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Students' Understanding of Diagrams for Solving Word Problems: A Framework for Assessing Diagram Proficiency

Apryl L. Poch

Delinda Van Garderen

Amy Scheuermann

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A visual representation, such as a diagram, can be a powerful strategy for solving mathematical word problems. However, using a representation to solve mathematical word problems is not as simple as it seems! Many students with learning disabilities struggle to use a diagram effectively and efficiently. This article provides a framework for supporting special educators' use of diagnostic assessment as a means of understanding and identifying areas of need for students with learning disabilities in order to promote diagram proficiency for solving mathematical word problems.

The use of a visual representation, such as a diagram, can be a powerful strategy for helping students with learning disabilities (LD) solve mathematical word problems. Indeed, for students with LD, research suggests that use of a diagram can increase mathematical achievement (Gersten et al., 2009). The advantage of a diagram is that it can serve as a "cognitive tool" to display information in a spatial manner, allowing the problem solver to better understand the problem, and, subsequently, solve the problem (Diezmann & English, 2001). However, the complexity of using representations both as a tool to solve mathematical word problems and as a means of instructing students how to use representations in this process is often underestimated.

In general, a *representation* is something that stands in place for something else—the expression of a concept, idea, relationship, or the like—typically in a visual format (e.g., a symbol, picture, diagram; Smith, 2003). In mathematics, a representation is both a process—the act of creating or expressing a mathematical relationship or concept—and a product—the object, or representation itself (National Council for Teachers of Mathematics [NCTM], 2000). Understanding and effectively using a representation, such as a diagram, as both a process

and a product requires the integration and application of a multitude of concepts and skills.

First, the process of using a diagram to solve a word problem requires *translation*. This process includes (a) identifying relevant and extraneous information in the problem; (b) recognizing that a diagram could help solve the problem; (c) accurately representing the relevant information with an appropriate diagram that depicts relational information; (d) accessing and merging background knowledge with new learning in order to reason; and (e) modifying, or even recreating, the diagram when necessary. After a diagram has been created, additional translation tasks occur, including establishing and explaining the obtained solution via the diagram and putting the diagram back into words to justify and explain the solution as well as the steps taken to obtain the solution (Diezmann, 2000; Diezmann & English, 2001; diSessa, 2004; van Garderen, Scheuermann, & Poch, 2014).

Second, a diagram must also be understood as a product of the process outlined (i.e., what was produced). As a product, then, students need to understand the two characteristics of a diagram: (a) what it is when it stands alone and (b) what it stands for or what it represents. In other words, a student needs to be able to recognize what a diagram is and to understand that it symbolically represents a mathematical idea, concept, or relationship (Uttal, O'Doherty, Newland, Hand, & DeLoache, 2009). Understanding a diagram as both a process and a product is critical if a student is to proficiently use a diagram to solve a word problem. However, using or developing adequate levels of diagram proficiency is not simple.

Findings from several studies (e.g., van Garderen & Montague, 2003; van Garderen, Scheuermann, & Jackson, 2013) clearly demonstrate that if students experience difficulty when using a diagram to solve mathematical word problems, they are more likely to get the problem incorrect. Although some of that difficulty can be attributed to lack of mathematical content knowledge, it also appears that poor use of a cognitive tool, such as a diagram, can be a contributing factor (van Garderen, 2006; van Garderen et al., 2013). Such difficulties suggest the need for a comprehensive assessment of how students use diagrams to determine if indeed some of the difficulty with solving word problems relates to how they use diagrams.

Understanding a diagram as both a process and a product is critical if a student is to proficiently use a diagram to solve a word problem. However, using or developing adequate levels of diagram proficiency is not simple.

Assessing Students With Learning Disabilities' Use of Diagrams to Solve Word Problems

Central to intensive instruction for students with LD is diagnostic assessment (National Center of Intensive Intervention, 2013). Diagnostic assessment is intended to help teachers pinpoint areas in which students require the greatest support during instruction. It also helps educators answer questions such as (a) What does the student know about this topic? (b) Where is the student experiencing difficulty? and (c) How has the student approached the problem?

