capital” would be used.

**Contemporizing Bourdieu**
Bourdieu refines capital when he attaches to the arts, highlighting his reasoning founded in the positive social attainment that an affinity for the arts offers (1984). As the Science, Technology, Engineering, and Mathematics (STEM) world continues to impact social outcomes in many areas, it seems reasonable to extend Bourdieu’s original concepts of capital attached to the arts that inform social advantage to mathematics attainment. The reason for this is that mathematics outcomes can substantially impact people’s futures. This was first extended by Archer and colleagues when they termed science capital (2015). Thus, mathematics capital is adopted as the overarching power that exists within mathematics education, where mathematics capital informs outcomes of people either reproducing advantage or disadvantage.

All components of mathematics education are presumed intertwined, and all forms of capital are intertwined (Bourdieu, 1984; Martin, 2009). It is theorized that this perspective will add to the literature by contemporizing Bourdieu and connecting what is said in current mathematics education research along with the results of the achieved success as a result of positive outcomes in mathematics education. One example is that currently, people who work in the STEM field (thus having been successful in math) typically earn substantially more income than their counterparts (Fayer et al., 2017). A theoretical and contemporary bridge connects capital, field, habitus, and doxa to a Bourdieusian perspective, to mathematics education today for mathematics capital to be operationally defined (Bourdieu, 1977, 1984, 1986, 1990).

Mathematics Education Research

Mathematics is a constant subject of conversation, as it has been for so many years. Nevertheless, this conversation is valid. A student’s outcomes in mathematics are directly related to their performance in many academic areas (Pong, 1998; Rose & Betts, 2001). Not only is mathematics a predictor of student performance in other areas, but lack of mathematics success also holds people back in STEM. One may question relevance again, but this author...
continues by highlighting income discrepancies between STEM occupations and non-STEM occupations by nearly half that of the first (Fayer et al., 2017; Rose & Betts, 2001) which means that mathematics success can impact how much money a person makes over a lifetime.

To date, researchers have considered math achievement and outcomes based on specific areas, considering items individually. Researchers have been focused on groups of people; how one area or characteristic might contribute to overall mathematics success (Boaler et al., 2016; Bulková et al., 2020; Jorgensen & Larkin, 2017; Jorgensen, 2018; Martin, 2013; Navarro, 2012; Singh, 2002; Xu et al., 2021). This work does not stand in contrast with previous work. Instead, this work is to creatively connect all previous work in mathematics education aiming to find out why each person is individually successful in mathematics.

Mathematics research often focuses on subgroups to analyze mathematics achievement. In this research, each person is considered individually as a player in the field, such that each person’s outcome remains unattached to any subgroup. This perspective offers the nuance to consider a holistic perspective on a person’s mathematics achievement; this is fluid and changing due to different people's mindsets, experiences, self-efficacy, and culture. This influences their mathematics capital and, thus, their outcomes in mathematics education.

**Mathematics Capital—Contemporizing Bourdieu’s Work—A Four-Part Model**

Bourdieu’s 1984 work, Distinction, describes how preferences, specifically taste (in art selection and more) can work to impact a person’s social mobility. Bourdieu categorizes and defines a person’s ability to exist in “higher” cultures based on an analysis of their taste. This is the first example of considering how people are treated individually to determine how the person fairs long term. Archer et al. (2014) introduce science capital showing that science capital is a relevant extension of capital, embodying cultural, social, and symbolic capital as science capital impacts a person’s financial outcomes. A year later, Archer et al. (2015) refined and
adapted Bourdieu’s work, to describe and quantify the term science capital. Within this, researchers use a three-part model, for means to consider each person individually and measure their levels of science capital to quantifiably verify whether this impacts their science achievement. These authors conclude by stating that science capital should not be conditioned; it should simply be understood so that the social structure in which science outcomes are so vastly divided can be better understood (Archer et al., 2015). For this paper, Bourdieu and Archer et al. interact to develop the four-part model to begin to describe mathematics capital. Mathematics capital’s four-part model includes the layer of mathematics mindset that is unique to mathematics education literature and developed by Jo Boaler (2015). The conceptualized four-part model is based on mathematics mindset, mathematics teaching and learning experiences, mathematics self-efficacy, and cultural experiences where each part of the model impacts a person’s mathematics capital.

Figure 1

The Conceptualized Model

| mathematics mindset | mathematics teaching and learning experiences | mathematics self-efficacy | cultural experiences | mathematics capital |

Mathematics Mindset

Jo Boaler studies high school mathematics while comparing the outcomes of males to females. Within this, she highlights the authoritative approach to mathematics teaching and learning in their setting, explaining that it impedes the high-achieving females from learning and that it did not for the males. This author closes by differentiating between mathematics and
school mathematics, stating that the beauty within mathematics is often eliminated as it is introduced in schools (Boaler, 1997).

So then, mathematics teaching has centered around a set of checklists and not on individualized understanding or connection to mathematics, as many subjects are taught, as described by Boaler (1997), creating a noticeable divide between the act of learning and what we define as mathematics in K-12. Brown (2010) continues to establish that mathematics is one of the only subjects that has not adapted to cultural changes; instead, the standards and content today is very much the same as what it was many years ago. He finds value in the fluidity of ideas taught as it establishes relevance to current events.

**Mathematics Teaching and Learning Experiences**

In 2014 the National Council of Teachers of Mathematics (NCTM) released a publication to specify recommendations for mathematics teaching and learning with six guiding principles for school mathematics and eight recommended mathematical practices for mathematics teachers. This guide explicitly details recommended teacher and student actions within the eight practices and research analyzed and executed mathematical tasks (NCTM, 2014). This publication followed NCTM’s 2000 book, where NCTM leads with six other, slightly different guiding principles. However, NCTM focuses here on the content and five process standards, what a mathematics learner should do while doing mathematics, to describe details of high-quality mathematics education (NCTM, 2000). Jo Boaler works reverently to research mathematics mindset and the impact that mindset has on mathematics (2015a). Boaler’s research extends to understanding how the brain works, specifically how synapses fire while working with mathematics, as evidence that all students can learn mathematics. Modifying the learning environment offers more opportunities for all students and publishes many examples of this work in action (Boaler, 2015b, 2019).
Mathematics teaching and learning experiences, embodied by cultural and economic capital (Bourdieu, 1986), will be defined as, but are not limited to: (a) a person’s school-aged experiences in mathematics learning via courses took toward graduation; (b) a person’s experiences in STEM courses that specifically teach mathematics content; (c) a person’s workplace experiences, while in school, where their position requires learning of mathematics or continued use of mathematics; (d) a child’s experiences with mathematics prior to attending formal schooling, in the home, in daycare or in informal preschool; (e) an adult’s experiences after leaving formal education; (f) a person’s workplace experiences, after leaving formal education, where their position requires learning of mathematics or continued use of mathematics; and (g) a person’s time devoted to strengthening mathematics skills in the way of homework practice, tutor, parental support.

Mathematics Self-Efficacy

Self-efficacy (i.e., a person’s belief or perception of their ability to complete specific tasks) through a Bourdieusian lens, connects the concentric spheres of social influence in Bourdieu’s theory of social capital (Bandura, 1977, 2002; Bourdieu, 1986). The synthesis of these two theories places the individual as the center most sphere. Within the primary sphere exists the person and their interactions with the various components that form Bandura’s theories of self-efficacy and one’s own sense of self (1997). The learners’ experiences form through practice and observation, mastery and vicarious experiences, and with formal instruction in school. Within social interactions, learners access resources available to them to support their learning.

Mathematics self-efficacy, embodied within Bandura’s theory of self-efficacy and Bourdieu’s theory of social capital, will be defined as, but is not limited to: (a) a person’s belief in themselves as mathematics learners, in and out of mathematics classes; (b) a person’s agency as it relates to mathematics and (c) a person’s identity in mathematics.
Cultural Experiences

Cultural capital can influence a child’s success in education. Sella & Kadosh (2018) question the biological, cognitive, motivational, and external factors for individuals who enter STEM fields using their cultural capital as a descriptor. Other researchers use empirical evidence to show that school transfers can impact social capital, increasing the likelihood that a student will drop out of school early (Teachman et al., 1996). Sella & Kadosh’s views parallel Turnbull et al. (2019) when they show that physics, the whitest, male-dominated STEM field, continues to increase the attrition of women which perpetuates the viewpoint that physics is a white, male-dominated field.

Archer et al., consider the crossover of cultural capital to science. Using Bourdieu’s preliminary thoughts from 2005 where science capital is first termed, these authors investigate:

- scientific forms of cultural capital (scientific literacy; science dispositions, symbolic forms of knowledge about the transferability of science qualifications),
- science-related behaviors and practices (e.g., science media consumption; visiting informal science learning environments, such as science museums),
- science-related forms of social capital (e.g., parental scientific knowledge; talking to others about science). (Archer et al., 2015, p. 929).

Cultural experiences, embodied within cultural capital (Bourdieu, 1986), will be defined as, but are not limited to: (a) a person’s economic background; (b) a person’s voluntary social activities; (c) the value placed on mathematics and on education as a whole in the home; (d) the value placed on mathematics and on education as a whole in the person’s social circle(s); (e) a person’s non-STEM educational experiences; (f) a person’s experience from the viewpoint of their race or ethnicity; (g) the influence of a person’s self-identified cultural group’s value of
mathematics; and (h) a person’s chosen role models and how these role models view mathematics.

Rationale

Immanent regularities suggest constants across all subjects that should be considered as we develop, criticize, and synthesize ideas. The goal of this research is to learn more, yet to learn it in a way that provides consistent, predictable understandings to either continue a discovered path of opportunity or to make corrections where paths of impossibility appear. And if there was a constant that existed to predict an individual’s likelihood for success, wouldn’t this be of interest? Furthermore, what if this constant existed in every form of success that a person could have? Not just social and financial, but for this work, what if it were mathematics?

Capital is a power that holds ultimate reign over every field (Bourdieu, 1977, 1986) where similar patterns emerge as evidence of capital, existing within all fields. Truthfully, mathematics does not necessarily exist self-contained (Lakoff & Núñez, 2000), but mathematics success and ideas are described and can be measured in a self-contained way in formal education, as it has been done. In that, consistency of nuanced containment exists such that mathematics capital is convertible, similar to the other forms of capital (Bourdieu, 1986).

If mathematics education is a field, then within this field, mathematics capital must be describable (Bourdieu, 1977, 1986). Acting exactly as social capital, higher amounts of social capital positively impact one’s ability to be successful in the social realm. It can be considered that higher levels of mathematics capital can positively impact a person’s mathematics education/outcomes. Social capital informs a person’s outcomes or successes in social spheres. Then, if we have adequately measured mathematics capital, mathematics capital should be directly related to mathematics success.
Coleman argues with sociologists who attempted to individually describe the economic, intellectual, and social organization as necessary to revise and reconsider a new framework that includes social capital. Struggling to tangible define social capital, Coleman instead describes social capital by its function. The function of social capital is not exclusive, instead inclusive and has a pair of common elements. These two elements are (a) aspects of social structures and (b) aspects of the actions of the actors within the social network (1988). Coleman continues by stating that “social capital is productive, making possible the achievement of certain ends that in its absence would not be possible” (Coleman, 1988, p. S98).

A continuous stream of information, including parenting strategies, floods families who actively engage in schools’ social networks. Pong (1998) recognizes that this stream creates a positive compounding effect for the students as parents gain knowledge and resources. College-educated parents continue to offer many different advantages to their children. For example, college-educated parents understand the collegiate experience as they experienced it, versus their counterparts do not, as they did not experience college. Godwin et al. (2015) conclude by noting the importance of further research on first-generation college students and the array of differences and experiences between first-generation college students and students who are not.

Social learning theory identifies learning through two pathways: direct experience and observing the behavior of others, where three regulatory processes, stimulus control and reinforcement control, influence a person’s choices. Bandura finds the nuances in action predictability as cognitive factors that always impact a person’s observations and feelings, which are influenced by past experiences. These nuances significantly impact a person’s learning (Bandura, 1977).
We learn from those around us, in what they do, say, and in their actions, founded by Bandura (1977) where either a person directly experiences something or observes a behavior; the brain works to process actions/words/reactions of others and to notice the environment. Learning by the example of others and with others offers us the ability to learn more with less. Mathematics researchers agree with this constructivist approach to learning by many researchers concluding that math learners must do the math themselves to understand mathematics as opposed to being told how to perform mathematics problems (Boaler & Greeno, 2000).

Writing from Williams & Choudry (2016) supports Archer et al. by clarifying the differences between Marx and Bourdieu where Bourdieu’s focus remains primarily with capital attached to a subject (e.g., social, cultural). Williams and Choudry establish a difference between the ability to apply mathematics learned in school as an adult to having been successful in mathematics to move the person through new, higher-level fields and employment positions (e.g., getting into an ivy league school requires high scores on the SAT but that student doesn’t necessarily need to remember the mathematics learned after the test has been taken).

**Research Questions**

The research questions evolve as the chapters progress. Each chapter contains a research question that will begin with: Is there an underlying, operationalizable phenomenon, *mathematics capital*, that has otherwise not been described, offsetting individual student outcomes in mathematics education? Then, each chapter will add a follow-up question, to the initial question to expose details and nuances. These follow-up questions are designed to allow the researcher to dig deeper as each chapter closes only to allow for another to open.

**Research Question One for Chapter 1: Introduction**
Is there an underlying, operationalizable phenomenon, *mathematics capital*, that has otherwise not been described, offsetting individual student outcomes in mathematics education?

**Research Question Two for Chapter 2: Proof of Concept**

Is there an underlying, operationalizable phenomenon, *mathematics capital*, that has otherwise not been described, offsetting individual student outcomes in mathematics education? Can *all mathematical interactions* be considered as one comprehensive unit that influences the learner?

Seeking an emerging concept such as mathematics capital calls for innovation through a thorough historical review. In this article, a comprehensive examination of publications in mathematics education are required so that the field of mathematics education can be reviewed. This review is to find emergent, cross cutting themes to see if it is possible for mathematics capital to be defined, and even more, explained.

**Research Question Three for Chapter 3: A Qualitative Approach**

Is there an underlying, operationalizable phenomenon, *mathematics capital*, that has otherwise not been described, offsetting individual student outcomes in mathematics education? Is mathematics capital observable within stories of people who have been successful in mathematics education such that these stories can be used to validate and inform survey items to measure mathematics capital?

After potentially finding mathematics capital in literature, asking people, qualitatively, about their lives so that their experiences and perspectives can be understood will improve the overarching understanding of mathematics capital.

**Research Question Four for Chapter 4: A Structural Equation Modeling (SEM) Approach**


Is there an underlying, operationalizable phenomenon, *mathematics capital*, that has otherwise not been described, offsetting individual student outcomes in mathematics education? Can mathematics capital be created as a variable and shown that it is distinctively different from other variables?

Another measurable component of this research and important step is to make sure that mathematics capital can exist by being able to quantify it. Without the ability to measure it, it would be difficult to argue that *mathematics capital* fluidly exchanges with other forms of capital and that mathematics capital exists.

**Research Question for Chapter 5: Concluding Thoughts**

Chapter 5 stands without a new research question. Instead, chapter 5, acts to synthesize ideas across all the chapters and to answer the research question introduced in this chapter, chapter 1 and carried through all the chapters: Is there an underlying, operationalizable phenomenon, *mathematics capital*, that has otherwise not been described, offsetting individual student outcomes in mathematics education?

**An Overview for the Remainder of the Study**

The remainder of the study will include three sequential chapters, written as articles, and a final chapter, Concluding Thoughts. The three articles will each exist individually, yet the ladder cannot exist without its antecedent. The final chapter will connect all of the ideas and offer a personal perspective on the study.

**Chapter 2: Proof of Concept, Mathematics Capital in Mathematics Education Documents**

Research routinely has the following structure: (a) a thorough examination of the literature and contributions made to date. The reason for this examination is to clearly vet the field in which the study is founded in so that all previous work can be considered in the presence of new ideas. We understand academia under the philosophy that it is and or instead of yes and
Most ideas are accepted that are clearly defined, the perspective in which we are taking is well explained, and we have thoroughly reviewed what has been said before. Thus, this work begins with a thorough review of the guiding documents of mathematics education. It is not possible to collect every research article. Instead, it is possible to review most and then consider the works thematically. Here, if a structure under which most mathematics education research can be described is found, then this will be present. This will act as chapter 2 where proof of concept will be established.

**Chapter 3: Theorizing Mathematics Capital, A Qualitative Approach**

The next step in research is to decide on a method by which the study will happen. This method is flexible depending on the type of research that we seek to explore. Still, our approach must be specific, detailed, and structured. The approach itself should follow pattern-like behavior so that we can check for accuracy and the interpretation of the results. Gathering data from participants will act to verify and update (if necessary) the findings in the literature from the preceding chapter. A self-contained literature review will also exist in Chapter 3 of this study.

**Chapter 4: Mathematics Capital, A Structural Equation Modeling Approach**

Suppose that mathematics capital is considered as all other forms of capital. In this case, Bourdieu states that it must have some quantifiable impact on the people in the field of mathematics education (1977). Thus, a third study must be performed where the quantitative factors are examined for statistical significance. If no, statistically significant results emerge from the quantitative research, then it is difficult to build the argument that mathematics capital truly exists. Instead, it remains theoretical and a plausible gap in the literature. This third article will be a guide for a structural equation modeling approach to analyzing mathematics capital. The reason for this is that literature already finds portions of mathematics to be related to each
other. It instead does not find how each portion is interrelated and how each portion plausibly, linearly, informs the next. For example, with a structural model, it can be shown that a first independent variable informs one dependent variable that then becomes another informant of a final dependent variable. This type of analysis provides specific and interrelated information if the information is there to describe. Chapter 4 is also self-contained, thus more specific literature will be highlighted here.

