Nonlinear Taylor rules: evidence from a large dataset

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Nonlinear Taylor rules: evidence from a large dataset

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Abstract:
In this paper we estimate nonlinear Taylor rules over the 1986–2008 sample time period and augment the traditional Taylor rule by including principal components to better model Federal Reserve policy. Including principal components is useful in that they extract information about the overall economy from multiple economic indicators in a statistically optimal way. Additionally, given that uncertainty may influence Federal Reserve decisions, we incorporate an uncertainty index in the reaction function of the Federal Reserve. We find substantial evidence that the Federal Reserve responded to increases in macroeconomic uncertainty by cutting the Federal Funds rate over the sample period. We also find evidence that the Federal Reserve responded aggressively to increases in capacity utilization, especially when the inflation rate was above 2%.

Keywords: monetary policy, principal component, Taylor rule
JEL classification: E52, C22
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1 Introduction

Since Taylor’s seminal 1993 “Taylor Rule” paper, the empirical monetary policy literature has developed numerous Taylor rules that forecast the federal funds rate based upon a measure of inflation, the output gap, and usually lags of the federal funds rate. However, Nobay and Peel (2003), Cukierman and Gerlach (2003), Gerlach (2003), and Ruge-Murcia (2002, 2004), Bec, Salem, and Collard (2002), Dolado, Dolores, and Naveira (2005), and Surico (2007a) argue that central bank preferences are likely not to be symmetric in their responses to deviations from inflation and output gap targets. If, for example, the Fed is more concerned about unemployment being substantially above the natural rate rather than below the natural rate or if the Fed cares more about high inflation rather than low inflation, the Taylor rule feedback rule will not be linear. Schaling (1999) and Dolado, Dolores, and Naveira (2005) provide a second rationale for a nonlinear Taylor rule. If the aggregate supply curve is convex, so that there is a non-linear relationship between output and inflation, the optimal feedback rule relating interest rates to output and inflation should also be nonlinear. Qin and Enders (2008) and Bunzel and Enders (2010) find substantial evidence that the Taylor rule is non-linear. Specifically, Bunzel and Enders (2010) argue that the Federal Reserve actually follows an “opportunistic policy” in which the Federal Reserve takes advantage of a low inflation rates (due to recessions or positive supply shocks) to support trend growth at the prevailing inflation rate. Bunzel and Enders (2010) argue that such an “opportunistic policy” implies a nonlinear, rather than a linear policy rule.

Our aim in this paper is to build upon the Bunzel and Enders’ (2010) framework. Our primary interest is in evaluating whether an output or employment principal component in the Taylor rule provides a better fit than the traditional output gap. We estimate a nonlinear Taylor rule using inflation as measured by the Personal Consumption Expenditures (PCE) index, an output principal component, and an employment principal component to model Federal Reserve behavior. The rationale behind our principal components approach is that the Federal Reserve likely takes advantage of a large amount of information in a variety of data series to study the economic conditions in order to form its policy responses. For example, while the Federal Reserve has explicitly targeted a 2% inflation rate in the PCE index, they have not specified an output or employment target. The closest the Federal Reserve has come in announcing an output target was in 2013 when Chairman Bernanke stated that an unemployment rate at 6.5% was a “threshold, not a trigger” for the Federal Open Market Committee (FOMC) to evaluate whether it was appropriate to adjust policy. Moreover, he additionally stated that asset purchases would likely come to an end when the unemployment rate was in the vicinity of 7%. However, he was forced to back away from the unemployment rate targets due to an unexpected fall in the unemployment...
rate despite weak economic growth and stated, “the unemployment rate is not necessarily a great measure, in all circumstances, of the state of the labor market overall.”

Given that the Federal Reserve examines a broad array of labor market variables in determining the appropriate prescription for monetary policy, employing factor analysis as a tool to forecast monetary policy may be useful in that principal components capture information from multiple economic variables. In this sense, our work is related to Favero, Marcellino, and Neglia (2005) who, focuses on the inflation part of the policy rule and employs factors as a proxy for the Federal Reserve’s inflation expectation based on multiple economic data series. We instead focus on the real economic activity in the policy rule given that the Federal Reserve explicitly targets PCE inflation. Additionally, given that uncertainty may influence Federal Reserve decisions, we incorporate the Baker, Bloom, and Davis (2015) uncertainty index in the reaction function of the Federal Reserve. Moreover, we also allow lags of the output and employment principal components and the uncertainty index to serve as the threshold variable in our nonlinear models.

As a preview of our results we find evidence that the Federal Reserve responded aggressively to increases in capacity utilization, especially when the inflation rate is above 2%. We find substantial evidence suggesting that the Federal Reserve follows a nonlinear Taylor rule and that lagged inflation served as the best threshold variable. Our estimates of the inflation threshold were 2.02% when using PCE inflation in the estimated Taylor rules. We find that nonlinear Taylor rules have a better out-of-sample fit than linear versions.