Diagnostic assessment essentially involves two key steps—collection and analysis of data. In some ways, the collection of data is the most "straightforward" piece. However, data can be collected in different ways. One such approach is flexible interviews. The interview can be used to determine what students understand about diagrams and how they use a diagram during the problem-solving process. In addition, samples of student work for solving mathematical word problems can be collected. Once all the data have been collected, they need to be analyzed to determine areas of difficulty and misconceptions in order to form a hypothesis about potentially effective instructional adaptations. A major assumption during this step is that special educators know what to look for, how to interpret the data, and how to develop inferences to inform their next steps of instruction. But the process of analyzing and interpreting data is much more challenging than it seems (Hill, 2014). Consequently, the information gathered may not be used effectively to guide or focus instruction (Vaughn, Swanson, & Solis, 2013).

Assessment Framework for Diagram Proficiency

The assessment framework for diagram proficiency includes analysis of five competencies: (a) conceptual understanding, (b) procedural fluency, (c) strategic competence, (d) adaptive reasoning, and (e) productive disposition (van Garderen et al., 2014). These competencies, based in part on the National Research Council's five strands of mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001), were identified in a research study that examined, via an interview process, how students with and without LD used diagrams to solve word problems and what difficulties or misconceptions they experienced while using diagrams (see van Garderen et al., 2014). These competencies provide a framework for analyzing student proficiency in using diagrams. Teachers can use the Checklist for Assessing Diagram Proficiency (Figure 1) when

working with students to identify challenges associated with diagram use and plan for instructional support.

Figure 1. Checklist for Assessing Diagram Proficiency

Student: _____ Grade: _____ Date: _____

Instructions: Collect samples of student work and conduct an interview to identify what the student knows and understands about diagrams. Using this information, check which challenges the student may be experiencing. Determine a strand the student needs the most support in and identify an appropriate instructional support to implement. Monitor performance over time and adjust support until the strands are mastered.

Strand	Challenges and misconceptions	Instructional supports
❒ Conceptual understanding	❒ Identifying what a diagram is ❒ Identifying why a diagram can be useful ❒ Providing examples of diagrams	❒ Help students define what constitutes a diagram, know diagrams are more than pictures, and identify how various types of diagrams present relational material ❒ Identify how diagrams are helpful, and how one decides how to visually represent mathematical concepts
❒ Procedural fluency	❒ Matching the representation to the problem ❒ Correctly translating quantitative details from problem to constructed diagram ❒ Correctly identifying key details central to solving the problem	❒ Help students accurately generate and represent quantities and relationships, understand they do not have to draw everything, understand visual representations do not have to look realistic, and to use codes and symbols to represent the problem
❒ Strategic competence	❒ Recognizing and understanding different uses of a diagram ❒ Generating an appropriate diagram for the problem	❒ Help students recognize they can use a diagram to organize information, track and solve the problem, and check and monitor problem solving ❒ Help students generate an appropriate diagram for the problem (i.e., matrices, part-whole, hierarchies, networks, etc.)

❒ Adaptive reasoning	❒ Explaining why they chose the diagram they did ❒ Using the diagram to problem solve ❒ Explaining their reasoning behind the steps taken in problem solving	❒ Help students understand a diagram is a tool that can aid them in the process of problem solving and reasoning and communicating that process ❒ Teach students how to use the diagram to track solutions, ideas, or inferences; decompose and recombine diagrams as needed; explain why they chose the diagram they did
❒ Productive disposition	❒ Generating and utilizing multiple representations to solve a problem ❒ Recognizing that a diagram can be a useful tool ❒ Confidently using diagrams	❒ Help students by encouraging and supporting the use of multiple representations to solve problems, scaffolding instruction to ensure success with diagrams and other visual representations, and providing problems that are sufficiently challenging and warrant the use of a diagram (i.e., contextualized problems) ❒ Help students understand that diagrams can be beneficial and worthwhile