**Chapter 5: Concluding Thoughts**

In the concluding thoughts, the evolution of the concept of mathematics capital across the chapters will be discussed. Here, one final, operational definition of mathematics capital will be presented. More, it will show the current state of mathematics capital and how this flexibility interchanges with financial capital. This important discussion relates to Bourdieu’s emphasis on the importance of the interchangeability of capital with other forms of capital. This chapter will show how mathematics capital is interchangeable with other forms of capital to strengthen the validity and necessity of the term mathematics capital being added to the field, highlighting the researcher as the instrument of this study to contextualize mathematics capital and to showcase the power of mathematics capital in an individual’s life.

This chapter will embed a personal narrative, highlighting pieces of my story as these relate to mathematics education. This story will detail personal challenges alongside the successes because of mathematics. The goal of sharing this story is to contextualize the power of mathematics capital and how mathematics capital can impact a person’s life.

**Conclusion and Implications**

If someone is both arithmetically competent and a master of extensive mathematical knowledge, then does this make them have a math mind? Stated another way, does
mathematical competence combined with mathematical expertise mean that a person has natural mathematical ability, or could it instead be the simple result of extensive time spent with mathematics? Sella & Kadosh’s (2018) findings lead to more questions on investigating whether cognitive differences exist between math experts and non-experts. Specifically, these researchers notice that the brain fires in the same way as math experts and non-experts.

And yet, the comments like, “I’m not a math person” still stampede our culture and classrooms (Hachey, 2009; Kimball & Smith, 2014; Willingham, 2009). Research shows that those who carry on in mathematics are not always the young children identified early on as mathematically gifted. Instead, students who persevere through high-level mathematics courses, and continue in a career in STEM, are self-regulated learners carrying a specific set of epistemic beliefs and these are those who sustain in mathematics. This author specifically identifies and profiles high-achieving mathematics students and yet, the question of context remains at bay for the author, wondering if the student profile would differ depending on the function of the domain in which the profile exists (Muis, 2008).

So then, consider exploring what early life indicators predict a student’s early life academic success. When this is unpacked, we instead notice not that intelligence, as difficult as it is to measure, is routinely not able to be disentangled from patterns of behavior. In fact, in educational psychology, a child’s ability to regulate themselves can either impede or expedite a child’s ability to learn. Self-regulation plays a critical role in the child’s ability to utilize their working memory (Shing et al., 2010). Even more, executive functions and attentional processes where both a child’s ability to pay attention and their ability to control themselves overlaps so vastly that they are measured together, as an umbrella term, in some studies (Poutanen et al., 2016), where executive functioning is a set of cognitive processes (e.g., inhibition, working memory) and attentional processes are related to a person’s ability to orient themselves.
Previous math success is a predictor of current and future mathematics outcomes, alongside the importance of preparedness in mathematics and mathematics success. So much so that early school success in understanding fractions and whole numbers outweighs and predicts a student’s success in mathematics over the intelligence quotient, socioeconomic status, and parental education (Siegler et al., 2012). Basque and Bouchamma study Canadian eighth graders alongside teachers and administrators to find two very specific things. The first is that by eighth grade, 50% of the variance that exists between students is determined by a student’s mathematical success in fifth grade. This supersedes other predictors like socioeconomic status and parents’ academic achievement. These authors close by stating the importance of early intervention for schools (2016).

Math readiness is a widely used term for students at many grade levels and is found to be the strongest predictor of success in mathematics courses (Li et al., 2013). Quality preparedness for collegiate mathematics remains a focal point of mathematics education research, especially due to the lack of success connected to remedial math courses in college (Xu & Dagar, 2018). Further, even when obstacles like remedial coursework to take college-level mathematics are removed, students who meet all necessary academic requirements and are able to begin in their college mathematics courses are more successful (Atuahene & Russell, 2016).

Lastly, common misconceptions in mathematics could become a topic of conversation as student mistakes made in mathematics rarely change; instead, the same errors are repeated at each level. In research, we find the topic of fractions studied at almost every grade level in K-16 (Clements & Sarama, 2004; Fuchs et al., 2016; Jordan et al., 2016; Li & Kulm, 2008). What does this mean if prior success is a predictor of future success and even more, all the same mistakes continue to be made from preschool through college?
Organization of the Remainder of the Study

The structure of this dissertation will not follow a standard, five-chapter dissertation. The first chapter included content descriptors and method explanations of what is to come in future chapters. In essence, chapter one in this dissertation parallels, although not closely, a standard dissertation by including content that would routinely be in chapter one and in chapter three. Similarly, chapter two in this dissertation is a document analysis that acts as proof of concept. This means that chapter two will serve as the literature framework for the study and as the rationale for the altered structure. Thus, chapter two in this dissertation includes components of chapters one, two, and three. In this, chapters one and two of this dissertation mirror more closely chapters one, two, and three of a standard dissertation.

Chapter three is a qualitative analysis where data was collected and reviewed. Results from this research is presented and analyzed. Then, these results are connected to the findings from chapters one and two. Chapter four is the research proposal for a lengthy structural equation modeling approach to mathematics capital. Lastly chapter five acts to connect all the chapters via personal narrative and closes with a call for more research on mathematics capital.
References for Chapter 1


Chapter 2: Proof of Concept, Mathematics Capital in Mathematics Education Documents

Abstract

Mathematics education continues to be heavily researched with the goal of supporting every learner. Now more than ever, a movement towards justice in education, serving all students, is being revealed as central to educational philosophy to positively influence learning and the learning experience (Safir & Dugan, 2021). Bourdieu captures the essence of holistic social conduct in many of his works by reimagining reality through capital, field, habitus, and doxa. Thus, a thorough examination of the literature was performed, through Bourdieu’s definition of capital (1977), framing this article from a Bourdieusian lens. Findings suggest a call for more research on the general existence of mathematics capital as it emerges, and more research needs to be done to ascertain the existence of mathematics capital.

Keywords: Bourdieusian, document analysis, mathematics education, capital, field, habitus, doxa.
Chapter 2: Proof of Concept, Mathematics Capital in Mathematics Education Documents

Many mathematics researchers have found attributes, personality, race, economic background, and more as reasons for influencing a student’s success in mathematics (Lee et al., 2020; Montague et al., 1991; Smith & Schumacher, 2006). These preceding studies centralize on one or two ideas and evaluate either students’ abilities or students’ deficits, addressing each item in relative isolation to try to impact mathematics education. As researchers continue to identify individualized items of mathematics learners’ success, the full picture may not be present. For example, both Smith and Schumacher (2006) and Montague and colleagues (1991) exclusively research attributes and cognitive differences of mathematics learners. Then, Lee and colleagues (2020) consider racial microaggressions and how these impact student achievement in mathematics. Others find that some concepts overlap and impact each other greatly, so much so that a body of research can be found connecting self-efficacy and mathematics achievement (Ayotola & Adedeji, 2009; Bonne & Johnston, 2015; Fast et al., 2010). This wide range of researchable topics in mathematics education calls to question what types of delineated topics could plausibly be excluded or included in mathematics education research without implications/limitations/delimitations forwardly mentioned.

The National Council of Teachers of Mathematics (NCTM) has published many texts. Most recently, the guiding text for mathematics teaching and learning is Principles to Action (2014) which is the overarching guide for effective mathematics instruction. This thorough review will serve as an attempt to consider mathematics education holistically, including mathematics teaching and learning. Further, framing will begin to describe mathematics capital, similar to Bourdieu, and Archer and colleagues (1977, 2014). Bourdieu’s writings on capital, field, habitus and doxa serves as the overarching theoretical lens from which this study is examined (Bourdieu, 1977, 1984, 1986, 1990). Then, Archer et al. 's (2014) relevant extension of
Bourdieu’s theories, introduces science capital. Science capital embodies cultural, social and symbolic capital and is defined and quantified by Archer et al.’s 2015 publication.

Problem Statement

This researcher aims to determine whether a collection and cross-analysis of documents in mathematics education will show evidence of mathematics capital by using Bourdieu and Archer et al.’s works, detailed above (Bourdieu, 1977, 1984, 1986, 1990; Archer et al., 2014; Archer et al., 2015). Mathematics education publications will be reviewed, beginning with texts from NCTM, then primary books from authors, and lastly some secondary books from authors.

Research Question

Is there an underlying, operationalizable phenomenon, mathematics capital, that has otherwise not been described, offsetting individual student outcomes in mathematics education? Can all mathematical interactions be considered as one comprehensive unit that influences the learner?

Understanding the Research Question

Consider this paralleled analogy to make sense of what is meant by all mathematical interactions as one comprehensive unit. In recent years, medical research on the human body has expanded to include the connective tissues named fascia or fascial tissues. This tissue was previously discredited. Researchers of the human body would perform dissections by cutting through the fascial tissues to get to the muscle and the other body components that were thought to be the more important. Now, research on the fascial tissues has become common due to the nature of unexplained back pain and more. Researchers began to take a closer look at the tissue that had previously been ignored with many questions about the human body being left unanswered. To their dismay, the fascial tissues seem to be answering questions that otherwise could not be answered with research on the other components of the body. Back pain
for example, as researchers test the fascial tissues in comparison to the back muscle, skeleton, and other structures, has been found to be localized in the fascia. Even more, the facial tissues have been revealed as completely interconnected and are “part of a body wide tensional force transmission system” (Schleip et al., 2012). This discovery is slowly completely changing the nature in which the human body is being researched (Adstruma et al., 2017; Chen1, May 21, 2019, 0-42mins). This discovery will not minimize previous research on the body, instead, it will add to the body of research and will guide future research on the human body. Similarly, *mathematics capital* will not minimize previous research in mathematics education, it will only add and guide the body of future research in mathematics education.

**Method**

**Bourdieu**

Through a Bourdieusian lens, it can be argued that researchers need to do more than investigate one concept and find how a few concepts interact. This perspective allows researchers to evaluate all parts as intertwined, to gather rich and impactful data, improving mathematics teaching and learning. *Mathematics capital* will be described using a four-part model, similar (but with some intentional differences) to Archer et al.’s (2015) three-part model of *science capital*.

Bourdieu claims that within every understandable container (i.e., system, structure, community), there exists capital, field, habitus, and doxa. Even more, there are multiple forms of capital, each of which can be described yet all of them convert in some way to financial capital (1986). Symbolic capital, for example, is sometimes referred to as capital of honor, attached to a family and prestige. This form of capital is readily convertible to financial capital. Depending on the season of life or time, it can be the most valuable form of capital. This speaks
to the transmutation of capital, the flexible value of capital, the relative values of different types of capital, and the depreciation of capital (1977).

**Operational Definitions**

**What is Capital?**

Bourdieu establishes that nothing is accidental, instead, everything in the social world appears in patterns of inherent design. Always named in conjunction with a field, capital, as he saw it in 1986, was masked in three ways: economic, cultural, and social. For Bourdieu, the mask titles: economic, cultural, and social, are all in relation to how efficiently each form can transform and produce benefits in the field of question (1986). The search to describe and provide details on these patterns was often the backdrop of Bourdieu's research. Some of these details include defining habitus, field, and doxa and how each of these relate to capital. Capital works in relation to a field and is strengthened through doxa and habitus (Bourdieu, 1977, 1984, 1990). Capital exerts ultimate power over the *field*, existing within each structure and acting as an overarching force, establishing acceptable regularities. Even more, capital reigns over any profits attached to the *field*, whether it be financial, cultural, educational, or other (Bourdieu & Wacquant, 1992).

**What is a Field?**

A field is any well-defined subject existing in self-containment and is integral to exploring capital. Without a field, there is no viable way to describe capital (Bourdieu & Wacquant, 1992). Clarification must be made to highlight capital's connection to culture, society, and economics (Bourdieu, 1984). Still, these connections that capital has to each only exist if there is a primary connection made to a well-defined, self-contained system. In Bourdieu's words, this is called a *field* (1977, 1984, 2005). For this work, the referenced *field* of this study will be defined as mathematics education, and all that encompasses it.
What is Habitus?

Bourdieu (1977) said it likely most eloquently when he defined habitus as:

Systems of durable, transposable dispositions, structured structures predisposed to function as structuring structures, that is, as principles of the generation and structuring of practices and representations which can be objectively “regulated” and “regular” without in any way being the product of obedience to rules, objectively adapted to their goals without presupposing a conscious aiming at ends or an express mastery of the operations necessary to attain them and, being all this, collectively orchestrated without being the product of the orchestrating action of a conductor. (Bourdieu, 1977, p. 72)

To understand Bourdieu’s definition of habitus, ideas can be delineated and rephased in the following ways:

1. Habitus is produced by a specific type of environment.
2. Habitus then is not fleeting, instead, it is sustaining in durability.
3. Habitus is fixed, preset, and designed to work systematically, reproducing the patterns that habitus first creates.
4. Habitus is the accepted ways of conduct to a generation that are not reinforced in any way.
5. Habitus is a function of each person in the group without any such person in the group holding the functions accountable.

Connecting habitus to present-day terminology, habitus is the systemic underpinnings of government, education, and policy that are not questioned yet still act inherently by a set of guidelines and rules between each of the people that operate within the government, education, and policy. For this work, habitus is operationalized and defined in all that is explained above, without the exclusion of any details.
What is Doxa?

Doxa can be explained as a relevant extension or reproduction of habitus through a cycle of agents, in the absence of intrigue it is strengthened, through group-accepted structures (collective understandings), and through individually, internalized structures (individual understandings). But doxa is not fully explained with only this sentence. Instead, elaborating, while habitus refers to the acceptable conduct of each person within a group (or field), doxa is more than this in that it is “the belief of a whole group in what the group believes” (Bourdieu, 1977, p.164). It is the construct of reality within a group’s understanding of what exists. It is “the agents’ aspirations have the same limits as the objective conditions of which they are the product” (Bourdieu, 1977, p.166). In other words, doxa is setting a dream, fulfilling that dream, and then deciding that because of being within the dream, there must be nothing other than what was initially dreamt.

The Definitions Applied

All told, a Bourdieusian lens for mathematics education exists with the following terms embedded within mathematics education: capital, field, habitus, and doxa. Capital in mathematics education is termed mathematics capital where mathematics capital is the overarching power existing within mathematics education, reproducing itself within the field and flexibly interchanging in forms of capital (Bourdieu, 1986). This is a power that inherently, continuously, and subjectively alters people’s outcomes/trajectories in the field. Matching Bourdieu (1977), and Fayer and colleagues (2017), mathematics capital is flexibly interchanged with financial capital through the continual reproduction of higher income levels in Science, Technology, Engineering, and Mathematics (STEM) fields.

The field is a describable “unit” in which people exist. For example, a club, government, country, or team. In this case, mathematics education is the field, and mathematics education
includes mathematics teaching and learning, education as it relates specifically to mathematics, and any government department that makes decisions involving education as it specifically relates to mathematics.

Lastly, habitus and doxa are closely related but distinctly different. Habitus, as it relates to mathematics education, is all the accepted opinions, ideas, decisions, and perspectives on mathematics education that are not argued. An outside example of habitus can be interpersonal skills alongside intrapersonal skills. Doxa, however, is the entire set of beliefs about mathematics education that no one person can necessarily answer why to, instead it may not even be asked why, nor would any differing answers be given should the question of why be asked. Habitus is contained within doxa but doxa encompasses more than habitus. Doxa, as an applied example, might be our adherence to time as it is socially construct or the date on the calendar.

Doxa is the beginning of “common ground”, habitus is the executed actions both unintentional and intentional within the common ground. Mathematics capital is the positive reproduction of favorable habitus existing in doxa. The more any single player that aligns themselves within this set of beliefs or naturally falls on the reproductive end of this belief, the higher the player will be in the field. Attainment refers to a player’s positionality and ability to exchange results in the field to function in areas of their life and family. Although spoken on the reproductive end of each of these defined terms, there exists the ladder of the binary option, as arbitrary as each of these concepts is. Reasonably, if it is possible to have attainment then it is possible to have naught. This naught is viewed as a field threshold when choosing the negated binary for doxa and habitus. Sitting in the negated binary does not positively reproduce capital, instead, it yields nonfulfillment of capital, meeting thresholds. See figure one for a visual
representation of doxa, habitus, field, and capital as explained. Lastly, it should be noted that the final arrow that indicates “no threshold” represents the cycle repeating itself.

**Figure 1**

*Capital, Field, Habitus and Doxa Transposed*

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**Literature Framework**

**NCTM Publications**

*Principles to Action*

Principles to Action identifies six guiding principles for school mathematics: teaching and learning, access and equity, curriculum, tools and technology, assessment, and professionalism. Within these six, each guiding principle carries relative importance and intentional guidance for mathematics educators.
**Teaching and Learning.** Mathematics teaching and learning captures a large portion of the work in Principles to Action, although not all. Specific attention highlighting the aspects of effective, equitable, and research-based best practice will be described below.

