Challenges students identified with a learning disability and as high-achieving experience when using diagrams as a visualization tool to solve mathematics word problems

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Abstract
This article addresses a much understudied topic and concern regarding how students of varying ability levels employ visualization as a strategy in mathematics learning. The importance of this topic can be found in its connection to students’ ability to solve mathematical word problems. Many students, particularly students with learning disabilities, often struggle to use visualization as a strategy and this impacts their mathematics performance. The purpose of this article is to present findings from a study that examined the challenges that students—those identified as learning disabled and high-achieving—displayed when using one visualization form, a diagram, to solve mathematics problems. Overall, nine challenges related to the use of diagram proficiency to solve problems were identified. Further, students with learning disabilities were found to be more likely than their high achieving peers to experience these challenges. Implications for practice are provided.

Keywords
Learning disability, Ability, Diagrams, Challenges

1 Introduction

Broadly defined a “representation” is a configuration that stands for something else (Goldin, 2002; Kaput, 1987). Although often considered an artificial dichotomy (Lesh, Behr, & Post, 1987) within mathematics, a representation could refer to both an
internal and external manifestation of a mathematical idea or concept (Pape & Tchoshanov, 2001). Further, it could also refer to the act of taking an internal representation or mental representation and externalizing it (Pape & Tchoshanov, 2001). Therefore, as noted within the National Council for Teachers of Mathematics [NCTM] Standards (2000) document, a representation “refers both to process and to product—to the act of capturing a mathematical concept or relationship in some form and to the form itself” (p. 67).

Many different representational forms exist (Zawojewski & Lesh, 2003), and all, to some extent, can be used to solve word problems. However, one often recommended strategy for solving mathematical word problems is to use visual (external) representations—in particular, a diagram (Lesh, 1999; Pape & Tchoshanov, 2001; Polya, 1957; Shigematsu & Sowder, 1994). In studying diagrams, it is useful to distinguish between different types that may be used for problem solving (Zahner & Corter, 2010). The first type is a diagram that frequently accompanies a text or problem (e.g., Mayer, 1989; Mayer & Gallini, 1990) or may support a particular type of mathematics problem using a given schematic format (e.g., Jitendra et al., 2013). The second type, and the focus of this study, is student-generated diagrams that are both prompted for and spontaneously initiated while engaged in problem solving (similar to studies conducted by Diezmann, 2000; Schwartz & Martin, 2004; Stylianou, 2002; Zahner & Corter, 2010). Furthermore, we are particularly interested in the use of student generated diagrams (e.g., tables, part-whole diagrams) that convey “information in a spatial layout” (Diezmann & English, 2001, p. 77) and cross various mathematical domains (e.g., algebra, number and operations, proportionality, probability).

2 Use of a diagram to solve word problems

The use of diagrams can be an extremely “powerful” visual representation strategy when solving word problems. In the research, this is evidenced in two ways. First, numerous studies that incorporated the systematic use of a diagram have led to substantial improvements in mathematics achievement for students with disabilities (e.g., Butler et al. 2003; Jitendra et al. 2002; Maccini & Hughes, 2000; van Garderen, 2007; Witzel et al. 2003). Second, a connection between the type of visual
representation generated and overall problem solving performance has been established (Brown & Presmeg, 1993; Hegarty & Kozhevnikov, 1999; Owens & Clements, 1998; Presmeg, 1986a, 1986b; van Garderen & Montague, 2003). Specifically, the use of imagery considered schematic (images that contain relational information of the problem) has been positively related to success in mathematical problem solving, whereas the use of imagery considered pictorial (images that depict the visual appearance of objects or persons in a problem) has been negatively related to success in mathematical problem solving (Hegarty & Kozhevnikov, 1999; van Garderen & Montague, 2003; van Garderen et al. 2012; Zahner & Corter, 2010). In addition, the more strategic ways in which a diagram was used (e.g., not only to understand the problem but also solve and monitor problem solving) has been positively correlated to higher performance for solving word problems (van Garderen et al. 2012).

Beyond the evidence related to mathematics performance, there are a number of other compelling reasons for using diagrams as a means to solve word problems. First, diagrams as a representation strategy demonstrate great versatility as they can be used for solving various types of problems for many topic areas (e.g., geometry, number and operations, probability) and at all grade levels (Diezmann & English, 2001; Ives & Hoy, 2003; Novick et al. 1999; Zahner & Corter, 2010). Second, diagrams are powerful ways to facilitate communication about critical ideas in mathematics as well as provide a platform for sharing problem solving strategies with others (Stylianou, 2010). As a result, both teachers and students can easily monitor the problem solving process with a diagram to evaluate and improve upon its use, thereby increasing problem solving performance (Kulm, 1994). Third, a diagram is a tool that can be used in multiple ways during the problem solving process. As Stylianou (2010, p. 329) notes, a representation—in our case a diagram—can serve (1) as a means to understand the problem situation, (2) as a way to record information both of the problem situation itself and of ideas as the problem is being solved, (3) as a tool to facilitate exploration of critical concepts of the problem being solved, and (4) as a way to monitor and evaluate progress.
2.1 Proficiency for using diagrams to solve mathematics problems

The ability to use a diagram as a tool for solving word problems should not be underestimated; it is a complex task. Representing the information from a mathematics problem as a diagram involves decoding the linguistic information that then is encoded into visual information (Diezmann, 2000). This “translation” process depends on the ability to selectively encode what is relevant, selectively combine the information into an integrated form, and selectively compare that information to identify new knowledge and connect to prior knowledge about the problem and, of particular interest to this study, the diagram (Sternberg, 1990, cited in Diezmann, 2000). Furthermore, encoding information from a mathematics problem into a diagram requires an extensive knowledge base. This includes knowledge related to the ability to select, produce and productively use a diagram as a problem solving tool as well as the ability to critique and modify or generate a new diagram where needed within the context of a problem solving situation (Diezmann & English, 2001; diSessa, 2002; diSessa & Sherin, 2000). The ability to use a diagram proficiently for solving a word problem would be the goal.