Conceptual Understanding for Diagram Proficiency

Conceptual understanding of diagrams involves a student's comprehension of what a diagram is, the relationships a diagram represents (i.e., quantitative relationships), and how a diagram can be used to solve a problem. Conceptual understanding is a foundational element of diagram proficiency, and it must be continually developed throughout the problem-solving process—first, as students begin generating diagrams and, later, when they begin to reason with their representation. A student who demonstrates conceptual understanding of diagrams can describe what a diagram is (e.g., a representation that depicts the different parts of a word problem and how they may belong together) as well as the ways in which a diagram can be used for solving problems (e.g., to organize data, keep track of what is being solved for, execute a solution, monitor and check the solution).

Two specific difficulties that students with LD may experience related to conceptual understanding include (a) being unable to define or having a limited definition of a diagram and (b) being unable to explain or having a limited explanation of how a diagram could be

used to help them solve mathematical word problems. These difficulties are illustrated in Figure 2 during an interview with two fourth-grade students with LD.

Figure 2. Noah's and Kate's Misconceptions of Diagram Proficiency in Conceptual Understanding

Conceptual understanding	
What is it? – Comprehension of what relationships a diagram can represent and how a diagram can be used when solving a problem.	
Noah	Kate
Problem: In mathematics, what is a diagram? In mathematics, why would you use a diagram?	**Problem:** In mathematics, what is a diagram? In mathematics, why would you use a diagram?
Interview: Interviewer: *Noah, what is a diagram?* Noah: *I know about it, but can't think of it.* Interviewer: *Can you describe a diagram?* Noah: *It's like the problem for the answer.* Interviewer: *Why might you use a diagram to solve math problems?* Noah: *I don't know.*	**Interview:** Interviewer: *Kate, can you tell me what a diagram is?* Kate: *A diagram is a kind of a table and it has all this stuff on it.* Interviewer: *What type of "stuff" does it have on it?* Kate: *How many stuff it is, and what it is.* Interviewer: *Why could you use a diagram to solve math problems?* Kate: *To see what you're solving the problem for.*

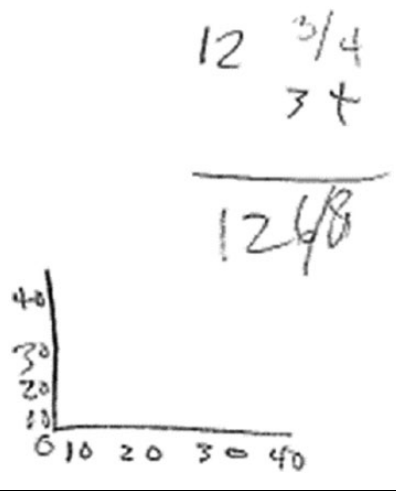
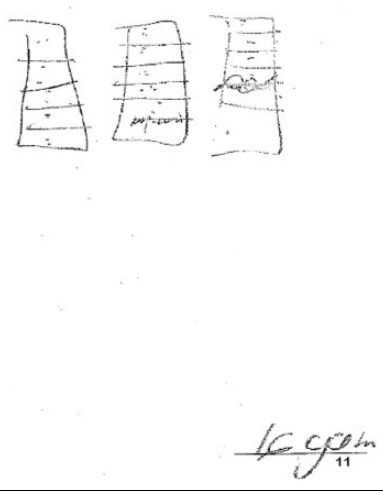
In analyzing Noah's responses, it is clear that Noah lacks a deep understanding of what a diagram is, and he has no sense of how it could be used to help him solve word problems. Noah needs explicit instruction in what a diagram is and the different forms a diagram could take, such as a line diagram, part-whole diagram, matrices/ table, or network/tree diagram.

Although Kate can identify a diagram structure, such as a table, and she recognizes that it can help her with "seeing" the problem, her understanding lacks depth and is limited. Kate needs additional instruction to help her expand her definition of a diagram and the different ways to identify what is known and unknown in the problem.