**The Five Interrelated Strands.** The five interrelated strands that constitute mathematical proficiency are (a) conceptual understanding, (b) procedural fluency, (c) strategic competency, (d) adaptive reasoning, and (e) productive disposition. These five strands will be discussed below in the National Research Council (NCR) (2001) publication.

**The Eight Mathematics Teaching Practices.** The eight mathematics teaching practices to guide instruction are listed below with a paragraph detailing each.

1. The first is to *establish mathematics goals to focus learning.* Teachers need to create clear goals to focus on student learning which provides students with the opportunity to master rigorous standards such as the Common Core State Standards for Mathematics. Elementary school mathematics educators, Clements and Sarama (2004) say that students and teachers must be able to answer four crucial questions: What mathematics is being learned? Why is it important? How does it relate to what has already been taught? Where are these mathematical ideas going? (NCTM, 2014, p. 13). The mathematics being learned needs to be clear so that the students know exactly what they are learning. These goals need not be written in daily task form and can instead be clarified with an essential question or overarching concept. A result of adequate planning with specified goals increases the teacher-to-teacher communication. This increases the foresight that teachers will have into common student misconceptions.

2. The second is to *implement tasks that promote reasoning and problem-solving.* There are three major findings in a mathematical task that can be summarized with the following statements: (a) tasks vary in opportunities provided for student thinking and
learning (Hiebert et al., 1997; Stein et al., 2009), (b) tasks should require high-level thinking to promote student learning, and (c) it is a challenge to properly implement a high-level task without transforming it into a task with less cognitive demand. The specifics behind this task selection refer to the foundational understandings of culturally relevant pedagogical decision-making as authors state context, culture, conditions, language, and experiences as ways to frame mathematical tasks (Brown-Jeffy & Cooper, 2011; Cross et al., 2012). High-demand tasks require the doing of mathematics. Doing mathematics has been described as the patterns of science, exhibiting mathematical habits of mind, or perhaps an act of sense-making (Hoffman & Even, 2021; Schoenfeld, 1994). Each of these calls for discernment and specification for mathematics in relation to the process of mathematics and in what is hoped to be accomplished within the study of mathematics.

3. The third is to use and connect mathematical representations. This mathematics teaching practice is founded in NCTM’s 2000 publication that details mathematical representations as a crucial feature of teaching and learning mathematics. Specifically references the process standards, the practices detail the importance of using diagrams while working with mathematics and the connections that must be made through these processes. More than diagrams, to use and connect mathematical representations is extended to include visual, symbolic, verbal, contextual, and physical representations citing that depth of understanding is directly related to the strength of the connections between these representations.

While visual representations are highlighted as they act in these ways to enhance student learning: (a) helping students advance their understanding of both concepts and procedures, (b) making sense of the problem, and (c) engaging in
mathematical discourse. These visual representations serve marginalized learners, specifically multilingual learners, students with special needs, and any other students who may struggle without visual aids. The reason for this deepened learning lies in the recognition and understanding of the structure of mathematics as the representations are compared (Zimba, 2011). Problem-solving, a crucial skill in mathematics, is related to a student's ability to recognize the transient nature of mathematical representations, thus moving fluidly between mathematical structures and representations.

Three strategies are highlighted to encourage students to develop representational competence: (a) purposeful selection, (b) dialogue highlighting the connections between representations, and (c) explicit reciprocal directionality between connections of representations.

4. The fourth is to facilitate meaningful mathematical discourse. Facilitating discourse in the mathematics classroom offers unique opportunities to students. Specifically, it allows students to speak up and contribute their ideas about mathematics in their classroom, and it allows students to make sense of their understanding as the student begins to speak. It requires students to present their whys and hows in mathematics, develop and express language of mathematical ideas and see mathematics from an array of different perspectives (NCTM, 2000). Through learning mathematics, an adequate amount of conceptual understanding and procedural fluency is required for continued advancement in mathematics. Discourse develops conceptual understanding more predominantly than any other activity in the mathematics classroom. Discourse also increases meaning in mathematics.

Although discourse is essential for conceptual understanding, there are some common pitfalls within classroom discourse to avoid. These are developed through
Smith and Stein’s five practices for orchestrating productive mathematics discussions, which will be discussed later in this chapter. Namely, the five practices are: (a) anticipating what students will do and what strategies they will use in solving a problem, (b) monitoring their work as they approach the problem in class, (c) selecting students whose strategies are worth discussing in class (d) sequencing those students' presentations to maximize their potential to increase students' learning, and (e) connecting the strategies and ideas in a way that helps students understand the mathematics learned (Smith & Stein, 2018).

5. The fifth is to pose purposeful questions. The type of questions asked in the mathematics classroom can significantly impact the richness of student discussion. Effective teachers ask questions to clarify student understanding, make changes to their lesson structure, increase the action of making mathematical connections, and facilitate students to ask their mathematical questions. Questions are designed to make sense of mathematics and to advance student reasoning. Four types of questions have been adapted from frameworks in principles to action. These four question types are: (a) gathering information, (b) probing thinking, (c) making mathematics visible, and (d) encouraging reflection and justification. While the type of questions are critical, it is also notable that the pattern in which the questions are asked is just as important. Namely, some of these patterns are Initiate-Response-Evaluate (I-R-E) (Mehan, 1979), funneling, and focusing (Herbel-Eisenmann & Breyfogle, 2005). These patterns either offer opportunities for learning or limit them. In the I-R-E pattern, student thinking is limited. Funneling pattern of questioning allows the teacher to initiate and execute their plan, although there is limited opportunity to negotiate student ideas/thoughts/opinions as they may or may not come up. Focusing pattern of questioning is sophisticated as it
accesses student thinking, encourages students to develop their thinking, requires them to activate metacognition, and offers an opportunity to evaluate the thoughts of their peers.

6. The sixth is to **build procedural fluency from conceptual understanding**. Conceptual understanding is the primitive skill developed in the mathematics learner. Then procedural fluency is mastered through flexibility. This happens due to the math learners' depth in conceptual understanding thus increasing their ability to recognize connections between forms. The connections increase a learner’s ability to navigate through common misconceptions; they can relate to and make connections between forms, allowing the learner to recognize their errors and quickly correct them. Another reason they correct errors is that mathematics and the values all have meaning through their conceptual development. Thus, the meaning behind the numerical values allows the learner to double back effortlessly to any procedural errors as the learner may ask themselves whether the answer is reasonable for the given situation.

7. The seventh is to **support productive struggle in learning mathematics**. The view of struggle is positive and an opportunity for growth through mathematics. Further, the emphasis is on the process when teachers support students through productive struggle instead of the correct answer. Teachers need to work with caution to avoid “rescuing” students, instead, the teacher can adequately prepare for student misconceptions. The prepared teacher in an environment that embraces productive struggle will be comfortable, allowing the student to solve the problem, possibly at a slow pace.

Other important facets of this practice include (a) The notion of mindset—which is discussed in detail below in Carol Dweck’s (2006) work, and (b) Self-efficacy and how it relates to student achievement. Lastly, NCTM encourages descriptive feedback in
mathematics teaching and learning so that students’ varied attempts are praised along with their thinking rather than the correct answer.

8. The eighth is to *elicit and use evidence of student thinking* which means to include student thinking in that a teacher’s real-time instruction may shift. Said differently, a teacher works to monitor a student’s mathematical progress by using the student’s thinking as a measure of formative assessment. This assessment is not left without action. In the act of “using” a teacher modifies their instruction continuously to provide extensionless student learning opportunities. Specifically, teachers monitor student thinking, find ways to collect student thinking, make decisions about the thinking and continue hereby responding to the student’s thinking and understanding (Jacobs et al., 2010). The timeliness of gathering data from students is crucial; it should not happen on removed days, instead, it should be ongoing, rigorously sought after, and frequent (Heritage, 2008; NCTM, 2014).

**Access and Equity.** Students need to be met through individualized needs. Equity is not equality. Instead, providing every student with whatever that student might need to be able to engage in high-quality mathematics is critical. Never lessening the curricula; instead, creating meaningful, rigorous opportunities to all in ways that support cultural differences, multi-language learners, historically marginalized/minoritized/underserved students, and more. Engaging solely in procedural fluency, or rote learning is one obstacle mentioned alongside tracking students that limits a teacher’s capacity to meet these above-mentioned learners. Tracking students includes, but is not limited to, placing students in courses that teach mathematics that do not develop the student for a future mathematics course.

**Curriculum.** Here the focus is on the process and practices identified in the standards for mathematical practice in the common core state standards. Mathematics curricula need to
be understood from a horizontal and vertical perspective. Mathematics should be considered by overarching ideas that carry the learner from kindergarten to twelfth grade as opposed to being taught delineated as checklists. Rushing through a pacing guide omits essential understandings that need to be developed in every learner (David & Greene, 2007).

**Tools and Technology.** Tools and technology serve similar purposes yet are very different. A tool in mathematics may be a hand-held manipulative, or it could also be an elaborate chart that visually displays mathematics in context, in another representation, or even compares representations. Updated, current technology is also critical in the classroom. Not only having access to the technology but also being taught how to use the technology to facilitate learning. This technology is not limited to what the students might do with it but instead also includes ways in which technology can be used to communicate with parents about the daily activities of the classroom. This engagement can provide additional critical layers of support to some students, enhancing their mathematical experiences.

**Assessment.** Assessment is not quizzes and tests. Instead, assessment is an opportunity to extend learning through connections and challenges. This piece ties closely to the eighth mathematics teaching practice, “elicit and use evidence of student thinking,” although it is more comprehensive than the eighth practice. Reconsidering assessment as such is a crucial component to effective teaching and learning.

The four functions of assessment are: (a) monitoring students’ progress to promote student learning, (b) making instructional decisions to modify instruction to facilitate student learning, (c) evaluating students’ achievement to summarize and report students’ demonstrated understanding at a particular moment in time, and (d) evaluating programs to make decisions about instructional programs. Some examples of assessments provided include having students explain concepts to others, and analyzing mathematical ideas. These assessments connect to
the five interrelated strands for mathematical proficiency, described below under the NRC (2001) publication.

**Professionalism.** Key ideas in this chapter include seeking, without isolation, collective efficacy, and teachers’ deepening their learning. Thus, professionalism within mathematics teaching and learning includes facets of all these and more. First, educators must continue to seek to level up their accomplishments in the classroom. Their lessons should never be jobs done; instead, the lessons can always act as opportunities to create more. To be without isolation means that teachers and educators will work in a culture of collaboration and mutual respect. Collective efficacy refers to the body of mathematics educators believing in the strength of each individual as a competent mathematician and mathematics educator. Finally, mathematics teachers and educators will seek to continually strengthen their understanding of mathematics and the pedagogy of mathematics teaching and learning.

**Catalyzing Change in High school Mathematics**

Equitable instruction is defined as (a) developing a positive mathematical identity, (b) having students develop mathematical agency, and (c) developing mathematical competence. Mathematical identity is defined as “the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives” (Martin, 2013, p.14). A positive mathematical identity means that people feel empowered by doing mathematics. They believe in themselves and see value in doing mathematics (NCTM, 2018). Mathematical agency is defined as students who engage positively in productive struggle, visibly illustrating their thinking, and can flexibly think about different approaches to mathematics. This flexibility allows students to use strategies and methods that work best for them. Agency in mathematics is also the way that a student will participate in the mathematics classroom. Categories of
mathematical competence can be interpreted as developing the essential categorial concepts in high school mathematics named (a) number, (b) algebra and geometry, (c) statistics and probability, and (d) geometry and measurement, where none of these are taught in isolation; instead, an enduring value should be retained through a student’s entire mathematics’ studies.

NCTM’s publications serve as the “foundation for mathematics education in the United States and Canada over the past three decades,” contributing to standards curricula, teacher professional development, and assessment (NCTM, 2018, p. xi). There are broader aims to high school mathematics than ever before. Ultimately, this publication is on improving instructional strategies in high school mathematics classrooms to actively engage learning, staking a claim at the deep-rooted difference that actively engaging students in mathematics does for each of these students.

A few chapters to highlight are as follows: Creating Equitable Structures and Implementing Equitable Instruction. Each of these discusses changes that need to be made to make mathematics accessible to more students. The first shines light on tracking and mathematics, which sends the message that some students are mathematically capable whereas others may not. This message is communicated by offering courses that do not prepare students for future work in mathematics. If the course does not prepare for future courses, then the course inherently provides a terminative quality in students’ capacities to learn mathematics. Unfortunately, this often happens to marginalized student groups (Boaler, 2011; Nasir, 2016). Noting the inclusion of teacher tracking in this chapter cannot be ignored, and that mathematics teachers in the same department should have a balanced course load with a mixture of upper-level and entry-level courses and/or changing teaching assignments every few years.
To provide equitable mathematics instruction means, but is not limited to, offering engaging mathematics that improves a student’s overall mathematical identity and mathematical agency. This immediately, and positively impacts student achievement. Much of this is due to the way a student will connect themselves to mathematics. When a strong and positive connection is made, a student sees themselves as capable in mathematics. Some students may see themselves as mathematicians. This positive self-portrait impacts their mathematics learning. The way a teacher teaches can create an equitable environment. An example shows the difference in questioning between two teachers, where a simple difference between a question can grow a student or impede a student (or many students) from growing with just one nuanced change in questioning. This nuance refers to reinforcing thinking in lieu of correctness in the way a teacher engages with student participation. This chapter also details the eight mathematics teaching practices by describing them as “equitable teaching practices” that develop identity, agency, and competence. These eight practices have been detailed above under NCTM’s principles to action publication (2014). Other highlights also include what teachers are doing in this chapter. Specifically, NCTM states that teachers must engage with each other collaboratively so that they are enacting the professionalism principle, highlighted in principles to action (and discussed above) (NCTM, 2018).

Although this is not all the content included in this rich publication, it highlights some of the most repetitive ideas. A quote that could not remain without mention was that right now, it is possible to “predict mathematics achievement and participation based solely on student characteristics such as race, class, ethnicity, sex, beliefs, and proficiency in the dominant language” (Gutierrez, 2002, p. 153).

*The 5 Practices for Orchestrating Productive Mathematics Discussions*
Anticipating what students will do and what strategies they will use in solving a problem. Teachers need to be prepared for what students might come up with in the classroom. Specifically, teachers may plan a list of anticipated questions, reactions, responses, contributions, or solution types that students may have. Planning ahead of time what students might come up with offers teachers the opportunity to prepare the math lesson so that the conversation continues to develop throughout the discussion as opposed to falling short. The richness in anticipation is the first step in ensuring a productive discussion ensues.

Monitoring their work as they approach the problem in class. Teachers should be circulating through the classroom as students are working on mathematics tasks or problems. As the teacher circulates, the teacher should be closely monitoring what the students are working on and how they are working. The teacher may enter and exit in conversations at tables, offering key suggestions to increase the level of cognitive demand in students, to monitor the level of struggle of the students, and to be certain that mathematics continues to be engaged in with all students. This act of monitoring is crucial that the teacher engages in not only to hold students accountable but also to ensure that conversation takes critical turns that the teacher has planned for.

Selecting students whose strategies are worth discussing in class. Selecting student work can be one of the most challenging steps along the way as it, along with the other practices that follow, that require real-time action on the part of the teacher. Monitoring, includes the teacher inserting themselves in group conversations, but selecting student work is different in that it is the first practice that requires the teacher to facilitate the classroom holistically in the discussion. The teacher needs to mindfully choose which works they will show and know exactly why each work was chosen.
Sequencing those students' presentations to maximize their potential to increase students' learning. The sequence cannot be ignored. The teacher needs to sequence their selections similarly to another teacher that may plan a direct instruction lesson. The teacher should consider how each work will build the overarching classroom understanding, being intentional with where they start and why. A routine pattern may be to start with a solution that appears most accessible but that is not the only way to start. The teacher needs to know why and how they have sequenced and be prepared to continue to facilitate discussion with that purpose in mind.

Connecting the strategies and ideas in a way that helps students understand the mathematics learned. This is the final action in the classroom: bringing all the pieces together. A teacher should work to make sense of the different types of solutions from the students and to build connections with and within students. The teacher works to highlight the thinking by comparing and contrasting among the students. The teacher should also include student voice in this step as it can likely increase student understanding when information comes directly from peers. The teacher continues to sustain original authorship in all the works from each student and builds each strategy as powerful, unique, and valuable.

NCR Publications

Adding It Up: Helping Children Learn Mathematics

Introduction. This book starts with, “Mathematics is one of humanity’s great achievements” (NCR, 2001, p.1). The authors continue almost setting up Su’s 2020 publication, Mathematics for Human Flourishing, detailed below, whereby they contest the beauty of deductive reasoning. The primary purpose of this report is to clearly establish the learning objectives within mathematics education for pre-kindergarten through eighth grade so that the teaching and learning of mathematics can then be decided from the learning objectives.
**The Five Strands for Mathematical Proficiency.** This publication highlights the five strands of mathematical proficiency in relative detail. First calling to action that the strands do not stand alone; instead, they work interconnectedly, interwoven, and in relation to each other. They should not be separated, instead, they should be considered in conjunction with each other. The strands create a framework for how educators can approach the teaching and learning of mathematics. This framework includes “knowledge, skills, abilities, and beliefs that constitute mathematical proficiency” (NCR, 2001, p. 116).