In examining what it means to be “mathematically proficient,” the National Research Council (NRC; 2001) suggested that it involves five core strands or components: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Each strand provides a framework for “discussing the knowledge, skills, abilities, and beliefs that constitute mathematical proficiency” (NRC, 2001) as a whole, but also to the various mathematical domains (e.g., algebra, geometry, number and operations, etc.) (NRC, 2001, pp. 141–142). We also contend, however, that such a framework can also be applied to using a mathematical process such as a diagram. Figure 1 describes the five strands of “diagram proficiency” along with the knowledge, skills, abilities, and beliefs associated with each strand. An important point to make about these five strands is that they are interwoven and interdependent. For example, to use a diagram in a strategic manner does require conceptual understanding of what a diagram is and how it may help to solve a problem. However, this framework functions as a way to guide instruction and provides a way to analyze student proficiency (e.g., student can generate a diagram accurately and efficiently for a given problem) (NRC, 2001).
2.1.1 Conceptual understanding

Diezmann and English (2001) define a diagram as a “visual representation that displays information in a spatial layout” (p. 77). Therefore, a diagram is not simply a picture or piece of artwork that portrays surface details; rather it depicts mathematical relations of the problem situation (Fosnot & Dolk, 2001; van Garderen, 2006). In addition to understanding what a diagram is, knowing how it may be of use in the process of solving a problem is important. As previously noted, diagrams can be used in multiple ways to understand, solve, and monitor progress when solving a problem (Stylianou, 2010).

<table>
<thead>
<tr>
<th>Strand</th>
<th>Description</th>
<th>Abilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptual Understanding</td>
<td>Comprehension of what relationships a diagram can represent and how a diagram can be used when solving a problem.</td>
<td>Can describe what a diagram is and the ways a diagram can be used for solving a problem.</td>
</tr>
<tr>
<td>Procedural Fluency</td>
<td>Skill in accurately and efficiently generating a diagram that represents the problem situation.</td>
<td>Can generate a diagram that represents the problem situation.</td>
</tr>
<tr>
<td>Strategic Competence</td>
<td>Ability to represent and use a diagram to solve a problem.</td>
<td>Can use a relevant diagram as a tool to represent, solve and monitor the problem solving process.</td>
</tr>
<tr>
<td>Adaptive Reasoning</td>
<td>Ability to justify and communicate an explanation of one’s use of a diagram to solve a problem.</td>
<td>Can explain and justify how the diagram that was used was appropriate to help solve the problem.</td>
</tr>
<tr>
<td>Productive Disposition</td>
<td>See a diagram as beneficial, worthwhile and sensible for solving a problem coupled with a belief and confidence in one’s own ability to use it.</td>
<td>Recognizes when a diagram can be beneficial for solving a problem and will self-initiate use, with confidence, as needed.</td>
</tr>
</tbody>
</table>

Fig. 1 Strands of diagram proficiency. (Note. Adapted from NRC, 2001 p. 117)

2.1.2 Procedural fluency

Procedural fluency in mathematics is the knowledge of procedures first in terms of knowing when and how to apply a procedure but also performing this procedure
flexibly, accurately, and efficiently (NRC, 2001). Likewise, procedural fluency in terms of a diagram requires skill in accurately and efficiently generating a diagram that represents the problem situation. This includes recognizing that a diagram needs to depict key components of the problem situation, but does not necessarily have to depict all objects or things stated in a problem, nor does it need to look realistic. In addition, it should not take long to generate which may mean using appropriate symbols or codes to represent the information (van Garderen, 2006).

2.1.3 Strategic competence

Critical to solving any problem is the ability to use a variety of strategies that might be useful for solving a specific problem (NRC, 2001). The “strength” of a diagram as a strategy is that it can take various forms depending on the type of problem to be solved (Diezmann & English, 2001; Novick et al. 1999). Novick et al. (1999), for example, identified three general-purpose diagrams (line diagram, matrices, hierarchies) and Diezmann (1999) identified one general purpose diagram (part-whole) that can be used to solve certain types of problems (e.g., line diagram or net-work that is best suited for problems that require objects or things to be put in order). While we acknowledge that generating a relevant diagram for a given problem does depend on understanding the mathematics content in the problem, we suggest that diagram strategic competence involves generating an appropriate diagram (i.e., using a form that best suits the quantitative relationships expressed in the problem) and also using the diagram as a tool whether to understand, solve, or monitor the problem solving process.

2.1.4 Adaptive reasoning

Presmeg (1986b) noted that for imagery to be useful it needs to be coupled with analysis and thought. When using a diagram to solve a problem, it should be used as a part of the reasoning process (i.e., “think logically about the relationship among concepts and situations” [NRC, 2001, p. 129]) in order to not only reach a solution, but also as a way to explain and justify (i.e., “provide sufficient reason for” [NRC, 2001, p. 130]) the solution. This suggests that students recognize that a diagram is a tool and can explain and reason how they used it as a tool rather than just an “end product” that
can aid them in the process of solving a problem (Pape & Tchoshanov, 2001). Further, this implies a need for accuracy and precision when using a diagram as a tool to aid the reasoning process (Diezmann & English, 2001; van Garderen, 2006).