Procedural Fluency for Diagram Proficiency

Procedural fluency with diagrams is defined and characterized by the ability to accurately and efficiently generate a diagram that represents the problem situation. A primary difficulty students with LD encounter is drawing inaccurate diagrams when compared to the information found in the problem. This difficulty includes (a) the notation of incorrect numbers and quantities, (b) the absence of key details central to solving the problem, and (c) a diagram that does not represent the situation presented in the problem. Difficulties two students with LD experienced with procedural fluency can be seen in Figure 3.

Figure 3. Misconceptions of Diagram Proficiency in Procedural Fluency

<table>
<tr><th colspan="2">Procedural fluency</th></tr>
<tr><td colspan="2">What is it? – Skill in accurately and efficiently generating a diagram that represents the problem situation.</td></tr>
<tr><td>Zack</td><td>Nora</td></tr>
<tr><td>Problem: Adams Middle School students made an Italian submarine sandwich that was 12 ¾ feet long. After making it, they decided to divide the sandwich into smaller portions to share with other students. If each portion was ¾ of a foot long, how many students would get a portion?</td><td>Problem: Robin has 3 packages of gum. Each package has 5 pieces. How many pieces of gum does Robin have altogether?</td></tr>
<tr><td>Interview:
Interviewer gives Zack, a seventh-grader with LD, the word problem to solve. After Zack solves the problem, the interviewer asks, “How did you solve this problem?”
Zack: The only way I would do it is like this, 6 over 8 and then 12.
Interviewer: Tell me about your diagram.
Zack: I drew it because I was told to. I don’t know how many students there are.</td><td>Interview:
Interviewer gives Nora, a fourth-grader with LD, the problem to solve.
Interviewer: Tell me how you got your answer to this problem.
Nora: I put three rectangles then I put 5 lines on it for the pieces of gum. Then I counted them all and I got 16.</td></tr>
<tr><td>Diagram:
 
</td><td>Diagram:
  </td></tr>
</table>

In analyzing Zack's work, he draws an inaccurate diagram; in particular, he generates a diagram that clearly does not match the problem situation. Zack needs to learn how to generate a diagram that matches the problem situation. For this problem, a matching diagram would include a "sandwich" (e.g., line, oval, bar) that is labeled as 12 3/4 feet long, which can then be divided into segments.

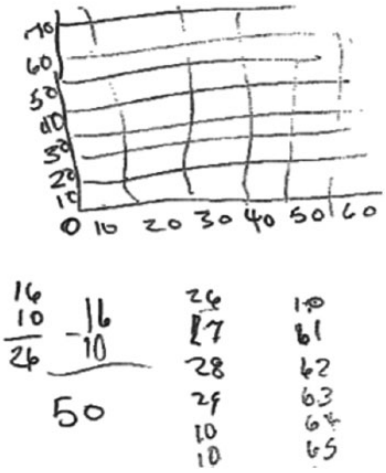
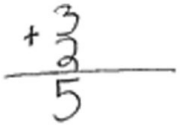
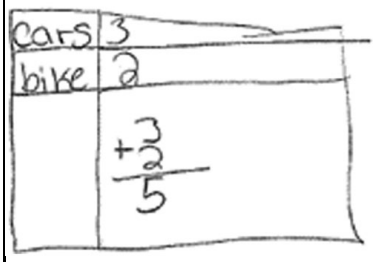
Although Nora also explains later in the exchange that her diagram was helpful in solving the problem, Nora divides up two of the packages of gum incorrectly. Although she tries to correct the diagram, the inaccuracies contributed to an incorrect count. Nora needs additional practice in learning procedures to generate an accurate diagram (e.g., how to divide a figure up for a desired quantity).