*Conceptual Understanding.* Students who understand mathematics conceptually have a holistic perspective of mathematics, monitoring their memory and making sense of mathematics. These students can work flexibly with different representations of mathematics. Strength in conceptual understanding works inversely with volume of computational errors. The stronger the conceptual understanding of a mathematics student, the fewer computational errors they may make.

*Procedural Fluency.* In this strand, students need to be able to perform operations with numbers flexibly. These operations, ideally, should be able to be performed by hand or even mentally. Students work with conceptual understanding to execute procedures. This duality allows students to not only be strong mathematicians procedurally speaking, but also increases their ability to estimate numbers and to make sense of their answers. In essence, this strand is not just being able to perform the operation asked but also to be able to perform the operation correctly and to be able to identify, in real-time, whether they have performed the operations correctly.

*Strategic Competence.* Formerly, a component of strategic competence would have been called problem-solving in mathematics. Now, strategic competence extends beyond problem-solving in that it also includes a student's ability to formulate a problem and to also,
within solving a problem, be able to represent the problem in multiple ways. Students begin to visualize problems as they solve them in the many ways the given situation is presented. The value of this approach increases a student's adaptability and flexibility. These two skills are critical in solving problems that require higher-level cognitive skills in mathematics. Strategic competence refers to all these facets of problem-solving, building, flexibility within representations, and fluidity in understanding mathematical situations.

**Adaptive Reasoning.** Considered the glue and lodestar of mathematics, adaptive reasoning comes with a strong reputation. A student must be able to make logical assumptions and understandings about mathematics. Adaptive reasoning refers to a student’s ability to generalize mathematics. This process is one method of inductive reasoning, which is another method for building mathematical arguments and proofs. One of the most important components in the practice of adaptive reasoning is for a student to be able to justify their work, or in other words, provide reason for their work. This alone is not enough. Adaptive reasoning must work in direct relationship to the other proficiency strands.

**Productive Disposition.** Here students see value in mathematics. Simply put, students have developed a sense of agency and identity within mathematics thus mathematics as a practice has usefulness to engage in. Productive disposition, like the other strands, cannot stand alone, instead it works in conjunction with the others. Students must see themself as able and flexible in their mathematical competence and not that their mathematical achievement is fixed, similar to Boaler and Dweck's works on growth mindsets, discussed below. Here students are excited about mathematics and are comfortable making mistakes through the journey of mathematical problem-solving. A student with a productive disposition is confident and sees mathematics as something that makes sense.
Other Considerations. “Mathematical proficiency goes beyond being able to understand, compute, solve, and reason” (NCR, 2001, p.133). Instead, students need to be dispositionally connected in a personal way towards mathematics. Students need to believe in mathematics and in themselves within the practice of mathematics. Each of the strands is integral and intertwined in making this happen.

Books with Two or Less Authors

Street Data

Safir and Dugan (2021) discuss empowering students to be involved in their learning and to learn things in their own way, on their own pace which can be achieved by allowing the students to learn from, and talk to, each other. Authors discuss the roots of street data and the epistemological basis that it has with an overwhelming connection to teaching and education as a practice. Educators are encouraged to avoid the common equity tropes as we attempt to do what is best for all students, schools, and communities. Antiracism and efforts to dismantle it are also discussed so that educators can become advocates and activists. The approach to learning is holistic, looking at the whole child and valuing and using their experiences as a resource, and developing skills from there. Students’ voice is important in street data practice, calling for more of their voices in the classroom and in their educational experiences.

Other ideas include community-based learning with an emphasis on relationships and beginning to develop an understanding of each student’s lived experience as strengths. This is the practice of “taking a step back” and “getting out of control” by giving your students autonomy and leadership in the classroom. The authors also detail the importance of classroom seating arrangements and how these impact student learning. For example, creating a circle of desks can make conversations more personal, creating complex, meaningful interactions that may go in directions that teachers were not expecting.
Street data thrives to capture data that a test could not capture, emphasizing the whole student rather than just their academic or "work" value. Street data is different from other forms of data because it focuses on the people from whom the data comes. Data is gathered from the street, listening to individuals' voices and actions, especially from those that often are silenced. Using data from the street, we strive for equity by researching and revealing the deep-rooted seeds of the injustices. Different learners superseded those who may learn traditionally, similar to how the varying types of cultures are welcomed into schools in the United States.

Ensuring equity within your classroom is vital. This does not mean lecturing at students, instead, to the authors, this means finding agency and identity within each classroom. Both inherently offer a feeling of value, respect, belonging, and importance to the student. The author emphasizes that what matters most in education is the human experience. This inclusive experience needs to be unapologetically rooted in equity embracing the pedagogy of student voice and developing agency in all stakeholders.

**Mathematical Mindset**

This book captures the essence of Boaler’s work through the publishing year of 2015. Jo Boaler works to expose the plasticity of the human brain and what it means for students and all people who want to learn mathematics. Building on Dweck’s work on mindset, holistically, Boaler highlights what is specifically different and perhaps unique in the practice of mathematics. Thematically, Boaler includes mistakes, engaging in mathematical work (calling back Gladwell’s 2011 publication, Outliers), teaching through rich mathematical tasks, the importance of all students engaging in high-level mathematics (consistent with Principles to Action), eliminating tracking (consistent with Principles to Action and Catalyzing Change), dynamic assessment, including self-assessment and to teach in ways to develop growth mindsets within children. This publication acts as a connector for many others and precedes the
publication date of some others. In essence, Boaler connects, recursively, many ideas and possibilities for mathematics education. These ideas have since been extrapolated, and refined, by being called required by some mathematics educators (Su, 2020).

**Accessible Mathematics**

A book of practice, Leinwand focuses on key instructional changes that classroom teachers can make to improve student learning. Some examples include starting class with ongoing cumulative review in every lesson. He keeps instructions simple and accessible to all teachers. Key concepts include teaching mathematics as connected, including language in the classroom, and adding questions like “How do you know?” or “Why do you think that?” to engage students with higher-level cognitive tasks. Leinwand also pushes back on some more traditional practices such as lengthy homework assignments and instead suggests abbreviated homework assignments that include an array of problem styles. Throughout his book, he includes a “so what” box asking “So what should we see in an effective mathematics classroom” where he challenges teachers to push past norms in each of the chapters (Leinwand, 2009).

**Mindset**

Dweck reiterates the value of a person’s mindset in her book, Mindset, where Dweck differentiates two mindset forms: (a) fixed, and (b) growth. Each of these carries a specific set of epistemic beliefs. The first, a growth mindset, does not find permanent barriers. Instead, a person with a growth mindset will be solution-oriented and find a way, whether that path is or is not clear. The second, a fixed mindset, suggests that a person will not be malleable, instead what they have now is what they will always have, independent of what may lay ahead.

Growth-minded individuals believe that natural talents, skills, and abilities are merely a portion of what brings success. Other portions of success include their dispositions toward failure; failure is a tool for success and is not viewed negatively. One of the most interesting
aspects of a growth mindset is that the person has the skill to take in the information of their choosing in each situation and, similarly, release any information that is not fruitful in any given situation. Referring to learning, a person with a growth mindset has a sheer interest in learning instead of passing.

Fixed-minded individuals believe that fundamentally you were born with what you have now; you cannot improve yourself. Your capacity does not increase, instead, it remains the same. Further, to a fixed mindset person, they would not gain new skills in new areas. Instead, they would only be able to be skilled in areas in which they have already seen success. Those with a fixed mindset often avoid adversity; when the time comes that a fixed mindset person is found incorrect, they will connect this incorrectness to a lack of intelligence and capacity. The response to failure from a person with a fixed mindset is to quit; continuing would not be an option. As learners, people with a fixed mindset will focus on getting answers correct with little to no attention given to the process unless the process is directly related to the correct answer.

Dweck ends with a firm ideology that reframing the mind to a growth mindset can significantly impact a person’s outcome. This new mindset can shift trajectories, and impact life satisfaction, in addition to having financial implications. These trajectories include what types of skills they might choose to obtain. Life satisfaction might include what types of new hobbies they choose to explore. Regarding finances, a shift in mindset could mean going after a new job in a different field. The impact of mindset can determine one’s success in a significant way as your basic qualities are cultivable through investments in these qualities. “The view you adopt for yourself profoundly affects the way you live your life” (Dweck, 2006, p.6).

*Outliers*

This book is selected due to the highlighted individualistic perspective that it offers on success stories to see if commonalities are seen within the outliers. From a Bourdieusian lens, it
seems reasonable to find the most holistic understanding to examine people who may have unique differences in their lives that set them apart from others. This aligns with the concept of seeking all areas that could influence how a person interprets their surroundings (Bourdieu, 1984). These could be used as future things to look out for or even more just things to be considerate of as research continues down this path of mathematics capital; it seems reasonable to give examples of exceptions to what would otherwise have been a routine or expected outcome for a person.

Some specifics to highlight:

1. Lower scores on convergent tests and the consistent misalignment to achievement.
2. “In the end, only one thing mattered: family background” (Gladwell, 2008, p. 111).
3. Parenting philosophies highlighted two distinctly different perspectives, divided by class lines.

These highlights emphasize the text’s frequent callbacks to family and the impact that this has on individual outcomes. In layman’s terms, Outliers can be summarized with this thought: No person is particularly special but instead, the context in which the person fell into and the given time was.

**Mathematics for Human Flourishing**

Su (2020) shifts ideologies by considering the virtues that can be learned through mathematics, intentionally considering how mathematics can be used to enhance our lives. This perspective is developed through the exploration of mathematics where readers can engage and innately experience his words through the journey for his readers. Foundationally, this author believes in every person’s capacity to do mathematics. This is not without intent.
Instead, this is for the human to flourish, as the title suggests. Specifically, the struggle that mathematics excites develops within twelve virtues that naturally increase through the practice of mathematics. Some of the virtues mentioned include recognition of beauty, the act of discovery, the answer to elaborate stories, to engage in inquiry and to find deep meaning, to struggle through work, to find freedom and permanence, justice and community, to be playful, and to love. Su highlights the value of mathematics in the lives of all people, but specifically the value of doing mathematics which includes deep exploration and discovery including, the power that mathematics can have to create justice or injustice within people. In essence, mathematics is all-encompassing, yet touches each of us differently and this should be considered carefully.

**Results**

Bourdieu’s writing on capital, field, habitus, and doxa serves as the overarching theoretical lens from which this study is examined (Bourdieu, 1977, 1984, 1986, 1990), alongside Archer et al.’s (2014) introduction and quantification (Archer et al., 2015) of science capital, embodied in cultural, social and symbolic capital. It was proposed that mathematics capital could be a reasonable parallel in mathematics education as science capital is in the field of science. To consider mathematics capital, four theorized categories were introduced, similar to science capital, with an added category of mathematics mindset (Boaler, 2015). The four-part model included mathematics mindset, mathematics teaching and learning, mathematics self-efficacy, and finally cultural experiences. See figure 2, where each part of the model equally informed mathematics capital and no particular order was considered. Instead, each part was introduced as though the contribution from each component would be similar and without many differences. Details about each part are explained in chapter one.

**Figure 2**
Emergence of the Research-Informed Model

Although the research highlighted that the model reasonably captured comprehensive themes, there were some nuanced details that had yet to be considered. The contents of the categories that emerged contained some differences. Further, the weight of each category's contribution was also a new finding that emerged. Specifically, cultural experiences thematically reigned over all other components of mathematics capital (Boaler, 2015; Gladwell, 2008; NCTM, 2014; Su, 2020). Each text included the undercutting theme that improvement in mathematics education was critical as the vast discrepancies between achievement exist within the cultural backgrounds of the mathematics learners. Specifically, the primary reason for this was due to the stark prediction nature of mathematics success of marginalized groups, including but not limited to racial differences, socioeconomic status, language learners, and citizenship status (Su, 2020). For this reason, cultural experiences are an informant of all other categories and the only one that directly informs mathematics capital. See figure 3.

Next, the model's order, meaning, the way in which each category is introduced, became a relevant consideration due to the nature of the impact of each category as referenced above. At the beginning of this study, the order was not initially considered, similar to how much impact each category would have. Now, the contribution of each category and the order
in which each category influences mathematics capital is being considered. A third nuance is that there was no initial consideration of the overlap and intertwined nature of the categories. This third, overlapping feature exposed that many categories work fluidly with others, similar to the eight mathematics teaching practices (NCTM, 2017) and the 5 strands of mathematical proficiency (NRC, 2001). Categories each contain segregated pieces although categories also significantly overlap. In the research-informed model, these categories are represented as circles to highlight the differences within them from the first, conceptual model. These three categories, mathematics mindset, mathematics teaching and learning, and mathematics self-efficacy, inform mathematics capital as they intertwine and overlap. See figure 3.

Next includes the components that each category contains. Most uniquely was the intertwined nature of all the theorized categories, with almost every category appearing intertwined within each text, even if the text captured the essence of one category. Said differently, even if a text was for mathematics teaching and learning, there were clear details and instructions on how cultural experiences inform mathematics teaching and learning alongside informing mathematics self-efficacy (NCTM, 2014). Similarly, changes in teaching and learning were found to alter a student’s opportunity for positive learning experiences (Safir & Dugan, 2021). It seemed that no one category could exist without some type of crossover with another. As serendipitous as this may seem as it falls under Bourdieu’s framework, this is instead in slight contrast to his work in Distinction (1984) where he is able to categorize taste thoroughly and with distinctiveness. To credit this nuance though, in Bourdieu’s model, it only exists on a spectrum. A spectrum insinuates a certain amount of fluidity as well and the fluidity in taste is described.

**Figure 3**

*The Research-Informed Model*
Cultural Experiences

Cultural experiences are embodied within cultural capital (Bourdieu, 1986). Justice and equity are the overarching themes in every text that was reviewed. Starting with principles to action, teaching all students to be successful in mathematics is highlighted beginning with the title “ensuring mathematical success for all”. The introduction highlights the importance of transitioning from pockets of excellence to “systemic excellence” and continues to highlight the structural misalignment in achievement in mathematics education. This is not the only text that roots their argument for mathematics reform in the name of justice and equity. Instead, principles to action merely follows what the other texts had already done. This means that all texts mentioned above all refer to creating a structure in which every student in the United States has an equitable, personable, flexible experience with mathematics (NCTM, 2018). Once this foundation is set, authors move into specific details of each text and into the other categories that are now backed by research.

Mathematics Mindset

Mathematics mindset adds the themes of agency, and identity alongside efficacy (NCTM, 2014; Safir & Dugan, 2021). How confident a student feels about executing a task is
directly related to how a student sees themself within the task. In this case, how confident a student feels about themselves in mathematics is related to how a student sees themselves as a doer of mathematics (Boaler, 2015). The details of mindset are bleeding in through other texts. Su’s conversation about building the virtues as opposed to anything else is certainly one that seems to support this idea of mindset and not of simply finding a good or bad. More principles to action includes charts of productive versus unproductive beliefs in each practice and in each section. Student and teacher dispositions are a large part of equitable mathematics instruction based on the text.

The growth mindset is understanding an evolving mind, one that changes, learns, adapts, and is flexible, as described by Dweck (2006). Boaler specifies Dweck’s mindset work for mathematics, in Mathematical Mindset (2015). Stakeholders would consider mathematics teaching and learning by taking a complete view of mathematics learners. The phrase “wow, that student is good at math”, implies that a student has developed an inherent characteristic towards mathematics. Similarly, then, the phrase “wow, that student is not good at math” can be considered inherent to both the outsider and the subject. In the immediate future, the purpose of this study is to have researchers in higher education begin the conversation on the theory of mathematics capital. This researcher will attempt to offer an initial description and design for mathematics capital.

This perspective seems timely as it is becoming increasingly more popular to state that all students can learn mathematics (Flores, 2007). Boaler (2013) and Sella and Kadosh (2018) find mathematics high achievers and underachievers have specific cognitive differences that are not a result of birth, but rather repetition and practice with mathematics; previously identified underachievers are using more parts of their brain to learn mathematics than their overachieving counterparts. Specifically, Boaler et al. (2016) find that when a student finger
counts, the learner is taking in information in a more complex way, seeing numbers both abstractly and visually. Historically, finger counting was viewed as a deficit.

In the eight mathematics teaching practices, a plethora of specific teaching strategies are listed. All require intentional methodology. Within these, concepts mentioned include productive struggle and task-based learning. The disposition that a student takes towards these practices can be referred to a student’s mindset in mathematics. More than what is discussed by Boaler and Dweck in that one layer is how the mathematics student will specifically respond to the eight practices.

**Mathematics Teaching and Learning**

*Mathematics teaching and learning experiences* are embodied within cultural and economic capital (Bourdieu, 1986) and are the most well defined in mathematics education. A plethora of research, along with the research presented all conclude with some key factors. Mathematics is a process of doing and is more than one skill. Instead, it is a network of practices that work together to build mathematically proficient students. Namely, the 5 strands of mathematical proficiency are named above (NRC, 2001) and are such that not one exists without the other. In this same vein, the eight mathematics teaching practices work so that they flexibly and fluidly interchangeable with each other to create effective teaching practices (NCTM, 2014).

**Mathematics Self-Efficacy**

Self-efficacy is a person’s belief in themselves to complete a task. Self-efficacy also relates to how a person might think of themselves as capable to complete a task (Bandura, 1997, 2002). Connecting self-efficacy to Bourdieu argues that self-efficacy can be seen as a relationship that a person might have with themselves as this person views their competence in task completion. The way a person interacts with themselves is dependent on how a person
may speak to themselves, which in turn is dependent on how a person might believe themselves capable or not. It’s in this spherical patterning that self-efficacy, that social capital is seen as the social relationship that exists with oneself (Bandura, 1997, 2002; Bourdieu 1977, 1984). This further infuses mathematics agency and identity within mathematics self-efficacy. All these implications are found and highlighted in the texts mentioned above.