2.1.5 **Productive disposition**

Critical to the spontaneous use of diagrams to solve mathematics problems is the need for confidence, along with the perception that diagrams are “easy” to use and not just a teacher strategy (Uesaka et al. 2007). Therefore, a productive disposition towards diagrams includes the recognition that a diagram can be beneficial and worthwhile to use as a strategy for solving a problem along with the belief and confidence in the use of a diagram. This also suggests that students will, as their conceptual, procedural, strategic, and adaptive reasoning abilities develop, self-initiate use of a diagram when needed and appropriately select diagrams for a given problem-solving situation.

2.2 Diagram proficiency of school-aged students with learning disabilities

While poor conceptual understanding of the mathematics has been cited as a reason for interfering with mathematics performance (e.g., Dufour-Janvier et al. 1987; van Garderen, 2007), poor problem solving performance may also be the result of limited diagram proficiency (Diezmann, 2000; Dufour-Janvier et al. 1987; Larkin & Simon, 1987). If a student does not have an adequate proficiency for using diagrams, a diagram as a tool will have limited usefulness (Pape and Tchoshanov 2001). As Dufour-Janvier et al. (1987) noted, a representation can only be useful to the extent it has been “grasped” by the child.

From the limited research available focused on school-aged children (K-12), three main findings emerge about the difficulty students with a learning disability (LD) have using visual representations, in particular diagrams. First, they infrequently, if at all, use any visual representation strategies when solving mathematical word problems (Montague & Applegate, 1993a, 1993b; van Garderen & Montague, 2003). Second, when a diagram is generated, it is often poor in quality and lacking in important relational information (van Garderen & Montague, 2003; van Garderen et al. 2012). Third, students with LD appear to use their visual representations in fewer strategic
ways (e.g., to organize, plan, monitor, compute, justify) than their peers when solving word problems (van Garderen et al. 2012).

Although it is clear that many students with LD appear to struggle to use visual representations for solving word problems, to date, no studies have examined the specific challenges these students may experience as they use diagrams as a tool to solve mathematics word problems. However, two studies, which focus on school-age students without disabilities, have examined student use of diagrams while solving problems and provide potential insights as to the difficulties students with LD may experience.

Diezmann (2000) examined 12 students’ (mean age of 10 years 3 months) responses to five novel problems during an interview and identified three categories of difficulties the students displayed. The first category was that students did not use a diagram. Reasons for this included a lack of understanding the term and concept of a diagram, and that a diagram is a representation that utilizes a scale. The second category related to generic difficulties in the generation of a diagram. Specifically students drew diagrams that were unusable as they were either too small or untidy to allowed for modification or manipulation, or contained inaccuracies related to quantities. The third category involved idiosyncratic difficulties related to the generation of a diagram. Specific difficulties included a lack of precision in the diagrams, overlooking the constraints in a diagram, incorrect labeling of the starting position of a diagram, and incorrect portrayal of a relationship within a diagram.

Smith (1999, cited in Smith, 2003) examined elementary-aged student responses focused on the creation and use of representations, in this case, diagrams, from a child’s perspective to solve non-routine problems. In a summary of four children’s responses to problems involving portioning, several interesting findings pertinent to the current study were identified. First, it is possible to overemphasize the context of a problem in the representation and not engage with the abstraction of mathematics (e.g., the quantitative relationship via an algorithm). However, it is also possible to overemphasize the abstraction of the mathematics to the exclusion of the context in the process of examining the reasonableness of an answer. Finally, the use of representations may be perceived as being designed only to solve a particular problem,
but not be seen as a tool for general use.

3 Purpose of the current study

Unfortunately, not all students with LD develop acceptable levels of proficiency in the use of a diagram (van Garderen & Montague, 2003; van Garderen et al. 2012). To overcome this requires the provision of instruction that supports the correct representation of problems (Rittle-Johnson et al. 2001). However, exactly how this should be supported in the instruction is less clear. One particularly useful approach to help guide instructional decision-making is examining students’ thinking—in particular their errors (Ashlock, 2006; Fennema et al. 1993). While errors in and of themselves are not bad as they can be seen as “an essential component of the construction of knowledge” and “indicators that learning is taking place” (Knight, 2002, p. 7), it is also possible that these errors may prevent or misdirect mathematical learning that, if not addressed, become entrenched and accepted as accurate. Therefore, errors can also be viewed as diagnostic tools that provide insight into students’ processes and thinking and suggest a focus for remedial instruction (Pressley et al. 1992). Instructionally, these gaps and misconceptions could then be considered as “sites for learning” that are essential for the student’s growth as a mathematician (Hiebert et al. 1997). Additionally, many studies compare how students of exceptional ability versus students of lower ability solve problems (e.g., Montague & Applegate, 1993a, 1993b). Through this type of comparison, it is possible to identify areas of weakness or challenge that the “novice” experiences as compared to the preferred ways in which an “expert” would solve problems.

The purpose of this study is to identify and compare what challenges students, specifically students identified as high achieving (HA) and students with LD, experience when using diagrams to solve mathematics problems.