Our results have some policy implications. For example, we find that the responses of the Federal Reserve to the output gap and capacity utilization are both insignificant when the inflation rate is below the threshold value 2.02%. This suggest that Federal Reserve does not respond as aggressively to the output target variable when the inflation rate is low relative to when inflation is high.

Additionally, we incorporate an uncertainty index into the reaction function of the Federal Reserve. We find substantial evidence that the Federal Reserve responded to increases in macroeconomic uncertainty by cutting the Federal Funds rate over the sample period.

The rest of the paper proceeds as follows. In Section 2, we provide an in-depth literature review. Section 3 describes the data we use in our analysis. Section 4 estimates and compares several linear and nonlinear specifications of the Taylor rule and Section 5 concludes.

2 Review of literature


Additionally, Svensson (2000, 2003) presents a significant argument in favor of including the exchange rate as a variable in the Taylor rule. Ball (1999) and De Belle (1999) argue that the inclusion of the exchange rate in the monetary policy rule plays a significant role in reducing output and inflation rate predictability; however, Lubik and Schorfheide (2007) find that this result is not consistent among all developed economies. In a similar vein, Taylor (2000, 2001), Edwards (2007), Mishkin (2007), and Garcia, Restrepo, and Roger (2011) argue exchange rates may only need to be included in monetary policy rules for emerging countries. Other studies investigate the impact and role that financial asset prices may play in explaining the central bank’s behavior. Cecchetti et al. (2000), Borio and Lowe (2002), Goodhart and Hofmann (2002), Sack and Rigobon (2003), Chadha, Sarno, and Valente (2004), and Rotondi and Vacciago (2005) argue that central banks should take into account the effects that asset price changes may have on the macroeconomy when setting central bank policies; Bernanke and Gertler (1999, 2001) and Bullard and Schaling (2002) argue that a central bank should only consider the inflationary effects of asset price changes when setting interest rate policy rather than the effect of asset price changes on the macroeconomy as a whole. Interestingly, Disyatat (2010) argues that financial stability should explicitly be taken into the central bank’s loss function. Driffill et al. (2006) finds evidence that the inclusion of financial stability improves the econometric fit of the Taylor rule. In the same line, Montagnoli and Napolitano (2005)
develop a financial indicator composed of the exchange rate, equity markets and house prices and finds that the inclusion of these variables improve the econometric description of the monetary policy. Shrestha and Semmler (2015) focus on five Asian economies and find that the inclusion of a financial instability measure improves the Taylor rule.

One common issue with the estimation of any Taylor rule is the uncertainty regarding the precise measurement of unobservable variables, such as potential output McCallum (1999). Orphanides (2002) and Orphanides and Van Norden (2002) present convincing evidence that systematic errors in the forecasting of the output-gap lead to mistaken monetary policy prescription. Moreover, aside from the measurement error, many authors argue that the monetary policy using the Taylor rule is better described using nonlinear rather than a linear specifications. With respect to non-linearities in the Taylor-rule, there are two primary ways in which non-linearity may arise. Schaling (1999), Svensson (1999), Nobay and Peel (2003), and Dolado, Dolores, and Naveira (2005) and others suggest that uncertainty and non-linearity in the underlying macroeconomic structure may provide a compelling case for the use of nonlinear Taylor rule. Moreover, Schaling (1999) and Dolado, Dolores, and Naveira (2005) argue that in the presence of a convex aggregate supply curve, a nonlinear feed rule would be optimal. In addition, Meyer, Swanson, and Wieland (2001), Swanson (2006), and Tillmann (2011) also argue that uncertainty about the appropriate variable selection, appropriate model specification of the macroeconomy and the nature of the monetary transmission mechanism could also lead to a nonlinear Taylor rule. Bec, Salem, and Collard (2002), Cukierman and Gerlach (2003), Nobay and Peel (2003), Ruge-Murcia (2004), Martin and Milas (2004), Kim, Osborn, and Sensier (2005), Taylor and Davradakis (2006), and Surico (2007a, 2007b), Petersen (2007), Cukierman and Muscatelli (2008), Hayat and Mishra (2010), and Castro (2011) also argue that the Taylor rule could be better specified as a non-linear relationship if central banks have asymmetric responses to positive and negative deviations in the inflation and output gap. Caporale et al. (2016) estimates a Threshold Autoregressive (TAR), exchange rate-augmented Taylor rule specification for five inflation targeting, emerging countries and find substantial evidence for non-linearity in the conduct of monetary policy. In addition, they also find that most of the central banks react to deviations in real exchange rates. Orphanides and Wilcox (2002) note that non-linearity in the Taylor rule may result if central banks react only to “large” deviations in inflation. Aksoy et al. (2006) provides evidence to support this claim. Interestingly, Martin and Milas (2004) finds that the Bank of England (BOE) reacts stronger to positive deviations from their inflation target over the period 1992–2000 time period. In a similar fashion, Petersen (2007) estimates a smooth transition logistic regression model of Federal Reserve behavior over the period 1985–2005 time period and finds that the Federal Reserve reacts stronger to inflation than to output deviations. However, Qin and Enders (2008) do not find any non-linearity in Federal Reserve’s monetary policy over the 1987–2005 period once interest rate smoothing and forward-looking behavior of the Federal Reserve is taken into account. Castro (2011) finds that European Central Bank (ECB) reacts to inflation only when inflation itself is above 2.5 percent and reacts to output gap deviations afterwards.