**Conclusion, Implications, and Next Steps**

Redefining mathematics success as the result of collective action, beliefs and experiences suggests the nature of mathematics learning is structured by intervening and intertwined variables. By looking at each piece of mathematics education as interlocking, intervening, and overlapping facets, a comprehensive design can be formed to describe mathematics education. Mathematics education is a highly debated topic, known as a gatekeeper for the STEM field (Douglas & Attewell, 2017). Yet, finding success in mathematics can substantially impact a student’s future; the national average wage for all STEM occupations is nearly double the national average wage for non-STEM occupations (Fayer et al., 2017; Rose & Betts, 2001).

Mathematics education remains a highly relevant area for research as success in mathematics continues to impact persons well into adulthood (Fayer et al., 2017; Rose & Betts, 2001). To date, mathematics research has focused on segregated areas; restructuring the literature to connect all areas by imagining mathematics education through a Bourdieusian lens, can shift the lens in which the literature is viewed (Ayotola & Amedeji, 2009; Boaler, 2013, 2015; Boaler et al., 2016; Bonne & Johnston, 2015; Bourdieu, 1986; Fast et al., 2010; Montague et al., 1991; Lee et al., 2020; Sella & Kadosh, 2018; Smith & Schumacher, 2006). Mathematics education is the *field* and *mathematics capital* has the overarching power over this *field*. This can call to question previous notions of mathematics ability, creating a conversation on
mathematics capital. There is no truth; there are only perceptions of what is seen (Bourdieu & Wacquant, 1992). Moving forward, it is reasonable to interview people who have been previously successful in mathematics. These interviews will work to deeply examine and potentially expose if there are intertwining stories that suggest that mathematics capital worked to facilitate their success in mathematics.

Thus, for future research, an in-depth qualitative analysis will take place. This analysis will include interviews of mathematics teachers that are currently in service. These mathematics teachers will be highly specialized mathematics teachers carrying dual degrees in both mathematics education and mathematics. The reason for selecting a limited pool of interviewees is to heighten the likelihood of being able to consider what similarities exist between these people who have found success in mathematics. Similar to Bourdieu in Distinction, who discusses taste, his considerations do not simply rely on one layer alone. Instead, he begins with taste and then dives into not just how a food tastes and what types of classes might choose to eat it, and he details how a certain type of food might be served. How the food is served is then also related to class and who might eat this food, which of course adds a layer of depth to his analysis. Still, he does not stop here, once he’s considered how the food is served and whether it can be easily portioned by the masses, he continues by considering body form and how a person’s shape may also inform what they might eat and their tastes. In this he looks to the body as consideration for what is needed in each class, the working class needed a strong body and then the bourgeois, not as much. He also details the differences between males and females. With all things Bourdieu, it is never a one-layered response. Instead, once one layer has been examined, a deeper layer must be explored. If this research is to have a Bourdieusian lens, it is not possible for the research to end here. Instead, this is considered as one layer, in this case, the first layer and will inform the next (Bourdieu, 1984).
The next chapter of research is in a similar fashion to the research presented in this chapter. Notice that figure 2 was referenced from chapter 1. Then, figure 3 was presented in this chapter which was informed by the findings of this chapter. It was impossible to present figure 3 without the research done in chapter 3. In this same vein, figure 3 from this chapter will be able to be further informed by the finding of chapter 3. Then, figure 3 will be referenced and either modified to include the findings of chapter 3 or solidified in what was found in this chapter.

The feasibility of this research is intuitive. The mathematics teachers that will be researched will be those that have graduated from the University of Nebraska’s NOYCE teacher scholar’s program. The NOYCE teachers have certain requirements to their funding; it should be manageable to obtain access and participation. Furthermore, getting data from them, will secure a pinpoint for successful people in mathematics. It seems reasonable that subject research starts from an asset or surplus perspective; a philosophy of if it is known what happened with those who advanced, maybe understanding can be made for those who did not advance, especially when the pool of persons who advanced are smaller than those who did not in mathematics education.

This will add to the body of knowledge in mathematics education. The perspective taken for this research is new; no studies specifically look to operationalize mathematics capital using Bourdieu’s framework and Archer et al.’s work on science capital. Further, it does not seem that a comprehensive document analysis of overarching texts, all from complementary perspectives, have been done in mathematics education research. This study along with the next is novel in that it works to connect, and intertwine the literature base through Bourdieu’s philosophies. No one idea can be as monumental as all ideas working together to inform the next. That is the purpose of this work.
References for Chapter 2


Chapter 3: Theorizing Mathematics Capital, A Qualitative Approach

Abstract

Six mathematics teachers with dual degrees in mathematics and mathematics education are interviewed following a semi-structured interview approach to continue to define mathematics capital. Data was analyzed through a deductive and an inductive approach. The cumulative nature of the work was kept in the forefront, to answer the research questions and the long-term goals of the research. Thus, the theoretical framework used to analyze the data was drawn from chapter 2 in this series, where the following four categories are defined: cultural capital, mathematics mindset, mathematics self-efficacy, and mathematics teaching and learning experiences. The four categories are pictured in a figure to describe the nature of mathematics capital and how mathematics capital interacts with other forms of capital. Once deductively categorized, an inductive coding methodology was used to find emergent themes via short phrases and keywords. Lastly, this article calls for future research as its purpose is to develop an itemized survey that will be used for quantitative research.

Keywords: Qualitative research, mathematics capital, mathematics education, cultural capital, mathematics teaching and learning
Chapter 3: Theorizing Mathematics Capital, A Qualitative Approach

What is Capital?

Seminal capital research explains capital as a predictor of a person’s maneuverability or outcome in a closed circuit (Archer et al., 2014; Archer et al., 2015; Bourdieu 1998; Coleman 1988). Coleman defines capital as a web-like structure where social, human, and financial capital overlap; each of these impacts the opportunities a person possesses within their lifetime (1988). Lastly, the transmutation of capital through its various forms is an important and inherent characteristic of capital that should be mentioned (Bourdieu, 1977). It is exactly this predictable interconnectedness and transmutation that makes capital a tangible and necessary descriptor for every field (Bourdieu 1977, 1998; Coleman, 1988).

Mathematics Education Research

For this article and the successive research, a person is exactly this: one person. This person is an individual and the goal is to plainly see them as such. And yet, this person presents with lived experience. The experience, just like the person, must be considered exactly as such: this person’s experience. In mathematics research, at times, there is progress through findings that can be attached to subgroups of people but may not apply to all persons in mathematics education (Lee et al., 2020; Montague et al., 1991; Smith & Schumacher, 2006). This is not to say that this research is without value. Instead, this research is to say that this research is of the utmost value. That is because understanding subgroups can lead to understanding the entire group. Without research on the subgroups, we cannot shine a light on all parties in the group, including outliers of such groups. In other words, this research does not stand to contradict all previous research, it instead stands to acknowledge all research and then dig into the corners of what has been revealed to discuss what has already been explored expansively.
Similarly, other research has strategically posited hierarchical angles (i.e., mathematics mindset, mathematics self-efficacy, and mathematics teaching and learning experiences) to improve academic outcomes for mathematics students (Ayotola & Adedeji, 2009; Boaler, 2015; NCR, 2001; NCTM, 2014). Again, the same can be said now that was said about the subgroups. Never should it be said that this research is not inherently informative and integral to the cumulative wealth of knowledge in mathematics education. Instead, it is exactly these researchers and ideas that build the foundation for which this research is founded.

Research Question

Is there an underlying, operationalizable phenomenon, *mathematics capital*, that has otherwise not been described, offsetting individual student outcomes in mathematics education? Is *mathematics capital* observable within stories of people who have been successful in mathematics education such that these stories can be used to validate and inform survey items to measure mathematics capital?

**Understanding the Research Question.** The goal of this research is to construct, with limited argument, the existence of mathematics capital. To do this, each piece of research in this series must inform the other. That means that this article, although self-contained, cannot exist without retroactively mentioning the preceding article and also, in tandem, forwardly referring to the future article. Therefore, this article moves to create a research-supported structure for the items to come in the future work, alongside being written from the lens found in the preceding document analysis article.

**Theoretical Framework**

From the overarching lens of Pierre Bourdieu in 1986, mathematics capital emerges by considering all mathematics research texts in intersection with each other alongside the financial differences that exist between those who have found success in mathematics and
those who have not (Fayer et al., 2017). Similar to how intersectionality is used to describe the complex, unique nature of each person’s lived experience when belonging to more than one marginalized group (Crenshaw, 1989), the intersection of all mathematics research will be used to provide a reason, mathematics capital, for a person’s otherwise presumed mathematics ability.

**Theorized Model to Describe Mathematics Capital**

The categories and their intersections emerged from research in chapter two. Cultural capital, first imagined as cultural experiences, became clearer when defined as cultural capital (Bourdieu, 1977), as these cultural experiences emerged within every other noted category that impacts mathematics capital (Boaler, 2015; NCTM, 2014; Su, 2020). The texts framed culture and context in a way that is interpretable, with a Bourdieusian lens, as cultural capital. Cultural capital then impacts each of the components of mathematics capital which in turn impact mathematics capital, which impacts mathematics education. This syllogistic pattern concludes that cultural capital informs mathematics education is not a new one. In 1977, Bourdieu claims, “academic qualifications are to cultural capital what money is to financial capital” (1977, p. 187).

**Figure 1**

*The Theorized Mathematics Capital Model*
Cultural Capital

Cultural capital described by Bourdieu plainly refers to a person’s family background, educational attainment, and perhaps social refinement (1977, 1986). Although this is not inclusive, these short phrases can act as descriptors to understand Bourdieu’s vision of cultural capital. In mathematics research, justice and equity are frequently discussed (Su, 2020). These two words highlight why teaching strategies must be implemented or considered alongside curricular decisions (NCTM, 2014). Some authors discuss the connection between cultural capital (in the many ways that this is understood) and mathematics achievement (Bills & Hunter, 2015; Huang & Liang, 2016; Tan, 2015).

For example, Bonne and Johnston (2015) found that students’ “mathematical deficiencies” were in part reflective of their social realities, wherein social and fiscal barriers continued to influence mathematical development regardless of intervention (Bonne & Johnston, 2015). These findings highlight that despite the intention to provide worthwhile mathematical experiences that enhance students’ learning and empower a sense of citizenship, teaching interventions cannot solely rely on the approach (Bonne & Johnston, 2015). Teaching
for equity must consider the broader political and social structures that impact students’ lives (Apple, 1992; Bonne & Johnston, 2015; Tate, 1997).

**Mathematics Mindset**

Students need to see themselves as mathematicians. This flexible thinking and seeing oneself as a mathematician encourages the student to engage with mathematics and to persevere when problems become more challenging and when problems require more than one step to complete. When a student identifies themself as a doer of mathematics, this allows them the tenacity to continue in mathematics (Boaler, 2015).

Su continues within this idea but develops it further through the conversation of virtue development in his text, abstaining from attributing value to right and wrong answers, and to instead accepting the process of doing mathematics as valuable. This allows students to seek more than just answers but fulfillment through mathematics. This fulfillment allows for mathematics to serve as a fruitful gift and as a function instead of being a hurdle for students to step over (2020). Furthermore, the way in which a student believes mathematics should or could be done is also an indication of mathematics mindset, which is well delineated (NCTM, 2014).

Bonne and Johnston challenge binary theories of intelligence (stable/malleable) to show that there exist incremental beliefs and ideas about intelligence such that beliefs about intelligence cannot be captured in one way, it may be able to be captured on a spectrum. Little research has been done to review this incremental understanding of intelligence and how it relates to mathematics outcomes (2015).

**Mathematics Teaching and Learning Experiences**

Mathematics teaching and learning experiences are influenced by cultural and economic capital (Bourdieu, 1986), as well as a network of teaching practices that contribute to the
development of mathematically proficient students. Accepting that mathematics is a process of doing, and is representative of more than one skill, highlights the importance of flexibility and fluidity within approaches for effective teaching strategies (NCTM, 2014). In fact, previous research has found that students assigned to the longest remedial math sequence did not benefit in terms of completing the gatekeeper math or associated degree (Xu et al., 2018), and that instead, one example of effective mathematics teaching might evolve to include multidimensions of learning experiences (Boaler et al., 2016).

Boaler et al. (2016) found that visuals can be used to explain or illustrate all mathematical concepts or ideas, though visuals are rarely integrated among mathematics instruction. From elementary to high school grade levels, mathematics instruction remains “obsessively numerical”, with few visual or creative representations of mathematical constructs, despite evidence of a positive relationship between visual thinking and high mathematics achievement (Boaler et al., 2016). Regardless, most mathematics teaching still does not yet encourage non-numerical conceptualizations of mathematics, highlighting the imminent importance of integration of its multiple representations.

Evolving the ways in which mathematics is taught must also include means of instruction, whereby research from mathematics teaching founded on collectivism, as opposed to individualism, better supports students across the achievement spectrum, with potential to provide equitable mathematics education (Boaler, 2011). For example, both Finland and Japan are considered to be at top the world in mathematics achievement, wherein both integrate peer support, teaching, and learning, and promote different achievement levels as a resource as opposed to a challenge. Specifically, students helping each other, and learning from each other was found to support mental, physical, and intellectual growth, and supported the notion of collective improvement and achievement of goals (Boaler, 2011).
Mathematics Self-Efficacy

This work is framed through a Bourdieusian lens. Yet, self-efficacy is well-defined by Bandura as a person’s perception of their ability to complete functions (1997). Thus, to view Bandura’s theory of self-efficacy through a Bourdieusian lens, understanding how social capital and self-efficacy intersect is relevant for operationally defining mathematics self-efficacy for this study.

Intersection of Capital and Self-Efficacy. Bourdieu discusses social capital as a person’s capacity to move, engage, relate, and function within social circles alongside a person knowing people within that same social circle (1986). Movement, engagement, and function relate to a person’s capacity to act programmatically through the social network and knowing people relates to a person being positively and potentially personally connected to those in the circle (e.g., at a tennis game, a person would know the rules of game and would know all, most, or at least some of the people at the game, and the person would be able to act in a way that would allow them to have conversations at the game) (Bourdieu, 1986). Still, as much as this is social capital, it is notable that within the definition of capital exists the intersection and overlap of capital. Some of what is described for social capital could also be considered cultural capital (Bourdieu, 1977). For this work, self-efficacy is contextualized under these two ideas. Self-efficacy, as it relates to a person’s belief of their capacity, and social capital, a person’s fusion within a social network such that the social network is the person, and networking is the person’s capacity to relate to themselves.

Mathematics Self-Efficacy. Mathematics self-efficacy layers agency and identity alongside general definitions of efficacy (Bandura, 1977, 1997; NCTM, 2014; Safir & Dugan, 2021). For mathematics, the confidence rooted from within a student is in direct relation to how a student pictures themself within the task. Bandura found that self-efficacy is an indicator for
academic achievement (1986, 1997) and other authors’ research concurred with this finding (Ayotola & Adedeji, 2009). Ayotola and Adedeji added to the body of research by specifically investigating the roles of school counselors. These authors found that school counselors should investigate each student’s beliefs about their capacity to engage in mathematics and called for future research to understand contexts that construct motivation (2009). Other researchers found high levels of mathematics self-efficacy impacted, positively, mathematics achievement, and found that how a student perceives the environment impacts their mathematics self-efficacy (Fast et al., 2010). In summary, mathematics self-efficacy directly correlates to mathematics achievement, from several angles and in varying degrees (Ayotola & Adedeji, 2009; Bandura 1977, 1986, 1997; Fast et al., 2010; NCTM, 2014; Safir & Dugan, 2021).

Method

Participants and Procedures

A semi-structured interview protocol was followed with exempt permission received from the NebraskaMath Omaha NOYCE Partnership. This research acts to continue the programmatic efforts as described in the grant 1852908 funded by the National Science Foundation, where the author is a member of the grant leadership team. Six in-service teachers who received scholarships from the grant to obtain their credentials volunteered to participate in the interview in the summer of 2022. Each of these six participants are highly specialized mathematics teachers with dual degrees in both mathematics education and mathematics. The reason for selecting a limited pool of interviewees was to heighten the likelihood of similarities emerging among these participants who have found success in mathematics.

Justification for Qualitative Research

The goal of this study is to answer the question, “Is there an underlying, operationalizable phenomenon, mathematics capital, that has otherwise not been described,
offsetting individual student outcomes in mathematics education? Is mathematics capital observable within stories of people who have been successful in mathematics education such that these stories can be used to validate and inform survey items to measure mathematics capital?” This qualitative approach follows a document analysis. The document analysis reviewed and primarily confirmed the conceptualized categories. The main differences were three-fold. First, the results of the document analysis called for restructuring the figure that described mathematics capital. It became evident that all the categories were interconnected and that together (mathematics mindset, mathematics teaching and learning, and mathematics self-efficacy) informed mathematics capital. Second, what was initially thought of as cultural experiences was cultural capital. It became clear that the cultural implications found in the documents were evidence of cultural capital, and thus the category was renamed as such. Lastly, it also became clear that cultural capital was the only category that singularly and directly informed mathematics capital.