4 Method

4.1 Participants and setting

The data utilized in this study comes from a larger study that quantitatively examined the understanding and use of diagrams to solve mathematics word problems.
Participants included 95 fourth through seventh grade students with and without LD from Ohio and Missouri (see van Garderen et al. 2012). Every student who participated in the original study returned a signed consent form and, according to school records, had English as his/her primary language. For this study, the data from 28 students, 16 students with LD and 12 students identified as HA in mathematics, was qualitatively examined in order to more clearly identify the challenges or misconceptions students experience when using a diagram to solve word problems. Students with LD met their local district eligibility criteria that follows the federal regulations (Federal Regulation 34 C.F.R § 300.8 (c)(10); see Appendix A for summary of criteria), and had a full-scale IQ score of 80 or higher on the Wechsler Intelligence Scale for Children—IV (2003). The students with LD did not have to have a specific mathematics learning disability to participate; all who were being served within the category of learning disability were included. However, on average, all students with LD were lower performing in mathematics than their peers without LD (see van Garderen et al. 2012 for more details regarding their mathematical performance) based on assessment scores using the Key-Math 3 Diagnostic Assessment (KeyMath 3; Connolly, 2007). The KeyMath 3 is a norm-referenced measure of essential mathematical concepts and skills across the mathematical strands.

To maintain balance between the number of students with a LD and those identified as HA, the strongest three students classified as HA in mathematics from each grade level were examined. Students identified as HA had to have scale scores of 15 or above on 2 or more of the KeyMath 3 subtests (numeration, applied problem solving, addition and subtraction, or multiplication and division). If more than three individuals in a grade level met the scale score criteria, the three with the highest total scores on the Non- Routine Word Problem Assessment (NWPA; van Garderen et al. 2012) were selected. Subject demographic data is presented in Table 1.

4.2 Research design and instruments

A qualitative analysis of the students’ responses to transcribed interview questions about diagrams, their use when solving mathematics word problems, and their actual problem solving was completed in order to identify the mathematical
understandings and misconceptions students with and without LD had. Data were collected from student interviews in which the students were asked: “In mathematics, what is a diagram?” and “In mathematics, why would you use a diagram?,” along with student work samples including their explanations of their mathematical problem solving and diagram usage when solving both prompted and unprompted word problems from the NWPA. In this measure, the students needed to solve up to 12 mathematical problems (minimum of 8) at various levels and explain their processing (e.g., “Tell me how you solved this problem?”, “In what way did you use a picture to solve this word problem?”). For the last six problems the students were prompted to use a diagram when solving. All problems were read to the students and written responses were collected via student workbook while verbal responses were audio recorded and then transcribed.

4.3 Data analysis
A content analysis approach was used to analyze the data. The analysis itself involved several steps that followed a constant comparative data analysis procedure in which data were analyzed systematically (Corbin & Strauss, 2008). At each step, data were analyzed independently then compared across researchers and re-analyzed to confirm findings. To establish a general sense of the data, students' written and verbal responses to each word problem, the unit of analysis, were read multiple times by the second and third author. First, they independently analyzed the data collected only from the students with LD and generated codes to categorize the data into frequently occurring challenges. While the solution accuracy of each problem was noted, the primary focus of their analysis was the difficulties and barriers these students displayed while using diagrams. The independent analyses were then shared and discussed by all of the researchers, who together identified shared similarities and differences in coding and categorization (Patton, 1990). Patterns related to student difficulties and barriers regarding diagrams emerged and were categorized into challenges. As the challenges were established, parallels to the five strands of diagram proficiency became apparent. These strands then became the overall themes by which the data was organized.
Table 1 Demographics of participants by ability level and means and standard deviations for IQ and KeyMath3 subtest

From: Challenges students identified with a learning disability and as high achieving experience when using diagrams as a visualization tool to solve mathematics word problems

<table>
<thead>
<tr>
<th>Variable</th>
<th>Ability level</th>
<th>High achieving (n = 12)</th>
<th>Learning disability (n = 16)</th>
<th>Overall (n = 28)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Age</td>
<td>11.17</td>
<td>11.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SD</td>
<td>1.55</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gender</td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Male</td>
<td>6 (50 %)</td>
<td>10 (62.5 %)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Female</td>
<td>6 (50 %)</td>
<td>6 (37.5 %)</td>
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<tr>
<td></td>
<td></td>
<td>Ethnicity</td>
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<tr>
<td></td>
<td></td>
<td>White</td>
<td>11 (91.7 %)</td>
<td>9 (56.3 %)</td>
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<tr>
<td></td>
<td></td>
<td>African-American</td>
<td>–</td>
<td>6 (37.5 %)</td>
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<tr>
<td></td>
<td></td>
<td>Hispanic</td>
<td>–</td>
<td>1 (5.3 %)</td>
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<td></td>
<td></td>
<td>Asian</td>
<td>1 (8.3 %)</td>
<td>–</td>
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<tr>
<td></td>
<td></td>
<td>Free and/or reduced lunch</td>
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<tr>
<td></td>
<td></td>
<td>Yes</td>
<td>2 (16.7 %)</td>
<td>13 (81.3 %)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No</td>
<td>10 (83.3 %)</td>
<td>3 (18.7 %)</td>
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<tr>
<td></td>
<td></td>
<td>Grade level</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Fourth grade</td>
<td>3 (25.0 %)</td>
<td>7 (43.8 %)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fifth grade</td>
<td>3 (25.0 %)</td>
<td>3 (18.8 %)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sixth grade</td>
<td>3 (25.0 %)</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Seventh grade</td>
<td>3 (25.0 %)</td>
<td>6 (37.5 %)</td>
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<tr>
<td></td>
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<td>IQ scores</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>M</td>
<td>131.86</td>
<td>94.06</td>
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<tr>
<td></td>
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<td>SD</td>
<td>14.44</td>
<td>8.11</td>
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<td>KeyMath3</td>
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<td></td>
<td></td>
<td>Numeration</td>
<td>15.33</td>
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<td></td>
<td></td>
<td>SD</td>
<td>1.50</td>
<td>1.87</td>
</tr>
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<td></td>
<td></td>
<td>Addition and subtraction</td>
<td>13.53</td>
<td>6.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SD</td>
<td>2.45</td>
<td>2.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Multiplication and division</td>
<td>13.33</td>
<td>6.19</td>
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<tr>
<td></td>
<td></td>
<td>SD</td>
<td>2.27</td>
<td>1.56</td>
</tr>
<tr>
<td></td>
<td>Applied problem solving</td>
<td>16.25</td>
<td>6.63</td>
<td>10.75</td>
</tr>
<tr>
<td></td>
<td>Applied problem solving</td>
<td>SD</td>
<td>1.29</td>
<td>1.82</td>
</tr>
</tbody>
</table>