Strategic Competence for Diagram Proficiency

Strategic competence with diagrams is the ability to represent and use a diagram to solve a problem. It is evidenced when a student uses a relevant diagram as a tool to understand (e.g., organize data, depict the quantitative relationships in a problem, identify what is known and what is to be solved), solve (e.g., calculate an answer, track a solution), and monitor (e.g., self-check, self-correct) the problem-solving process. When a student has created a diagram that accurately represents the problem situation, it is more likely that the student can solve the problem and monitor the steps needed to obtain a solution using the diagram. Unfortunately, some students with LD inaccurately believe that one representational form is appropriate for all problems (e.g., bar graph) or particular types of problems (e.g., pie graphs = fractions). The creation and continuation of these mathematical and diagrammatic misconceptions not only creates confusion for students, it limits their mathematical understanding and progression. Consider the scenarios involving Alejandro and Sara provided in Figure 4.

Figure 4. Misconceptions of Diagram Proficiency in Strategic Competence

Strategic competence	
What is it? – Ability to represent and use a diagram to solve a problem.	
Alejandro	Sara
Problem: My younger brother has $16 more than my sister Sue. Sue has $10 more than my brother Paul. The three of them have a total of $66. How much does Paul have?	**Problem:** A school parking lot has only bicycles and cars. On Monday there were 3 bicycles and 2 cars in the parking lot. How many wheels were in the parking lot?

Interview:	**Interview:**
Interviewer gives Alejandro, a seventh-grader with LD, the problem to solve. After solving it, the interviewer asks Alejandro, "Explain to me how you solved the problem." Alejandro: *I just put 16, 10, and that came out to be 6. And they have $66 altogether. I did that, and I couldn't do it without making it even by 10's, so I did 27, 28, 29, and then I put 10s right here, 30, 40, 50, 60, and then 61, 62, all the way down to 66. Then add them up by 10, 20, 30, 40, then I added these and these and got 50.* Interviewer: *How did the picture help you solve the problem?* Alejandro: *In no way; I was just drawing it to draw the diagram.* Interviewer: *Why wasn't it helpful?* Alejandro: *Because it was sloppy. It's not even; the lines are all messed up.*	Interviewer gives Sara, a fourth-grader with LD, the problem to solve. After Sara solves the problem, the interviewer asks, "How come you didn't use a picture or diagram to solve this problem?" Sara: *I didn't really need to, because I would come up with the same answer.* Interviewer: *Why don't you think a picture or diagram would have been helpful?* Sara: *Because I would have wrote down the same thing.* Interviewer: *If you had drawn a picture or diagram, what would it have looked like?* Sara then draws a table. The interviewer then asks, *Why do you think a diagram was helpful last time, but not on this problem?* Sara: *Because, I thought I would need to use it on that one, but on this one I didn't think I needed to use it.*
Diagram:  	**Diagram:**  

Based on the interview with Alejandro, it appears that he does not recognize the strength of a diagram as a tool to both understand the problem and develop a solution; it is just something he is supposed to do. Clearly, a place to start with Alejandro is instruction to help him recognize the way a diagram can help him understand the problem by drawing what is known and generating a symbol to represent what is unknown.

Sara's approach to solving the problem—with her accompanying diagram constructed

as an end product—suggests that she does not seem to recognize that a diagram can serve as a tool to help her understand, solve, and monitor her work. It would be helpful for Sara to see a teacher modeling how to use the diagram throughout the problem-solving process for organizing the data from the problem to understand what is being solved for, for tracking a solution on a diagram as it is being solved, and as a self-check to make sure that the unknown has been determined.

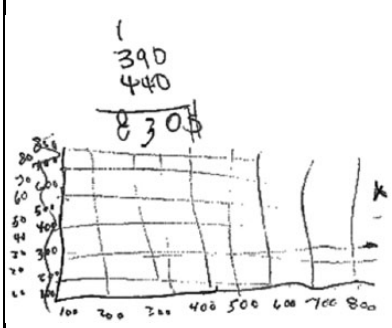
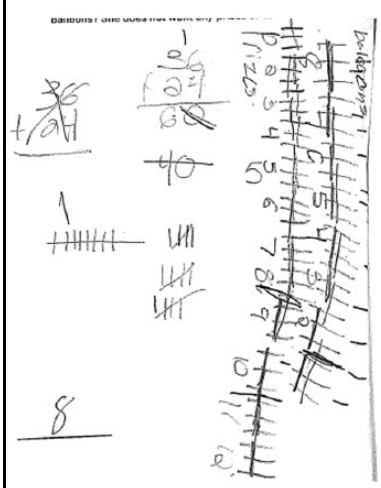
> *When students are cognizant of the reasons behind constructing a diagram, they understand and can communicate the purpose(s) behind using the diagram.*