The document analysis offered descriptive information and insight, although it did not provide data from subjects as a qualitative interview study data does. Qualitative data can be considered timely, descriptive data that allows for current-day perspectives on topics that emerged in the literature. It seemed reasonable, while being informed from the document analysis, to collect data from the highly specialized mathematics teachers to get a different perspective on mathematics capital and how it permeates from people’s stories.

**Analytic Approach**

The research method for this article was a combination of deductive and inductive coding (Miles et al., 2014). First cycle coding with the data was completed by using a deductive approach. All six participant transcripts were reviewed. Each category was well-defined, referring to chapter two. The cumulative structure within this dissertation exists so that
framework and results from each chapter can be used in the successive chapter to capture the long-term vision for this research. Cultural capital, mathematics mindset, mathematics self-efficacy and mathematics teaching and learning experiences were defined although it is also understood the overlapping nature of each of these categories. Thus, when each passage was analyzed, it was not always put into one category. Instead, the passage was placed into each category that it could fit in. This was also done because longer passages were retained to provide context while reviewing the data.

For example, the excerpt “Okay, I remember it being really, really, really easy with whatever we were doing [in class]. I mean, this was grade school. It was really, really easy, and so if somebody doesn't understand something it’s kind of like, “How do you not get it?” was coded in mathematics mindset, mathematics self-efficacy, and mathematics teaching and learning experiences. Mathematics mindset came up when the participant said, “How do you not get it?” referring to the participant’s capacity to be confused with how a person would not be able to be successful in mathematics. Mathematics self-efficacy was coded due to the sentence, “I remember it being really, really, really easy with whatever we were doing [in class]” and lastly mathematics teaching and learning experiences came up due to the last portion of the quote, “This was grade school.” Each of these sentences, removed from the entire quote, provides very little evidence and contextualized meaning. Yet, in context, the quote falls into each of the named categories for different reasons, as listed above. Further, this same excerpt highlights the possibility that one sentence could fit into more than one category. For example, in this same quote, the sentence, “Okay, I remember it being really, really, really easy with whatever we were doing [in class]” equally fits in mathematics teaching and learning experiences because this was how the participant is recounted what happened while they were
at school and it also fits in mathematics self-efficacy because the participant is stating that the mathematics was easy for them.

Second cycle coding began by analyzing the data inductively in each sub-category, cultural capital, mathematics self-efficacy, mathematics mindset and mathematics teaching and learning experiences. The cultural capital category was analyzed first. As common themes emerged, keywords were searched and tabulated, then the passages containing the keywords were reviewed together. Within each of the keywords, themes and particularly descriptive passages were noted for further review and attention (either at this time or for a future study). This approach allowed for the cycling of deductive and inductive throughout the coding process to exhaust all potential themes and their counterparts. For example, researcher subjectivity could assume mothers of all six participants positively influenced participants. By returning to a deductive approach and searching for mom after inductively noting mom as a keyword under one common theme in the cultural capital category, mom was reviewed again with all the independent all mentions of mom to decide whether, in isolation, mom was referred to as positive, negative, or neither.

In the mathematics self-efficacy category, common themes were in closer proximity, intuitively providing justification for an immediate step to searching keywords, without a thematic breakdown as done in the cultural capital category. These keywords were also less descriptive of the category’s name like cultural capital. This may be because words that describe self-efficacy, for example, the word capacity, when searched revealed zero mentions in the document. This was assumed to be due to the mention of self-efficacy in more tangible contexts. Specifically, the teachers spoke about what they could do as opposed to simply referring to it. So, the keyword “can” alongside “math” and “easy” produced reasonable results. From here, the same process of reviewing the quotes in isolation with the keywords took place
and summarized descriptions within each of the keywords took place until the summaries reached a saturation point.

Mathematics mindset proved a significant challenge as the specific mentions of mindset are interpretable. Similar to self-efficacy, all data was in close proximity thus, subcategorization was counterintuitive. Differently from self-efficacy, as much as the data seemed to say similar things, data in this category seemed not to provide predictable patterns. Still, the notable key phrase searched in this category was “math person” as it was possible to review each participant's perspective on what it meant to be a math person. From there, other keywords were searched and reviewed just like the other three categories.

Lastly, mathematics teaching and learning experiences was analyzed. The inductive approach began, like the other categories, by reviewing the data comprehensively. When this began, it was clear that much of the data in this category had been reviewed in previous categories. Although the data was the same data, the lens of mathematics teaching and learning experiences was taken to consider emergent themes. Keywords were also searched here and reviewed in isolation to assure consistency.

**Instrumentation**

The nature of this research is qualitative, which makes the researcher the instrument in this study. It is stated that “differing views of evidence influence the role of the researcher” (Xu & Storr, 2012, p.3). This is not to say that this research is limiting, instead it is to clarify the perspective and this researcher’s experiences. As much as this research follows a Bourdieusian lens, it is all interpreted by me and thus, I am the instrument. My subjectivity cannot be ignored. Instead, I can present some influential experiences to highlight my motivations for this research and continue to remain consistent from these fundamental experiences that have informed my dispositions.
I grew up in poverty, minoritized due to my race, sex, and athleticism—African American student-athletes aren’t perceived as competent in the classroom (Stone et al., 2012), and in a single-family household. Statistics would suggest that these factors would have made me more likely to not have been in the STEM field (Basque & Bouchamma, 2016; Boaler, 1997; Lee et al., 2020). Instead, I excelled in mathematics, from an early age, and when faced with challenges throughout my mathematics career, I preserved through them. Now, I am the university’s secondary mathematics methods instructor, a leader in mathematics education in my area, and a math enthusiast in my spare time. The fact that I persevered through mathematics wasn’t necessarily a surprise until I met students with similar backgrounds as me, a mathematics teacher. I watched student after student with similar stories not have the same outcomes and I began to wonder why I positively persevered through mathematics. Thus, this research began with a fundamental question, “Why me? How am I different?”. As I searched for nuances in exceptional mathematics students to start with these six participants, I am hopeful that this might lead to understanding all mathematics students with future research.

Still, I know that I am “not capable of an omniscient point of view” (Xu & Storr, 2012, p.14) but I know that I am an outlier when I align my experiences with my mathematics outcomes (Basque & Bouchamma, 2016; Boaler, 1997; Lee et al., 2020). With this elementary knowledge, I seek to find out more about why all students are successful in mathematics because what I knew was that there were differences, but I could not yet describe them. The six participants interviewed had the same profession as me, but all had different backgrounds from me, at least at first glance. When I began to dig deeper into their backgrounds, I noticed that there were some key similarities between all six participants and myself, aligning with Bourdieu’s discussion of the transmutation of capital (1977).

Results
The framework used to analyze the results was from chapter two where the following four categories were used: cultural capital, mathematics mindset, mathematics self-efficacy and mathematics teaching and learning experiences. Lastly, some of the interviews were left uncoded if the quote did not fit in one of the four categories. These uncoded quotes were uncoded not due to findings that did not fit, instead these quotes contained no information. Every piece of data was able to be merged into at least one category. This provided additional evidence that the framework was suitable for the data.

After this initial deductive coding process, each category was reviewed inductively to review emergent themes from within. Four tables follow to see the themes emerged from each category alongside a short description of content within the emergent themes.

**Table 1**

*Cultural Capital*

<table>
<thead>
<tr>
<th>Themes</th>
<th>Short description content within themes (<em>keywords, count, if available</em>)</th>
</tr>
</thead>
<tbody>
<tr>
<td>engaged parents/family</td>
<td>Mention of mom being good or bad at math. Dad mentioned being good at math. Parents engaged in their education. Close relationships with parents. Mention of involved extended family and involved friends. <em>(Mom, 17; dad, 9; parents, 13; friends, 9).</em></td>
</tr>
<tr>
<td>embedded math opportunities</td>
<td>Specific mention of math flash cards used at home (two participants), counting at home, balancing checkbooks, parents who encouraged doing mathematics and being in mathematics. Friends who also do mathematics with participant. Science kits at home. A variety of board games and cards played at home from all participants. <em>(Board games, 5; math, 28; science, 5)</em></td>
</tr>
<tr>
<td>cultural experiences</td>
<td>Participants all noted positive experiences in their upbringing. No mention of trauma or any negative experiences in their youth. All participants seemed to feel that their home life activities were all normal and made casual mention that these experiences were not “a big deal”. All participants who discussed homework clearly stated that their parents reinforced and required them to do their homework prior to any other activities. 3 of 6 participants reference specific extended family as active players in their daily lives. <em>(Home, 7; backyard, 2; homework, 4; family, 10; fun, 3; awful, 0; horrible, 0; don’t like, 0; abuse, 0)</em></td>
</tr>
</tbody>
</table>

**Table 2**
**Mathematics Mindset**

<table>
<thead>
<tr>
<th>Themes</th>
<th>Short description content within themes (keywords, count, if available)</th>
</tr>
</thead>
<tbody>
<tr>
<td>math person</td>
<td>Definitions of what it means to be a math person are varied. Some define it as doing mathematics in general, whereas others define it as some success in mathematics. All relate in some way to being a math person. From the teacher’s perspective, discussion around their perceptions of what students would or would not enjoy and how the student’s experience influenced learning math. <em>(Math person, 10; math, 44)</em></td>
</tr>
<tr>
<td>can do mathematics</td>
<td>Students can learn math, if you’ve ever done math than you are a math person, everyone can do math.</td>
</tr>
<tr>
<td>good and mathematics</td>
<td>Having early success with mathematics, being told that they are good at math, having a good experience with math dictates current success in mathematics, belief in being simply good at math <em>(good, 19)</em></td>
</tr>
</tbody>
</table>

**Table 3**

**Mathematics Self-Efficacy**

<table>
<thead>
<tr>
<th>Themes</th>
<th>Short description content within themes (keywords, count, if available)</th>
</tr>
</thead>
<tbody>
<tr>
<td>math was easy</td>
<td>All participants report mathematics being easy (or extremely easy) at some point in their academic path. Some report this through college.</td>
</tr>
<tr>
<td>participant sees themselves as a doer of mathematics</td>
<td>All participants reported being competent as a math teacher in a math classroom. Recognize that input in mathematics means a positive output in mathematics. Do not see failure in mathematics as a reason to leave mathematics. <em>(Do,15; fail, 3)</em></td>
</tr>
<tr>
<td>general mathematics as it relates to self-efficacy</td>
<td>Capacity to move forward through advanced math courses and the positive perspective of self that this brought on. Confusion on why other people would or could struggle with the same math content (early years). General alluding to confidence in mathematics. Reporting not feeling nervous. Enjoying mathematics.</td>
</tr>
</tbody>
</table>

**Table 4**

**Mathematics Teaching and Learning Experiences**
The research question asked: Is there an underlying, operationalizable phenomenon, *mathematics capital*, that has otherwise not been described, offsetting individual student outcomes in mathematics education? Is mathematics capital observable within stories of people who have been successful in mathematics education such that these stories can be used to validate and inform survey items to measure mathematics capital? To answer this question, some specific results are included:

1. The framework is suitable as all descriptive data fit into the categories.
2. Cultural capital influences mathematics education.
3. Mathematics teaching and learning experiences, mathematics mindset, and mathematics self-efficacy are all interconnected, interdependent and intertwined.
4. This emergent infrastructure allowed for a recall and definition of all three of the categories to one comprehensive idea: *The collection of mathematics framework* that includes all three named categories.
5. Failure in mathematics was not a reason to leave mathematics.
6. Specific details emerged within participants that would inform item questions due to content that overlapped between participants.

It seems reasonable that these research results can be used to: (a) continue to validate the existence of mathematics capital, and (b) to develop an itemized survey to measure a person’s mathematics capital, positively answering the research question.
Discussion

This research began with a general curiosity point, beginning with the self, “Why did I have different outcomes than my contextually described peers?” Peers, at this point, were those who had the same marginalized and socio-economically opportunistic background. Even within this, I knew that there were differences that I experienced from these individuals and these I could recount vividly. One most obvious difference was my Canadian education that I knew set me apart from others that surrounded me. This is when the idea of mathematics teaching and learning was first born, alongside being a mathematics teacher. I knew, as a teacher, that I could impact a student’s outcomes. I also knew of teachers that I had who had impacted my outcomes. This is where the first portion of my research question, “Is there an underlying, operationalizable phenomenon, mathematics capital, that has otherwise not been described, offsetting individual student outcomes in mathematics education?” developed and the original, conceptualized model from chapter 1 was imagined. (See figure 2).

Figure 2

*The Evolution of the Mathematics Capital Models through the Chapters*
In this figure, I realized that there were different touch points for every person and that creating a more comprehensive picture and collecting all the research in mathematics education seemed relevant. Also, reviewing Bourdieu’s work, noticing the depth of analysis of one area alongside the reasons for the depth of analysis became interesting. This curiosity ignited the research and organized the document analysis from chapter two. Through chapter two, a
research-informed model emerged, where the categories could no longer be represented equally and linearly. Instead, it became critical that I collapse three of the categories and differentiate one, cultural experiences (see figure 2). It was in this model that it became clear that cultural experiences were thematically underlined in all mathematics education research and why cultural experiences were set apart from the rest.

Next, contrasting Figure 1 from this chapter to the research-informed model from chapter 2 (figure 2), there is a small difference between the figures. With further review and rereading of Bourdieu’s texts and mathematics education research, I noticed the clear, thematic undertones of cultural experiences. Cultural “experiences” would not suffice as the category name because I realized what I saw was not “experiences”; it was capital in the form of cultural capital. It is this cultural capital that created the initial underpinnings to develop the distanced and comprehensive analysis of the participants in this study. (See figure 3).

There was an interconnectedness of the results such that a saturation point was hit. This saturation point happened when, as rereading the results, the same themes continued to emerge, but all from different perspectives. It seemed no longer as relevant to describe only mathematics teaching and learning experiences, mathematics mindset, and mathematics self-efficacy to understand each participant but instead it seemed more relevant to recognize, holistically, the collection of all of these (recalling figure 1). This collection will be named the collection of mathematics framework as it is all three of these categories and their intersections that frame a person’s comprehensive experience, perception, and beliefs about mathematics. This issued, not a redesign, but instead a 30-foot view instead of a 10-foot view of the previous figure, which was a development from the initial 1-foot view in chapter one. This more comprehensive figure does not stand in contrast to any of the previous figures. Still, it instead
more deeply describes the nature of mathematics capital and how it relates to and interacts with the other forms of capital. (See figure 3).

Figure 3

*The Transmutation of Cultural, Financial and Mathematics Capital*

This figure highlights that mathematics teaching and learning experiences, mathematics mindset, and mathematics self-efficacy is seen as *the collection of mathematics framework* such that it is cemented in a Bourdieusian lens. Meaning that each of these categories do not exist without the other and each of these have pieces of different types of capital as underpinnings and each category is never mutually exclusive. Next, it is notable that cultural capital is the only category that is set apart from the collection, and it directly informs *the collection of mathematics framework*. Furthermore, mathematics capital then, is defined as a person’s cultural capital, as cultural capital is directly and indirectly related to mathematics alongside a person’s collection of mathematics framework where the framework inclusive captures teaching and learning experiences, mindset, and self-efficacy. Said differently, this research argues that mathematics capital is positively related to both cultural capital and the collection of a person’s
mathematics framework but is distinct and measurable. Mathematics capital can directly impact a person’s mathematics outcomes, as seen in the six participants in this study who are both mathematics majors and mathematics education majors and who have sustained professional careers as mathematics teachers.

The framework that structures this research views each “section” of the collection of mathematics framework: teaching and learning experiences, mindset, and self-efficacy, as an informant of mathematics capital but is distinctly different from mathematics capital. The collection is set apart from mathematics capital, but it is the intersection of the collection that can positively inform mathematics capital. It is also such that cultural capital directly and positively informs mathematics capital.

As more layers are added to create the 30-foot view, it is noted that mathematics capital informs cultural capital. In that same way, financial capital is added to the image such that financial capital interacts with cultural capital and mathematics capital. The interaction between cultural and financial capital is readily discussed in the literature (Bourdieu, 1977, 1986). Adding mathematics capital as another form of capital that interweaves fluidly with the definitions of capital means that mathematics capital is being understood as a form of capital. Capital networks and interconnectedness is discussed as the transmutation of capital (1977, 1984, 1986).

For example, Bourdieu selects one idea in some texts to specifically dive into only to use that one idea as a tangible, relevant example of capital in its many forms but also to better clarify the interconnected nature of capital alongside capital’s capacity for transmutation (1977, 1984, 1986). While this may seem unexpected, it is exactly as expected in the following way: Bourdieu defines capital’s relevance in relation to how quickly capital is directly convertible to financial capital, alluding to different forms of capital have different values at different times. In
1977, Bourdieu states, “symbolic capital is perhaps the most valuable form of accumulating in a society in which the severity of the climate and the limited technical resources demand collective labour” (p. 179). This is an example of the varying value of the forms of capital, depending on what is relevant at the given time. Currently, conversations on mathematics education dominate American culture by mathematics education making national news (Anderson, 2022; National Center for Education Statistics, 2022), and mathematics education, in tandem with literacy, being often used as an overall marker of student achievement (Pong, 1998). Lastly, mathematics outcomes are an indicator of long-term income earning (Douglas & Attewell, 2017).