*Means of 100 and SD of 15
**Means calculated on scale score (M = 10, SD = 3)

The second step involved the first and second author independently confirming the alignment of the challenges within the five strands of diagram proficiency and finding corroborating examples. At this point, the challenges were further refined.
The third step then involved the second and third authors examining the data from the students identified as HA. These data were then used to find additional challenges that may not have emerged from the data on students with LD and to find examples that matched the challenges already identified.

In the final step, all of the authors identified which students exhibited each challenge within the five strands of diagram proficiency generated from the data of both students identified as HA and students with LD in order to compare how the groups differed across the various strands. During this step, the challenges were further refined and collapsed.

5 Findings

The purpose of this study was to first, identify what challenges students experienced related to diagram use and understanding, and second, to compare these challenges experienced between students identified as LD or HA in mathematics. In the end, nine challenges were identified that aligned to the five strands of diagram proficiency (see Fig. 1). We first report the different challenges that emerged from the examined data. Following that discussion, we compare how the two groups of students differed on those challenges.

5.1 Challenges students experienced

5.1.1 Challenges related to conceptual understanding

Two challenges related to conceptual understanding emerged from the data. First, students were unable to define a diagram, and, in some cases, appeared confused by the term. For example, in response to the question “In mathematics, what is a diagram?” students responded:

I haven’t studied them yet. Diagram? Like those little dots? Oh, those are decimals. Sorry. I don’t know what they are, but I think that they help you when you’re doing a math problem.

The distance, height, and length. … Surface area, I don’t know.

S: I don’t know.

I: You don’t know? Have you heard that word before?
S: Yes, in class, but I don’t know what it is.
I: Can you remember when or why she used that word?
S: When, was before yesterday. And then she used it on the board and I did not know what she meant on the board.

Further, several students were unable to provide any reason or were confused as to why a diagram could be used to solve a mathematics problem and, how it could help. When asked how a diagram could help one student responded, “Because you need it to answer a problem, and if you don’t, you can’t finish the problem. You don’t know what the answer is.” Another stated, “Maybe to measure things.”

Although some students could provide a definition of a diagram and a reason for using them, a second challenge involving the conceptual understanding of a diagram that emerged was a lack of depth or understanding of the mathematical use of the term diagram. In many of their definitions, students simply referenced a diagram form such as a “table”, “chart”, or “graph”. Others indicated it was a “picture” or “drawing,” lacking any connection to a representation that depicts relational elements. Several students suggested it was something that organizes or gives you information. For example, “A diagram is like a graphic, a visual, that organizes data.”

Likewise, reasons given for how a diagram could be of benefit frequently lacked depth and were primarily limited to one main idea. For example, the majority of students suggested that a diagram is a good way to organize and see information to understand and remember the problem. When asked how a diagram could help, one student stated, “So that you can organize your thoughts and you don’t get numbers lost.” A few students indicated that it could be used to solve a problem and/or check work such as, “To help me solve problems, so I could find the right answer. And also to check my work.”

5.1.2 Challenge related to procedural fluency

Within procedural fluency, one primary challenge emerged— students drew diagrams that contained inaccuracies when compared to the information found in the problem. This included notation of incorrect numbers and quantities or the absence of
key specific details central to solving the problem as seen in the examples found in Fig. 2. As Diezmann (2000) notes, while this may reflect carelessness, because these inaccuracies were not detected and resulted in an incorrect answer, it is considered an irrecoverable error.

Fig. 2 Inaccuracies in diagram generation

5.1.3 Challenges related to strategic competence

Two main challenges emerged within strategic competence. First, students did not use a diagram to understand, solve, or monitor the process of solving one or more problems. Figure 3 provides examples of this challenge. Most notably, when prompted to use a diagram to solve a problem, some students generated it after they solved the problem or only when they recognized they were having difficulty solving the problem using other strategies. Interestingly, when some students recognized the problem was difficult to solve, a diagram was not seen as a tool that could be used to help even to understand the problem. As one student expressed in response to a series of questions when he was having difficulty solving a problem:
Did computations first then drew boxes with numbers in them as student notes, *Oh, I gotta draw a picture.*

The rent for an apartment on the 10th floor is $390 per month. The rent on the 15th floor is $440 per month. At this rate of increase what is the rent for an apartment on the 30th floor?

Student initially tried solving this problem by trying to solve for “x”. After solving the problem, the student was reminded to draw a diagram. In doing this, the student then obtained the correct answer.