Adaptive Reasoning for Diagram Proficiency

Adaptive reasoning with diagrams is the ability to justify and communicate an explanation of one's use of a diagram back to the problem while problem solving. Students who possess adaptive reasoning demonstrate the ability to explain and justify how they selected an appropriate diagram and how they used it to solve the problem. When students are cognizant of the reasons behind constructing a diagram, they understand and can communicate the purpose(s) behind using the diagram. Students with LD who struggle with adaptive reasoning may experience some of the following difficulties as they try to reason with a diagram: (a) getting "lost" in the process of trying to use a diagram to solve a problem; (b) getting overly focused on a diagram and not connecting back to the problem situation; (c) perceiving a diagram as being helpful though they are unable to explain how they used the diagram and may, instead, refer to the computation they used to solve the problem; or (d) incorrectly using an appropriate diagram as they solve the problem. Chloe and D'Quajanay also struggle with these same difficulties as demonstrated in Figure 5.

Figure 5. Misconceptions of Diagram Proficiency in Adaptive Reasoning

Adaptive reasoning	
What is it? – Ability to justify and communicate an explanation of one's use of a diagram to solve a problem.	
Chloe	D'Quajanay
Problem: The rent for an apartment on the 10th floor is $390 per month. The rent on the 15th floor is $440 per month. At this rate of increase, what is the rent for an apartment on the 30th floor?	**Problem:** Meredith is planning a Halloween party. She has 36 prizes and 24 balloons. What is the most number of children she can invite so each child gets an equal number of prizes and an equal number of balloons? She does not want any prizes or balloons left over.

Interview: Interviewer gives Chloe, a seventh-grader with LD, the problem to solve. Chloe immediately writes the number 390 over 440, and proceeds to add the numbers. The interviewer prompts Chloe to use a diagram to solve the problem. After Chloe solves the problem, the interviewer asks, "In what way did you use a picture or diagram to solve this problem?" Chloe: *I just did it by 100's, and then go up by 10's, and then you can stop at 800, go up by 10, and stop when it says 30 and put a mark there.* Interviewer: *You had your answer here, so did you find it helpful to draw your diagram?* Chloe: *It was a little bit helpful. Just by saying 830.*	**Interview:** Interviewer gives D'Quajanay, a fifth-grader with LD, the word problem. After she finishes it, the interviewer asks, "How did you solve the problem to get an answer of 8?" D'Quajanay: *Ok, um, I had some spares left over so I made them share.* Interviewer: *Ok, so give me a little more explanation on what way you used your picture to help you solve this problem.* D'Quajanay: *Um, it showed me how many prizes I had and how many balloons I did and then I took them all up in 3's. Well, first I did 4's, but that didn't work and then I put them in 3's and it worked and then I saw that I had more prizes than I did balloons and I had some spares over so I made them share 4 and then 4 and then 4 and it worked that way if I did it like that. If I didn't do it like that I would have a whole bunch of these spares over and then down here it says leftovers, no leftovers.*
Diagram:  	**Diagram:** 

Although Chloe explains that her diagram is helpful, in her brief response, she does

not use her diagram to communicate how she solved the problem. Therefore, Chloe needs to learn how she could use her diagram to help her reason (e.g., "If I look at my diagram, to solve this problem means I have to find this unknown first followed by this unknown") and explain her approach to problem solving (e.g., "I had two unknowns in my diagram, which meant I had to do several calculations").