Success in mathematics is related to outcomes in the STEM field (Atuahene & Russell, 2016). Math is known to keep people out of majors like engineering, which keep people out of STEM fields altogether. This hurdle has become so clear to the public that universities are working diligently to rework their entry-level mathematics courses making math more accessible to first-year college students (Augustyn, 2022).

For mathematics capital to exist, it must transmute, as the other forms of capital do. In the participants, it is difficult to differentiate between cultural, financial, and mathematics capital. It is also difficult to differentiate between cultural capital and the collection of mathematics framework and how these inform mathematics capital. Was it mathematics, financial, or cultural capital that allowed the participants to persist in mathematics education even in the face of failure?

A more directed question on the overlapping nature of capital embedding mathematics capital exposes the situational facts of the interviewees. These participants received well over $25,000 of funding to complete their degrees. Their local counterparts who are not mathematics teachers did not receive funding. Did the participants receive funding as a result of
their mathematics capital or their cultural capital? Then, once the funding was received, did the participants have financial capital or financial, cultural, and mathematics capital? These questions speak to the interconnectedness of capital and the importance of understanding the transmutation of capital in its many forms, mathematics capital being one of them.

**Conclusion**

Driving to work can be a process of being on autopilot. When I say this, some may smile, and others may immediately agree. It is likely that everyone has experienced some version of auto-pilot as they have made themselves to a familiar place. But what if you were on auto-pilot more frequently? The concept of doxa assumes exactly this. We live in a world of automated responses, most of which we do not know that we are in action and reaction to. We place on ourselves fundamental beliefs, or “autopilots” to make our way through life and to exist. For some, especially those strongest in math, advancing through the grades, levels, and schools, and seen in the six participants, it was almost a shrug-like, a basic expectation, a way of knowing, without knowing how to operate in mathematics classrooms. Their dispositions, self-efficacy, and mindset, all informed how they did things. This altered their teaching and learning experiences. Each of these previously mentioned is rooted in their cultural capital that drives, most rampantly their experience within all of it (Bourdieu, 1984). And yet, reviewing the data, it is assumed that not everyone would have all these tendencies developed so that these are ingrained within them so profoundly that it is part of their actions without knowing, it is a part of their autopilot in math class. It is this, this auto-mated action in mathematics classrooms that is a portion of the essence of mathematics capital. This, as seen in the participants, is mirrored through what their parents did while they grew up, how they acted and reacted to failure in a math class, they simply, “took it again”, without wavering question. And yet, I have seen so frequently the absence of this autopilot towards math, wondering myself why it isn’t intuitive,
using my own mathematics capital against my students as I couldn’t delineate from what I knew, without knowing.

My hope is that I can continue this research, that I can continue to explore what levels of without knowing are contributing to mathematics capital, but even more, backtracking to cultural capital to examine what layers of cultural capital are directly informing a person’s capacity to begin to develop this doxa within mathematics education. This is the road that all mathematics researchers have been traveling, they simply may have been on autopilot. This work’s goal is to bring light to the fact that this is where we have been and to arm us each as acting drivers of self into every mathematics room.

**Limitations and Next Steps**

It is important to note that while it is valuable to have been able to review participants that have what can be theorized only at this point as high levels of mathematics capital, it is also notable that this means that I could not analyze the contrary, binary perspective. That means, that although it can be considered what it means to have high levels of mathematics capital with this data, the only way that not having high levels of mathematics capital with this data is absence of what is not seen as opposed to descriptively looking and considering what factors could linearly connect with low levels of mathematics capital in adults. Further, this research only covered six participants that attended one institution.

In the literature review, reconciling needs to be made with what it means to be a math person and/or what it means to have a mathematical mind. As these theories continue to develop, they will need to be added to the collection of mathematics framework. Similarly, any mathematics research developed ideas should be folded into this model. That means that the collection of mathematics framework is a living, breathing model and is not fixed. This presents multiple challenges as mathematics capital is assessed because of the continuous updating that
the framework requires. It is also important to note that this research is qualitative and researcher objectivity can never be guaranteed.

Next steps include using the details from these interviews to develop an itemized survey. This survey will be used to develop questions that future participants will be able to answer on a 5-point Likert scale alongside their outcomes in mathematics so that mathematics capital can be measured and then connected to their mathematics outcomes. These participants will be freshmen in college. Thus, their mathematics success will be measured in relation to their outcomes as a freshman in their mathematics course. The longitudinal data will also review each of the categories and what happens within their semester as a measure of their active mathematics capital and its influence on their mathematics outcomes in the course that they are taking. This research will take place with an established research team such that data collection will exceed 300 data points making structural equation modeling (SEM) a possibility. The reason for a SEM approach is to analyze the data in a more detailed and descriptive manner to consider the relationships within the facets of capital alongside the relationships within the collection of mathematics framework and how each of these influence mathematics capital and how each of these influence mathematics outcomes.
References for Chapter 3

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http://www.theconversation.com/


Chapter 4: Mathematics Capital, A Structural Equation Modeling Approach

Abstract

A format for a structural equation modeling (SEM) approach is developed. Researchers show the nature of SEM as a method of quantitative analysis to explicitly describe relationships between variables. SEM allows researchers to analyze variables and to show that variables are distinctly different from other variables or created latent variables. This method will deepen the discovery of mathematics capital by showing that mathematics capital is distinctly different from cultural capital and the collection of mathematics framework.

Keywords: mathematics capital, structural equation modeling, the collection of mathematics framework, cultural capital, mathematics education
Chapter 4: Mathematics Capital, A Structural Equation Modeling Approach

Specific early years mathematics skills predict outcomes in mathematics in middle school. The authors present a multilevel model to describe the specific differences among mathematics topics that a prekindergarten student shows proficiency in and how these proficiency markers improve their mathematics outcomes as they progress through fifth grade. Lastly, their findings highlight incongruence between early childhood choice topics and required skills as students progress (Rittle-Johnson et al., 2017).

Specific details like these are critical for the development of mathematics students. Understanding as intentionally as possible what will set students up for long-term mathematics success is critical for their futures (Fayer et al., 2017). SEM is a specific type of data analysis that allows a researcher to create a model to represent the data. Fit indices are the key differences between multiple regression and SEM. Similar but more powerful than path analysis, SEM also allows for the creation of latent variables, variables that are in essence twice removed from the items on the questionnaire (Hox & Bechger, 1998). Furthermore, “SEM provides a particular powerful test of the equivalence of solutions across multiple groups” (Marsh & Yeung, 1998, p. 718). SEM allows researchers to collapse and compare differences within and between groups such that subtle nuances can be described with statistical significance.

Other strengths and differences of SEM refer to SEM’s capacity to analyze longitudinal data sets. For example, when applied to retention in the nursing force, SEM can identify key predictors associated to early career nurses’ turnover rates (Brewer et al., 2015). Another example is dynamic structural equation modeling (DSEM) that allows for researchers to connect time-series models to subjects. In DSEM, researchers can make choices with their analysis that will yield descriptive statistics to validate or deny the most detailed questions about groups through several time points (Asparouhov et al., 2018).
Research Question

This chapter will address the following research question: Is there an underlying, operationalizable phenomenon, mathematics capital, that has otherwise not been described, offsetting individual student outcomes in mathematics education? Can mathematics capital be created as a variable and shown that it is distinctively different from other variables?

Determining, whether mathematics capital is a variable that is distinctly different from other variables, is a task suited for SEM. SEM will be able to provide details and descriptors to show that mathematics capital is distinctively different from the other variables and latent variables. Hypotheses for the chapter are below.

Hypotheses

Hypothesis 1

A variable of mathematics mindset can be created with an eight-item questionnaire from the questionnaire used by Cribbs, Huang and Piatek-Jimenez in 2021. These researchers’ items were created from Jo Boaler’s widely accepted research (2000, 2015). Their reported Cronbach’s alpha value was 0.90 which supports the items’ internal consistency. Review table 1 for a list of the survey items.

Table 1

Tentative Survey Items for Mathematics Mindset
Hypothesis 2

A variable of mathematics teaching and learning can be created with the seven items used to create and validate the variable “Instructional Practices for Teaching Mathematics” (Ottmar et al., 2014). In this study, researchers validated these items as a reasonable measure for the named variable. The research used to define instructional practices for teaching mathematics is based off of the 2000 publication by NCTM where researchers consider the proficiency strands and teaching methods as a measure of instructional practices for teaching mathematics (Ottmar et al., 2014). This is not to say that “Mathematics Teaching and Learning Experiences” is the same variable but it is reasonable to begin to describe mathematics teaching and learning experiences from this variable as there are similar literature bases. Small modifications were made to the questions and the header “while growing up” is added. There is also an additional layer to which includes not just experiences in the classroom but also experiences that may have occurred in the home, or other, which is different from the original variable. These items also align relatively closely to the eight mathematics teaching practices (NCTM, 2014).

Table 2

<table>
<thead>
<tr>
<th>Item Number</th>
<th>Survey Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>You have a certain amount of math intelligence, and you cannot really do much to change it.</td>
</tr>
<tr>
<td>2</td>
<td>Your math intelligence is something about you that you cannot change very much.</td>
</tr>
<tr>
<td>3</td>
<td>No matter who you are, you can significantly change your math intelligence level.</td>
</tr>
<tr>
<td>4</td>
<td>To be honest, you cannot really change how intelligent you are in math.</td>
</tr>
<tr>
<td>5</td>
<td>You can learn new things, but you cannot really change your basic math intelligence.</td>
</tr>
<tr>
<td>6</td>
<td>No matter how much math intelligence you have, you can always change it quite a bit.</td>
</tr>
<tr>
<td>7</td>
<td>You can always substantially change how intelligent you are in math.</td>
</tr>
<tr>
<td>8</td>
<td>You can change your basic math intelligence level considerably.</td>
</tr>
</tbody>
</table>
### Tentative Survey Items for Mathematics Teaching and Learning Experiences

<table>
<thead>
<tr>
<th>Item Number</th>
<th>Survey Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>While growing up, I discussed solutions to math problems with my peers.</td>
</tr>
<tr>
<td>2</td>
<td>While growing up, I worked and discussed math problems that reflect real life situations.</td>
</tr>
<tr>
<td>3</td>
<td>While growing up, I solved math problems in small groups or with a partner.</td>
</tr>
<tr>
<td>4</td>
<td>While growing up, I wrote a few sentences about how to solve a math problem.</td>
</tr>
<tr>
<td>5</td>
<td>While growing up, I used visual representations (e.g. diagrams, tables, models).</td>
</tr>
<tr>
<td>6</td>
<td>While growing up, I learned how to communicate ideas in mathematics effectively.</td>
</tr>
<tr>
<td>7</td>
<td>While growing up, I worked with manipulatives (e.g. geometric shapes).</td>
</tr>
</tbody>
</table>

**Hypothesis 3**

A variable of mathematics self-efficacy can be created with a 12-item questionnaire used by Cribbs, Huang and Piatek-Jimenez in 2021. These researchers adapted their scale from Fennema and Sherman’s 1976 research. Their reported Cronbach’s alpha value was 0.94 which supports the items, internal consistency. Review table 3 for a list of the survey items.

**Table 3**

*Tentative Survey Items for Mathematics Self-Efficacy*
Hypothesis 4

A variable of cultural capital can be created from an 11-item questionnaire developed from the cultural capital scales in literature (Dumais & Ward, 2010; Noble & Davies, 2009; Zabihi & Pordel, 2011). These items were collected due to the wide variance between how authors measured cultural capital. Necessary items will be deleted during analysis depending on the related Cronbach’s alpha values. Review table 4 for a list of the survey items.

Table 4

Tentative Survey Items for Cultural Capital

<table>
<thead>
<tr>
<th>Item Number</th>
<th>Survey Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Generally, I have felt secure about attempting math.</td>
</tr>
<tr>
<td>2</td>
<td>I am not good at math.</td>
</tr>
<tr>
<td>3</td>
<td>I am sure I could do advanced work in math.</td>
</tr>
<tr>
<td>4</td>
<td>I am sure that I can learn math.</td>
</tr>
<tr>
<td>5</td>
<td>I am not the type to do well in math.</td>
</tr>
<tr>
<td>6</td>
<td>I think I could handle more difficult math.</td>
</tr>
<tr>
<td>7</td>
<td>For some reason even though I study, math seems unusually hard for me.</td>
</tr>
<tr>
<td>8</td>
<td>I do not think I could do advanced math.</td>
</tr>
<tr>
<td>9</td>
<td>I can get good grades in math.</td>
</tr>
<tr>
<td>10</td>
<td>Math has been my worst subject.</td>
</tr>
<tr>
<td>11</td>
<td>I have a lot of self-confidence when it comes to math.</td>
</tr>
<tr>
<td>12</td>
<td>Most subjects I can handle ok, but I have a knack of mucking up math.</td>
</tr>
</tbody>
</table>
Hypothesis 5

A variable of mathematics capital can be created from results of a 15-item questionnaire developed by the content in chapters 1-3 of this study. Exploratory factor analysis will be performed, and necessary items with inappropriate eigenvalues will be removed (Zabihi & Pordel, 2011) until an acceptable Cronbach’s alpha value of above 0.65 is seen.

Table 5

*Tentative Survey Items for Mathematics Capital*
Hypothesis 6

A latent variable, “the collection of mathematics framework” can be created with reasonable alpha values from the following 3 variables: mathematics mindset, mathematics teaching and learning experiences, and mathematics self-efficacy. Creating the latent variable of the collection of mathematics framework is a critical piece of verifying this framework. The latent variable verified the overlapping nature of the 3 factors, in the way these characteristics emerged in chapter three from the qualitative interviews.

Hypothesis 7
In relation to the structural model, there is a positive relationship between cultural
capital and the collection of mathematics framework.

**Hypothesis 8**

In relation to the structural model, there is a positive relationship between cultural
capital and mathematics capital.

**Hypothesis 9**

In relation to the structural model, there is a positive relationship between the
collection of mathematics framework and mathematics capital.

**Literature Review**

**Cultural Capital**

Cultural capital continues to be researched by many authors and in different ways
(Dumais & Ward, 2010; Huang & Liang, 2016; Noble & Davies, 2009; Tan, 2015; Zabihi & Pordel,
2011). For this work, cultural capital does not exclude any of the previously used definitions of
cultural capital nor does it exclude any of the other ways that cultural capital is measured. The
lens for this study remains on Bourdieu’s original concepts of cultural capital, thus cultural
capital will be considered from his perspective, where cultural capital is cultural capacities as
these capacities are integrated and valued by the culture in which the capacities are being
expressed (1977, 1993). As other authors more specifically update conceptions of cultural
capital, these broaden the understanding of “cultural capacities”. For example, cultural
capacities might mean parents’ achieved education levels or parental educational expectations
of their children (Tan, 2015).

**Mathematics Mindset**

Mathematics mindset refers to a learner’s understanding of mathematics as being fixed
or changing (Dweck, 2006). Mindset also refers to how a student pictures themselves in the
process of doing mathematics (Boaler, 2015). It is also particularly powerful if a learner can understand the powerful strengths that learning mathematics can develop: virtues (Su, 2020). Developing a growth mindset in mathematics can lead to more students choosing science, technology, engineering, and mathematics (STEM) pathways (Cribbs et al., 2021).

**Mathematics Teaching and Learning Experiences**

Mathematics teaching and learning experiences refer to a student’s experiences, in classrooms with mathematics. The Nation Council of Teachers of Mathematics (NCTM) discuss teaching practices that increase student learning (2014). This discussion does not exist without considering the 5 interrelated strands for proficiency developed by the National Research Council in 2001. The types of experiences that a mathematics student has can significantly impact their learning and their joy of learning in the math classroom (NCTM, 2014; NRC, 2001; Ottmar et al., 2014).

**Mathematics Self-Efficacy**

Self-efficacy refers to a person’s ability to see themselves complete tasks (Bandura, 1997). In mathematics, the same is true. More, mathematics students with positive mathematics self-efficacy see better outcomes in mathematics (Ayotola & Adedeji, 2009). Lastly, developing agency and identity in mathematics is also an integral to the development of mathematics self-efficacy and outcomes in mathematics (Boaler & Greeno, 2000).

**The Collection of Mathematics Framework**

The collection of mathematics framework is a theorized latent variable created by the union of the three mathematics variables: mindset, teaching and learning experiences, and self-efficacy. This collection is called a framework due to the unique nature in which all three components of mathematics interact in mathematics learners to create positive outcomes in mathematics. This collection also uniquely transposes mathematics with a Bourdieusian lens as
the collection recognizes the transmutation of each of these mathematics variables. The framework emerged from the findings in chapter three of this study.

**Using SEM**

Davadas and Lay (2018) illustrate SEM as a tool to find predictors of a concept by using SEM to find predictors of attitudes towards mathematics. Their findings suggest that perceived parental influences, teacher affective support, and classroom instruction all significantly predict attitudes towards mathematics (Davadas & Lay, 2018). Other authors note the well-researched term “attitudes towards mathematics” and their study focuses on the affective factors that are precursors to “attitudes towards mathematics”. These authors’ desire to describe in a more detailed way “attitude towards mathematics” highlights yet another strength of SEM and how SEM can be applied to observe details within and precursing concepts (Khine et al., 2015).