**I:** *How did you solve this problem?*

**S:** *First, I tried writing an equation. I found the slope, and then I solved for b, which is the y-intercept. But then when I checked it with this number, it did not add up to 440. So I figured it must have been wrong. Then, when I drew a table, I wrote the 10 and the 15 and their prices. I noticed that the difference in price was 50, and there are 5 floors in between them, so each floor should be $10. So there are 15 more floors to get to the 30th floor, so I multiplied that by 10, for the cost. That added up to 590.*

The rent for an apartment on the 10th floor is $390 per month. The rent on the 15th floor is $440 per month. At this rate of increase what is the rent for an apartment on the 30th floor?

![Diagram](image-url)
S: I don’t know that one, that one is hard.
I: What about if you used a picture to solve this? S: It still won’t help.
I: Why not?
S: Because I don’t know the problem, so I don’t know how to do a picture.
I: Can you tell me what you don’t know about the problem? Tell me what you tried here.
S: I tried counting down to see if I got a low numbers, and that’s too high to divide it from all 4 friends. So I knew that one wasn’t going to work. Then I tried that one.
I: What this one?
S: 64 minus 4. And I knew that wasn’t going to work. I: A picture?
S: A picture isn’t going to work either.
I: Are you seeing something in your mind? S: No.
I: Why do you think a picture won’t work?
S: I don’t have the answer to draw the picture.

Second, students did not generate an appropriate diagram to solve the problem as they either used the same diagram form (e.g., table/chart) to solve all the problems they were presented with or resorted to a specific inappropriate form as they considered them easier to use. See Fig. 4 for examples. In addition, some students tended to think that specific diagram forms were the only forms to be used when solving specific types of problems. For example, one student explained:
We learned in third grade that if you have a fraction problem, any type of fraction problem, you can draw a pizza and cut it into the amount it was.

5.1.4 **Challenges related to adaptive reasoning**

Adams Middle School students made an Italian submarine sandwich that was 12 \( \frac{3}{4} \) feet long. After making it, they decided to divide the sandwich into smaller portions to share with other students. If each portion was \( \frac{3}{4} \) of a foot long, how many students would get a portion?
Two main challenges related to adaptive reasoning emerged. First, although students used a diagram to solve the problem whether in a limited or extensive way, many students were unable to provide an explanation as to how a diagram was used to solve the problem even when use of the diagram was perceived as being helpful. Rather, students often referred to the computation they used in the process of solving the problem. Several examples are provided in Fig. 5.

| I: Did it help you to draw the picture? | S: Yes. |
| I: How did it help? | S: I can do this draft um I can do this draft any day. |
| I: Any other way this helped you solve the problem? | S: Yes, 30 plus 4. |

The teacher has 30 feet of rope. She cuts off pieces 4 feet long to make jump ropes for her class. How many jump ropes can she make?

| I: Did you find it helpful to draw your picture? | S: Yes. |
| I: How did it help? | S: Because 7 plus 4 is 28 will help me. |
| I: How did it help you? | S: I don’t know. |

Fig. 5 Unable to explain how the diagrams helped solve the problem
Second, students had difficulty using the diagram as a tool to reason with as they solved the problem even when it appears that they understood what they were solving for. In some cases, students got confused within or “lost” when using the diagram to explain their problem solving process as they had difficulty keeping track of what they were solving for or the diagram became too unwieldy as seen in the example of the student attempting to solve the Halloween party problem found in Fig. 6. Alternatively, as they used the diagram to explain their problem solving process, some students overly focused on the diagram and did not connect back to the problem. To illustrate, a student drew a 30-foot rope and segmented the rope into sections, but stopped drawing because “there was no more space in the 30 feet rope” and incorrectly stated the answer was 6 in the rope problem found in Fig. 6. Finally, some students used their diagram incorrectly even though they had generated an appropriate diagram as they reasoned about the relationships between the concepts and situation while they solved the problem. This is seen in the example of the student solving the card problem, where the student represents the total number of cards (64) as six groups of ten and four more. Each group of 10 is assigned to a friend and 12 (a 10 and 2) are circled. The student then incorrectly splits the remaining cards among the friends resulting in an incorrect answer.
<table>
<thead>
<tr>
<th>Halloween Party Problem</th>
<th>Rope Problem</th>
<th>Card Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>I:</strong> Did you find it helpful to draw the picture?</td>
<td><strong>I:</strong> Tell me how you solved this problem.</td>
<td><strong>I:</strong> Can you tell me how you solved this problem?</td>
</tr>
<tr>
<td><strong>S:</strong> No.</td>
<td><strong>S:</strong> Um, I made my own imaginary 30 feet rope um I made 4 feet and I made 6 ropes and that made 6 jump ropes.</td>
<td><strong>S:</strong> I drew out cards with number tens on them to represent 10 cards, and then drew 4 more and circles 12. Then I gave 10 to his friends, then I split the other 10 with his friends, and I got 12 remainder 2.</td>
</tr>
<tr>
<td><strong>I:</strong> Tell me why it didn’t help.</td>
<td><strong>I:</strong> How did you figure out you had to stop after this many ropes?</td>
<td><strong>G:</strong> Greg had 64 baseball cards. He gave 12 cards to his sister. Then he divided the remaining cards equally among his four friends. How many cards did each of his friends get?</td>
</tr>
<tr>
<td><strong>S:</strong> Because I was confused. Every time I crossed out one on each one, there would be tons left. Sometimes it doesn’t help to draw a picture, but sometimes it does. It depends on the problem you’re doing.</td>
<td><strong>S:</strong> Because there was no more space in the 30 feet rope.</td>
<td><strong>I:</strong> Any other way you figured that out?</td>
</tr>
<tr>
<td></td>
<td><strong>I:</strong> Any other way you figured that out?</td>
<td><strong>S:</strong> No.</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>S:</strong> No.</td>
</tr>
</tbody>
</table>

Meredith is planning a Halloween party. She has 36 prizes and 24 balloons. What is the most number of children she can invite so each child gets an equal number of prizes and an equal number of balloons? She does not want any prizes or balloons left over.