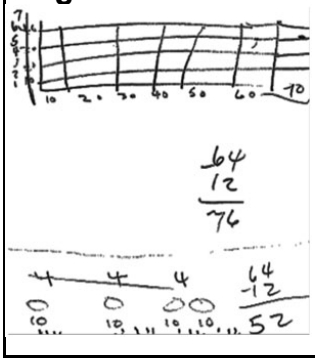
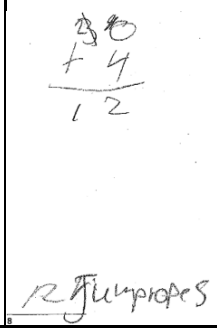
In reviewing D'Quajanay's diagram and response, she seems to get lost in the process of solving the problem with the diagram; though she recognizes that there cannot be any balloons or prizes left over, she does not equally share the prizes. When it comes to using her diagram as a way to reason to solve the problem, she needs to be taught helpful ways to track and organize her solution process (e.g., tally marks) as she uses the diagram.

Productive Disposition for Diagram Proficiency

Productive disposition with diagrams involves seeing a diagram as beneficial, worthwhile, and sensible for solving a problem, coupled with the belief and confidence in one's own ability to use it. When students feel competent using diagrams, they are both more likely to continue using them as a representational form, thus reinforcing their effectiveness, and more likely to further refine their skills through continued use and exploration. Unfortunately, many students with LD do not perceive a diagram to be of benefit, nor do they have confidence in using one (van Garderen et al., 2014). When students perceive a mathematical problem as too difficult, and they possess an incomplete understanding or misconception surrounding what a diagram is and how it can be used, they will struggle to see the diagram as a useful tool. See Jeremy's and Lara's struggle and disposition toward using a diagram to solve word problems in Figure 6.

Figure 6. Misconceptions of Diagram Proficiency in Productive Disposition

Productive disposition	
What is it? – See a diagram as beneficial, worthwhile and sensible for solving a problem coupled with a belief and confidence in one's ability to use it.	
Jeremy	Lara
Problem: Greg had 64 baseball cards. He gave 12 cards to his sister. Then he divided the remaining cards equally among his four friends. How many cards did each of his friends get?	**Problem:** The teacher has 30 feet of rope. She cuts off pieces 4 feet long to make jump ropes for her class. How many jump ropes can she make?

Interview:	**Interview:**
Interviewer asks Jeremy, a seventh-grader with LD, to solve a problem. After Jeremy solves the problem, the interviewer asks him, “Did you find your picture to be helpful to solve the problem?” Jeremy: *Yes.* Interviewer: *In what way?* Jeremy: *I don’t know. Not that many ways.* Interviewer: Rereads the problem to Jeremy and asks him to draw another picture. After he is done, interviewer asks, “Just out of curiosity, what do you think this is?” Jeremy: *The people. And then how much I gave them.* Interviewer: *Do you know what I would call this?* Jeremy: *What?* Interviewer: *A diagram. Nobody has pointed that out to you?* Jeremy: *Some people have.* Interviewer: *Did you find that helpful to do?* Jeremy: *Yes!* Interviewer: *Why?* Jeremy: *Because it was easier.* Interviewer: *In what way was it easier?* Jeremy: *Because all I had to do was make faces and put tally marks by it.* Interviewer: *Is there any other reason why this is easier?* Jeremy: *Because it’s separate, you know what each of them get.*	Interviwer asks Lara, a fourth-grader with LD, to solve a problem. After Lara solves the problem, the interviewer asks, “How did you solve this problem?” Lara: *I put 30 plus 4, and then I got the answer of 12.* Interviewer: *In what way did you use a picture to solve this problem?* Lara: *I don’t know.* Interviewer: *Did you use one?* Lara: *No.* Interviewer: *Can you tell me why you didn’t use one?* Lara: *It would be easier to make 30 plus 4, because it would be easier instead of making a line that’s 30 feet long on the piece of paper.*
Diagram: 	**Diagram:** 

In analyzing the conversation, it is clear that Jeremy struggles until he creates his own “memory” tool to solve the problem. It also becomes clear that Jeremy has been restricting himself

to using what he thought a diagram had to be, in this case, a graph that always took the same shape. Although he concludes that his new diagram helped him, he also portrays a lack of confidence in his ability to use a diagram throughout his response. To help Jeremy gain more confidence for using diagrams, it may help to provide simpler problems that enable him to focus on successfully generating diagrams that effectively represent the problem situation.