It should also be noted that SEM has described forms of capital as these relate to socioeconomic status and other factors, at times, named sociocultural level (SCL). Menardo and authors (2022) confirmed the multidimensionality of cultural and social capital with SEM and develop a detailed model including age, sex, educational level and occupational prestige in how these relate to cultural and social capital. This study aimed to delineate and better describe the structure of SCL to describe better each component of SCL and their interactions (Menardo et el., 2022). SEM is a powerful quantitative research technique that has been used to provide descriptive information on theoretical and conceptual terms (Davadas & Lay, 2018; Khine et al., 2015; Menardo et el., 2022). Thus, using SEM to continue to describe and define mathematics capital seems appropriate.

**Mathematics Capital**

Mathematics capital functions as an overarching power in mathematics education, which alters a person’s capacity to be successful in the field. Mathematics capital is different
from mathematics mindset, mathematics teaching and learning experiences, mathematics self-efficacy, and cultural capital but each of these can positively inform mathematics capital.

Mathematics capital, similar to cultural capital is not just competence in mathematics, although relative competence is one factor of mathematics capital. Instead, mathematics capital includes the specific type of competence that is widely accepted and rewarded in our society such that the success found in mathematics is often complimented, looked at with high regard, or offers financial gain (Bourdieu, 1977, 1993).

**Method**

In chapter three, figure 1 was used to describe the transmutation of cultural, financial, and mathematics capital, and how it interacts with the collection of mathematics framework (see figure 1). Notably, each of the arrows represents the relationship that exists from one concept to the other. For example, between cultural capital and the collection of mathematics framework, the arrow represents that cultural capital can positively influence the collection of mathematics framework. A similar relationship exists between cultural capital and mathematics capital but there is an important addition to the relationships between the capitals, as noted in the figure. For the relationships between the capitals, the arrows not only indicate that one form of capital may positively influence the other form of capital but also that each form of capital has the capacity for transmutation which means that the capital contained in one of the forms may change state and be in a new form (Merriam-Webster, 2023).

**Figure 1**

*The Transmutation of Cultural, Financial and Mathematics Capital*
Figure 1 illustrates the theoretical underpinnings for this chapter and is only valid with the context from the three preceding chapters. For this chapter, only a portion of this figure will be prepared to be verified and used as the conceptual model. While figure one is comprehensive and detailed, there is too much to verify in one study. Attempting to verify all aspects of figure one in one article would emphasize breadth rather than depth. Thus, similar to how the figure was slowly theorized beginning with a 1-foot view and then broadening to figure 1’s 30-foot view, the first quantitative analysis will attempt to verify the relationship between cultural capital and the collection of mathematics framework, the relationship between cultural capital and mathematics capital and the relationship between the collection of mathematics framework and mathematics capital (see figure 2).

**Figure 2**

*The Conceptual Model*
Recalling the components of the collections of mathematics framework as: mathematical mindset (MIND), mathematics teaching and learning experiences (TEAC), and mathematics self-efficacy (SE). Simply put, the collection of mathematics framework captures the following: (a) how a student thinks about math learning, (b) how a student sees themselves as a math learner, and (c) the experiences learning mathematics that a student has had.

It is important to highlight that the content included in each of the components of the collection of mathematics framework may vary depending on a person's age and what is included within the sections may also change over time. In this study, data from at least 300 early-career college students are theorized to be included. These surveys will be conducted in the fall of 2023, with a final measurement model created as alpha values are calculated, verifying construct validity and reliability. This final measurement model will be some variation of the tentative measurement model shown in this chapter (see figure 3). The measurement model will be used to either support or deny the hypotheses.

Figure 3
**Conclusion**

This is a comprehensive research proposal that includes SEM as the method for analysis and serves as the guide for ongoing research. It acknowledges that future research should work to define mathematics capital in two ways. The first is to differentiate mathematics capital from cultural capital and the collection of mathematics framework. The second is quantifying mathematics capital by creating mathematics capital as a variable through the SEM analysis. Future research will include how the other forms of capital inform, transmute, and how these other forms differ from mathematics capital. The reason for this is to validate that mathematics capital is distinct and measurable as a unique variable. Verifying mathematics capital transmutation is a critical component of confirming its existence.
**References for Chapter 4**


Chapter 5: Concluding Thoughts

Mathematics capital emerged through comprehensively reviewing mathematics education research and understanding Bourdieu’s definitions of field, capital, doxa, and habitus. Yet, I would be amiss if I neglected to note my personal narrative that connects me to the foundational concepts of mathematics capital and its development. Mathematics capital evolved from an innate desire to understand my mathematics successes through the many predetermined factors that would statistically suggest I be more likely to be unsuccessful in mathematics. Recognizing myself as the instrument, I begin by concluding with the personal experiences with the collection of mathematics framework categories and with cultural capital.

The Collection of Mathematics Framework

Mathematics Teaching and Learning Experiences

NCTM has listed the eight teaching practices that lead to increased student outcomes (2014). Each of these encourages classroom discourse, high-level tasks, student interaction, multiple representations, and more. In my experience, mathematics was not taught this way. Mathematics was taught through quietly listening to the teacher, taking notes and then figuring out homework problems at home. I loved working on homework at home but class was always boring to me. So much so, that the thought of being a math teacher was not optioned; instead I pursued math education to receive a master’s degree and not to use it for teaching. Teaching math, repeating the same words to many different students day after day as these students copied down notes, was not something that I could commit to doing. It wasn’t until I reached my math methods course in my graduate program where I was introduced to the habits of mind, mathematical processing, and task-based learning that math teaching became an engaging and endless way to dive into mathematics each day. It wasn’t until I learned that teaching math was writing a narrative for each math learner that becoming a math teacher became what I wanted
to do. Suffice it to say, that my mathematics teaching and learning experiences were not those supported by research as effective practices. This, however, did not remove me from success as a mathematics learner.

**Mathematics Self-Efficacy**

Self-efficacy is well-defined in the literature (Bandura, 1997). Self-efficacy and how it relates to mathematics, is also well-researched (Ayotola & Adedeji, 2009; Bonne & Johnston, 2015; Fast et al., 2010). In specific reference to mathematics self-efficacy, researchers include agency and identity as an invaluable components of mathematics self-efficacy which permeates to positive mathematics outcomes (NCTM, 2014; Safir & Dugan, 2021).

I believe myself to be mathematically competent. As a mathematics teacher, I certainly never knew all the answers at any given moment, but I felt confident that I could figure out nearly any given math problem at any given moment. Further, I remember the day where this confidence developed. In 1st grade, my teacher handed me some “special problems” as I had completed the day’s work and was doodling on my paper. She asked me if I had ever seen problems like this before. I glanced down at the worksheet to see double-digit addition with three numbers to add as opposed to two. Within only a moment or so, I got to work and quickly solved the problems that had not been taught and were well above grade level. On this day, I went home and told my parents that I was a math superstar. And from that moment on, I never thought any differently. I continued to be successful in mathematics through high school and beyond, so there was no reason to disagree with that foundational belief that began in 1st grade.

**Mathematics Mindset**

For this work, Jo Boaler’s publication Mathematical Mindsets (2015) which describes students understanding of mathematics learning as flexible, creative, and accessible to anyone, serves as a foundational understanding of mathematics mindset. More, how a person utilizes
the term “math person” (found in chapter 3) is also seen as an indicator of a person’s mathematics mindset. Lastly, what a person describes as mathematics can also be considered mathematics mindset (Su, 2020).

Interestingly, my mindset has changed over the years. As I grew up, I was confident in myself and considered myself a math person. I relished within this label and never saw myself seeing things much differently. I looked at others, those unsuccessful at math as less than me and me more because of my atypical competence in such a prestigious skill. Considering Dweck’s 2005 interpretation of mindset, this mentality would be considered a “fixed mindset” which does not offer the most opportunities for success in mathematics (Boaler, 2015). This mindset persisted through my formal experiences as a mathematics student.

However, through concluding my teacher preparation program and through years of teaching mathematics, my mindset has shifted drastically to be much more flexible in how I understand math learning and math outcomes. This plasticity is consistent with Boaler’s teaching on having a flexible mind for learning mathematics and flexibly considering who can learn mathematics. Further, thinking about what it means to do mathematics has changed for me; I have developed a deeper understanding of the doing of mathematics (2016).

**Summary**

Comprehensively, it seems that my collection both informs positively and negatively my mathematics capital, as it relates to the findings in literature and in this study. This is similar to what I had noticed, prior to mathematics capital being completely described. I knew that I had a fixed mindset throughout learning mathematics and that my mathematics teaching and learning experiences had been without research-informed practices. Still, my mathematics self-efficacy was quite high and was developed at a young age. It is these types of nuances that support the call for future research, specifically, a SEM approach to best describe what types of experiences
within the collection of mathematics framework that will positively impact mathematics capital and, ultimately, mathematics outcomes.

**Cultural Capital**

Cultural capital is not cultural competence, although cultural competence is the beginning of *possibly* having cultural capital (Bourdieu, 1977). Instead, cultural capital is both cultural competence (which is a flexible and comprehensive term) and cultural competence’s connectedness to habitus and doxa in a society’s field of financial reproduction as much as the cultural competence is reproducible, desired, and utilized in a given society (Bourdieu, 1977, 1993). Paralleling this to mathematics, mathematics capital is not the execution of basic mathematical procedures. Instead, it is proficiency in the types of mathematics that results in qualification in the STEM field which nearly guarantees the convertibility of mathematics success into financial success (Fayer et al., 2017). This type of mathematics would be considered school mathematics as school mathematics is the gatekeeper for a career in mathematics (and STEM). This is how mathematics capital differs from a person’s capacity to perform mathematics. Construction workers must make cuts, measure, and count, all skills that are related to mathematics but not tested in school mathematics. Thus, this is not the type of mathematics that puts a person into a math-related profession that increases their financial gain.

I had an astute awareness of the benefit of school mathematics when I chose to select a graduate degree. I decided to become a math teacher because I watched all too many physical education teachers remain substitute teachers and fight for teaching positions. There is almost always a shortage of mathematics teachers due to teacher retirement and due to math teachers leaving the teaching profession for other professional positions (Ingersoll & Perda, 2009). Having
knowledge of the difference between earning a math teaching degree and a different teaching degree was taught to me.

This was not the only type of cultural competence that I had; instead, my background offers a unique and vast expression of culture and cultural competence. This diversity allowed me to move fluidly in many places where others many have not been able to. Growing up in southern Ontario with a backyard bolstering fruit trees, Lake Ontario, and the Ganaraska river, I was able to develop key skills that have carried me through a lifetime.

Northumberland County contains a small town called Port Hope which is where I resided from the ages of two to ten years old. Port Hope was about one and a half hours from the big city of Toronto and boasts in proximity to Toronto by providing key competitive opportunities, but the small town of Port Hope allowed for tight-knit community networking which was the start of skill building in the culture of a small town. The Ganaraska River runs right through Port Hope and the edge of Port Hope sits on Lake Ontario. A river running through a town might seem like a beautiful site to see but it was so much more. It was a playground beyond all others, offering me key foundational development. The river allowed me to understand tangibly current, depth, and variation. It also allowed me to understand context, and seasonality with the different fish spawning as the seasons changed and the different fishermen utilizing the different fish for varying reasons.

So much play happened outside, in the backyard and the front yard. My siblings, the neighboring children, and I would run through the plum trees, up the alleyway playing all through the days and sometimes evenings. As the youngest of the group and often the only girl allowed to play, I quickly learned what I needed to so that I never got left behind. I worked harder, ran faster, processed more quickly, and adapted to chosen likes and dislikes to all to keep up with and to fit in with the other, older children.
At the age of 10, my family and I relocated to Northern Ontario, a unique French-Canadian community, where we knew no one. There were few cultural competencies from my former life that carried into this new life. Yet, the one that did may have been the most important one: the ability to engage and interact with new people (which was what I had learned by the river). Without missing a beat, I was integrated in the culture and the community by learning new ways that were culturally accepted by the Northern Ontario French.

Continuing through high school was no different for me. I engaged in both sports and clubs sustaining a vital place in my high school as a governing member of student council and the repeated athlete of the year. Not only was I a part of the setting and did I gain skills, it was these sports and opportunities that provided me with financial mobility and opportunity. I earned an NCAA, division 1, full scholarship in track and field which allowed me to graduate with an undergraduate degree debt free. Still, my undergraduate degree offered a new opportunity to earn even more cultural competencies. I learned how to adapt to a new country and the nuances of racial tensions in the United States on a track and field team. As a mixed-race student-athlete, with time, I learned to navigate across all cultural groups. This was a slow process but what I came to realize was to embrace who I was and through self-acceptance came social acceptance and cultural awareness.

With a lavish variety of lived experiences, when I started teaching at Omaha Central in my adulthood, I imagined that there was little left for me to learn about the culture. And yet, I couldn’t have been more inaccurate. I fought to learn the culture of the school that integrated urban students who lived in short proximity of the school and other students who opted in from the suburbs of Omaha to prestigiously attend the International Baccalaureate (IB) program at Central. Similar but vastly different from the nuances of racial tension on the track existed nuances in educational tensions between the students who lived close to Central and those who
opted into Central from the suburbs. I taught classes that contained both types of students and again learned the subcultures. It is in these experiences and the product of these experiences that I believe to be positively influenced by my cultural capital.

**Mathematics Capital**

The research question that followed this study was, “Is there an underlying, operationalizable phenomenon, *mathematics capital*, that has otherwise not been described, offsetting individual student outcomes in mathematics education?” Based on the preceding four chapters, there is evidence to support that there is an underlying operational phenomenon, mathematics capital, that is offsetting individual student outcomes in mathematics education. After a preliminary review in chapter one, it seemed clear that more research needed to be done to discover through a review of mathematics education documents whether mathematics capital emerged. The document analysis in chapter two confirmed the initial allusions of mathematics capital and the research-informed model emerged. This model describes the way that mathematics capital is developed within individuals. Next, in chapter three six participants had a *je ne sais quoi* factor, or more clearly stated *mathematics capital*. This capital carried them easily through elementary school, sustained several participants through failure in mathematics, and offered financial gain to all participants as they received scholarships for their mathematics teacher preparation. In chapter four, a SEM model is presented so that SEM analysis can be done which would allow details of factors within mathematics capital to be described and to further unitize mathematics capital as independent from any other descriptor. This describes the impact of mathematics capital and what leads to the development of mathematics capital but does not directly define mathematics capital. In this chapter, chapter 5, my story and how it relates to the categories of the mathematics capital measurement model is detailed. A personal perspective is used to strengthen and personalize the understanding of mathematics capital.
Mathematics Capital, Defined

Relying on Bourdieu’s definition and conception of cultural, social, and financial capital (1977), mathematics capital is a set of skills, related to mathematics, that are valued by the field of mathematics education, such that these skills allow a person to earn income or to exchange their mathematics capital for other forms of capital. Said differently, mathematics capital is mathematics education’s valued skill in mathematics such that this skill is used for the transmutation of other forms of capital.

A Summary of Mathematics Capital

Mathematics capital mirrors all forms of capital, and is in relation to having skill/expertise in the skill but is not this alone. Where Bourdieu defines doxa and habitus and how it relates to capital is exactly what needs to be incorporated to understand mathematics capital. Yes, it is having skill and experience in mathematics, but it is having skill and expertise in the specific form of mathematics that the field agrees both consciously and subconsciously as valuable and the person with mathematics capital can draw from the mathematics capital in a way that is homogenous with the field of thought embedded within the field of mathematics education. This is the reason for capital, including mathematics capital only being able to be described with a well-defined field.

Take for example, Hippasus, now famous for the discovery of irrational numbers, was not always valued. Yet, it would be false to recognize him in any other way than as a mathematical genius and as a man who changed history. Yet, through the discovery of irrational numbers, he no longer aligned with his present-day doxa and habitus. With that, he is legended to have been drowned at sea. He would not have mathematics capital, although he had mathematical skill. His skillset was set apart from the structured and accepted belief system of
his time. This irradicated him of any type of plausible benefit from his skill. For this reason, he did not have mathematics capital.

On the other hand, consider the six participants in chapter three. These six people all successfully completed advanced mathematic courses, earning double majors in mathematics and mathematics education. These six participants then have mathematical expertise, based on their ability to complete the mathematics coursework. Their mathematics competencies also allowed each of them to receive scholarship money and stable, life-long employment. Their skill in mathematics agrees with the contextual conscious and subconscious values in the skills of mathematics. It is in these that lie their mathematics capital, of which had been impacting them from a young age, as seen in their interviews. Each participant moved through their teaching and learning experiences in mathematics not without failure but when faced with failure, still succeeded. Similar to not just having money and spending it, but instead having money and knowing what to do with it as it adheres to what society agrees with what should be done with it: this is financial capital. Having mathematics skill then is knowing what to do with it as it adheres to society’s agreement with the skills: that is mathematics capital.

**Conclusion and Next Steps**

As the instrument of the study, this research is limited to my interpretation of the data and the findings. As a leader in mathematics education, it is also plausible that my inherent subjectivity and experience impact my interpretation of the definition of mathematics capital (Bourdieu, 1984, 1993). Still, rooted in Bourdieusian teachings, this does not mean doing without attempting to define capital in all contexts, as capital’s inherent power in all contexts cannot go without acknowledgment (Bourdieu, 1977). A thorough analysis of mathematics education documents, and hearing from mathematics teachers, this study provides a starting
point for understanding, describing, and defining mathematics capital, and then, intuitively exists as a call for more research on mathematics capital.
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