![Diagram](image1)

![Diagram](image2)

![Diagram](image3)

**Fig. 6** Difficulty using a diagram as a tool to reason with as they solved the problem

5.1.5 **Challenges related to productive disposition**

Within productive disposition, two main challenges emerged. First, some
students perceived diagrams not to be useful or worthwhile for solving the problems. Often students indicated that diagrams were something they used to do when they were “little” or in elementary school. Although, in some cases, generating a diagram was not necessary as the student could solve the problem accurately and efficiently without it; some students did not think they needed to use a diagram at all because a different strategy (typically computation) was easier to use even when it was clearly apparent that they did not understand the problem. Examples of this include:

S: Because I don’t really use pictures in my math, I just use pictures to draw what I like to draw. I don’t use pictures in my math.
I: Did you find it helpful to draw the picture?
S: Not really, because I don’t use pictures and it doesn’t help me anymore.
I: What do you mean by it doesn’t help you anymore? S: A long time ago I used to use pictures because I didn’t get math as much. But now I’m really good at it, so I don’t need pictures anymore.
[After incorrectly solving the problem]
I: Can you tell me why you didn’t use one [a diagram]?
S: It would be easier to make 30 plus 4, because it would be easier instead of making a line that’s 30 feet long on the piece of paper.
I: How did you know what to do here? [Referring to the diagram the student drew]
S: I did it when I was little.
I: But you don’t do that anymore?
S: No.
Second, students did not self-initiate the strategy of using a diagram when solving a problem. In fact, without prompting 17 out of the 28 students did not use a diagram as a strategy to solve a single problem. Further, even with prompting to use a diagram, six students (out of 28) chose not to draw a diagram for one or more of the problems they were asked to solve.

5.2 Comparison of the students across the challenges experienced

Using the five strands of diagram proficiency, several differences between the students classified as LD and HA were identified for each challenge. A summary of these differences for each challenge by ability level can be found in Table 2. Several interesting findings between the two groups deserve to be highlighted.

Table 2 Percent and number of students with challenges by ability

<table>
<thead>
<tr>
<th>Strand</th>
<th>Challenge</th>
<th>LD % (n)</th>
<th>HA % (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptual</td>
<td>Unable to define</td>
<td>37.5 (5)</td>
<td>8.3 (1)</td>
</tr>
<tr>
<td></td>
<td>Unable to provide a reason</td>
<td>43.6 (7)</td>
<td>0.0 (0)</td>
</tr>
<tr>
<td></td>
<td>Limited definition</td>
<td>62.5 (12)</td>
<td>91.7 (11)</td>
</tr>
<tr>
<td></td>
<td>Limited reason(s)</td>
<td>56.3 (8)</td>
<td>100.0 (12)</td>
</tr>
<tr>
<td>Procedural</td>
<td>Inaccurate diagram</td>
<td>31.2 (5)</td>
<td>0.0 (0)</td>
</tr>
<tr>
<td>Strategic</td>
<td>Did not use the diagram to understand, solve, or monitor while solving the problem</td>
<td>59.6 (9)</td>
<td>90.0 (6)</td>
</tr>
<tr>
<td></td>
<td>Inappropriate diagrams for the problems</td>
<td>25.0 (4)</td>
<td>0.0 (0)</td>
</tr>
<tr>
<td>Adaptive reasoning</td>
<td>Unable to explain, despite perceived helpfulness, how the diagram was used</td>
<td>51.3 (5)</td>
<td>8.3 (1)</td>
</tr>
<tr>
<td></td>
<td>Difficulty using a diagram as a tool to reason with</td>
<td>31.3 (5)</td>
<td>41.7 (5)</td>
</tr>
<tr>
<td>Productive disposition</td>
<td>Diagram not perceived as useful or worthwhile to use</td>
<td>50.0 (8)</td>
<td>41.7 (5)</td>
</tr>
<tr>
<td></td>
<td>Diagram not used</td>
<td>68.8 (11)</td>
<td>50.0 (8)</td>
</tr>
<tr>
<td></td>
<td>For any unprompted problems</td>
<td>25.0 (4)</td>
<td>8.3 (1)</td>
</tr>
</tbody>
</table>

First, a higher percentage of students with LD than students identified as HA were unable to provide a definition or reason for using a diagram. Notably, unlike the students with LD, all of the students identified as HA were able to give at least one reason for using a diagram. Second, no student identified as HA generated an inaccurate diagram whereas 31% of students with LD did. Third, no student identified
as HA had difficulty generating a diagram considered appropriate for the problem situation whereas 25 percent of students with LD did. Interestingly, however, 50% of the students identified as HA did not use a diagram in the process of solving a problem as compared to only 18% of students with LD. Fourth, a higher percentage of students identified as LD than HA perceived the diagram as helpful, but had difficulty explaining how a diagram helped them solve a problem. However, both groups of students struggled to use a diagram to reason with as they solved the problem. Finally, within both groups of students, a similar percentage of students did not perceive the diagrams to be worthwhile; however, students with LD were more likely than students identified as HA to not use a diagram to solve problems.