> *Helping students with LD see diagrams as more than "pictures" can be a transformative process that opens up future mathematical development and understanding.*

Lara does not perceive the diagram to be of benefit because she solves the problem using computation. What she is unaware of is that her computation does not match what the problem is asking her to solve for. She needs to be shown how a diagram can be beneficial and worthwhile, particularly as a way to ensure that she is answering the question the problem is presenting.

Final Thoughts

A critical component of a mathematically powerful classroom is the use of assessment. The importance of assessment cannot be overstated. It is through assessment that solicits information about student thinking that teachers can design and provide powerful instruction that is targeted to meet students' needs (Schoenfeld, 2014). However, when examining student work, it is not always clear or easy to determine student need. In this article, we provided an assessment framework that can be used as a way to examine how well students with LD can use a diagram as a tool for solving mathematical word problems. We highlighted several misconceptions and challenges students with LD might experience for each competency within the framework. Although the difficulties and misconceptions of using diagrams presented here were restricted to students with LD, we want to acknowledge that students without disabilities may also experience some of these difficulties and need instructional support as well (see van Garderen et al., 2014). Not only can the framework help identify areas of difficulty, but it provides a platform from which to provide targeted instruction. For additional resources and supports for teaching students with LD how

to use diagrams for solving word problems, we refer readers to Figure 7. Helping students with LD see diagrams as more than "pictures" can be a transformative process that opens up future mathematical development and understanding.

Figure 7. Resources to Support Instruction for Developing Diagram Proficiency

Diezmann, C. M., & English, L. D. (2001). Promoting the use of diagrams as tools for thinking. In Albert A. Cuoco and Frances R. Curcio (Eds.). *The roles of representation in school mathematics: 2001 yearbook* (pp. 77-89). Reston, VI: National Council of Teachers of Mathematics.

This chapter provides a summary of four different general-purpose diagrams that students can use when solving word problems. Examples of student work are provided along with good general classroom instructional recommendations for teaching students how to generate and use the diagrams for solving word problems.

Hudson, P., & Miller, S. P. (2006). *Designing and implementing mathematics instruction for students with diverse learning needs*. Boston, MA: Allyn & Bacon.

This book provides information on using visual representations, including diagrams, for various content areas in mathematics. Specific suggestions for using visual displays are found in Chapter 3 related to promoting and developing conceptual understanding in mathematics. Information related to the Concrete-Representational- Abstract instructional process is provided throughout this text.

Jitendra, A. K. (2007). *Solving math word problems. Teaching students with learning disabilities using schema-based instruction*. Austin, TX: Pro-Ed.

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Van Garderen, D. (2006). Teaching visual representation for mathematics problem solving. In M. Montague and A. Jitendra (Eds.) *Middle school students with mathematics difficulties* (pp. 72-88). New York, NY: Guilford Press.

This chapter provides specific instructional suggestions for teaching students with disabilities how to generate diagrams and use them appropriately when solving word problems. Emphasis is placed on developing conceptual understanding, diagram generation, and reasoning with a diagram. A four-part cognitive strategy for solving word problems that incorporates diagrams is provided along with sample lesson scripts and examples of student work.

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Apryl L. Poch, *Doctoral Candidate, Department of Special Education, University of Missouri – Columbia , and* **Delinda van Garderen**, *PhD Associate Professor of Special Education, University of Missouri – Columbia, Department of Special Education, University of Missouri;* **Amy M. Scheuermann**, *PhD Assistant Professor Educational Studies: K-12 and Secondary Programs, Minnesota State University, Mankato, Educational Studies: K–12 and Secondary Programs, Minnesota State University.*