6 Discussion

According to the Standards for Mathematical Practice in the Common Core State Standards for Mathematics (National Governor's Association, & State Education Chiefs, 2010), all students are expected to be able to model with mathematics using a tool such as a diagram. Specifically, it is expected that students

... are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams .... [Using the model, t]hey can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. (p. 7)

However, as the findings of this study suggest, not all students are necessarily proficient in using a tool such as a diagram and may experience any number of challenges when using them. Based on the findings of the study, two main conclusions can be drawn.

First, students of all abilities experienced a range of challenges while using diagrams. The challenges that were identified were varied and of concern. It is clear that not all students were able to identify what a diagram is and how it could be of benefit. For a number of students, generating and using a diagram as a tool to solve a problem was difficult. Further, many of the students did not perceive diagrams to be a worthwhile tool to use. Similar to Diezmann’s (2000) conclusions, drawing a diagram, despite its
potential, is not necessarily an effective problem-solving tool for many students. This is of concern as a diagram, as has been previously noted, can be a powerful tool given its versatility and has been demonstrated to aid in problem solving performance.

Second, while both students identified as LD and HA experienced challenges using diagrams, it appears that students with LD are more likely to experience these challenges. For example, more students with LD had difficulty defining a diagram and providing reasons for using a diagram. The findings in this study also suggest that students with LD may experience a greater range of challenges as compared to their HA peers. Various studies have suggested that students with LD differ in the quality of how they use their problem solving strategies rather than in the quantity (e.g., Montague & Applegate, 1993a, 1993b; van Garderen et al. 2012). Students with LD in those studies typically use strategies in a less effective and efficient manner. In this study, this also appears to be the case. Unlike other studies, however, the findings of this study provide more detail as to what explicit challenges they might experience when using diagrams. Specifically, students with LD are more likely to have difficulty generating a diagram that accurately represents the information presented in a problem and frequently struggle to generate an appropriate diagram for a given problem.

There were three interesting exceptions to the second main finding that should be noted. First, a higher percentage of HA students than students with LD did not consistently utilize a diagram as a strategy to solve a problem. However, a number of those students also indicated that they did not need to use the diagram, as they, in most cases, correctly solved the problem. This is not surprising, as a problem situation must warrant (e.g., be of sufficient challenge) the use of a given strategy. Second, a high percentage of students identified as LD and HA did not perceive diagrams to be worthwhile. In some cases, students indicated they did not need it as they had already solved the problem using another strategy, resorted to other ways (e.g., computation) to solve the problem, or the problem was too easy and thus did not warrant a diagram. A productive disposition towards mathematics is critical (NRC, 2001); therefore, if students do not see value or use in a strategy, such as a diagram, it may not be implemented most effectively or students may not find it useful to spend time learning to or improving on their use of diagrams. Third, a higher percentage of students identified as HA than
students with LD had difficulty using a diagram to reason with as they solved the problem. It is unclear as to why this may be the case. However, many students with LD had difficulty within the procedural fluency and strategic competency strands that precede adaptive reasoning and, therefore, more students identified as HA got to this stage. It is also possible that reasoning with a diagram is a difficult process that students may need more time and experience to develop.

Although several interesting findings were revealed in this study, there are a number of limitations that need to be acknowledged. First, it is possible that there are additional challenges that students might experience beyond those described in this study. Diezmann (2000) for example, notes other challenges specific to using diagrams classified as networks that were not identified here. Clearly, there is a need to verify the current findings as well as look for other possible challenges. Second, the data was collected in individual sessions not representative of a typical mathematics classroom setting. It would be interesting to examine how the students use diagrams when solving problems in the classroom as additional challenges may arise that were not found in the current data. Third, no attempt was made to break down and examine differences of diagram use by age (or grade) of the students. Differences in challenges that students experience may vary depending on age. Further research is needed to closely examine how use of a diagram may gradually develop over time as a problem solving strategy (e.g., idiosyncratic to conventional application) along with how tasks may impact that development (e.g., learning trajectory).

7 Implications for practice

A highly recommended strategy to use in mathematics instruction for both struggling learners and students with disabilities is the use of visual representations such as a diagram (Gersten et al. 2009). Although this is preferable, for some students, in particular students with LD, it needs to be recognized that they may lack knowledge about diagrams, one visual representation form, which may interfere with their development and understanding of key mathematical ideas and problem solving performance. Therefore, as Rubenstein and Thompson (2012) note, "When we acknowledge that reading [and, we contend, using] charts, diagrams, graphs, and other
visual representations may be less transparent to students than to us as teachers, we can begin to identify and adopt strategies to support students’ “reading” [and “use”] of those displays” (p. 550).

While many students, given sufficient time and opportunities, develop diagram proficiency, for some students additional targeted, explicit instruction for developing diagram proficiency may be necessary. In this study, we identified nine possible challenges that students may experience that could be areas for instruction. For example, instruction may be needed to help develop a clear understanding of what a diagram is and how a diagram may be used to solve mathematics problems. As Diezmann (2000) poignantly notes, “Advocating that students draw a diagram without addressing their difficulties and educating them about diagrams is quite simply the waste of a very good tool!” (p. 247).

Appendix A
 Learning disabilities:
  1. a disorder in one or more of the basic psychological processes, including visual, auditory, or language processes;
  2. academic achievement significantly below the student's level of intellectual functioning;
  3. learning problems that are not due primarily to other disabilities; and
  4. ineffectiveness of general education alternatives in meeting the student's educational needs.

References
instruction for students with mathematics disabilities: Comparing two teaching


In *Proceedings of the 22nd Annual Conference of Mathematics Education Research Group of Australasia*, (pp. 185–191), Adelaide, Australia.