Because the Taylor rule is a description of the central bank’s behavior, it is not unreasonable to believe that the coefficients of the Taylor rule may change over time. Judd and Rudebusch (1998) is a seminal paper demonstrating the possibility of time varying parameters (TVP) in the Taylor rule. Cogley and Sargent (2001), Canova and Gambetti (2004), and Mesonnier and Renne (2007) estimate Vector Autoregressive models (VARs) with a Kalman Filter to estimate a TVP Taylor rule to illustrate the time-varying nature of Taylor rule parameters. Orphanides and Williams (2005) and Sims and Zha (2006) also document time-variation in the parameters of Taylor rules using a VAR model. Elkhoury (2006), Kuzin (2006), and Trecroci and Vassalli (2006) shows that TVP of Taylor rules outperform Taylor rules with fixed parameters for a substantial number of developed and non-developed countries. Jalil (2004), Boivin and Giannoni (2006), and Kim and Nelson (2006) use the Kalman Filter to illustrate that parameters of the Taylor rule change gradually over time. Moreover, Kim and Nelson (2006) find evidence of at least three changes in the Fed’s reaction to inflation since 1970s.

3 Data

Our primary extensions of Bunzel and Enders’ (2010) model are as follows. First, given that Bernanke announced a specific inflation target of 2% in PCE inflation, we use the PCE inflation rate rather than the GDP deflator. Second, because Bernanke does not explicitly announce a target for employment or output, we employ an output principal component as well as an employment principal component in place of the real-time output gap ($y_t$) in (1). Third, whereas Bunzel and Enders (2010) employ lags of the output gap and inflation as threshold variables ($x_t$), we believe that because Bernanke states that the FOMC evaluates a broad range of employment and output variables in making monetary policy decisions, an output or employment principal component may serve as a better threshold variable rather than lags of the traditional Taylor rule variables. Fourth, given the recent importance of macroeconomic uncertainty, we include Baker, Bloom, and Davis (2015) uncertainty index in the reaction function as well include it a potential threshold variable.

In order to extract an output and employment principal component, we utilize the new macroeconomic dataset developed by McCracken and Serena (2016). McCracken and Serena (2016) develop a large (135 variables), monthly dataset that has several appealing features that spans the 1960–2015 time period. First, the dataset can be updated in real-time using the FRED database; second, the dataset is publicly available. McCracken and Serena (2016) split up the variables into 8 groups: output and income, labor market, consumption and orders, orders and inventories, money and credit, interest rates and exchange rates, prices, and the stock market. As noted above, our primary interest is in evaluating whether an output or employment principal component in the Taylor rule provides a better fit than the traditional output gap. As such, we use the variables in the output and income group and the variables in the labor market group to generate an output and employment principal component. The variables included in the output principal component were the following: real personal income (RPI), real personal income excluding transfer receipts (WP875RX1), (IPFPNNS) IP: Final Products and Nonindustrial Supplies, (IPFINAL) IP: Final Products, (IPCONGD) IP: Consumer Goods, (IPDCONGD) IP: Durable Consumer Goods, (IPNCONGD) IP: Nondurable Consumer Goods, (IPBUSEQ) IP: Business Equipment, (IPMAT) IP: Materials, (IPDMAT) IP: Durable Materials, (IPNMAT) IP: Nondurable Materials, (IPMANICS) IP: Manufacturing, (IPB51220s) IP: Residential Utilities, (IPFUELS) IP: Fuels, ISM manufacturing (NAPMPI), and capacity utilization (CUMFNS). The variables used in the employment factor were: help wanted index (HWI), ratio of help wanted / number of unemployed (HWIURATIO), civilian labor force (CLF16OV), civilian employment (CE16OV) all employees: total nonfarm (PAYEMS), all employees: goods producing (USGOOD), (CES1021000001) All Employees: Mining and Logging, Mining, (USCONS) All Employees: Construction, (MANEMP) All Employees: Manufacturing, (DMANEMP) All Employees: Durable goods, (NDMANEMP) All Employees: Nondurable goods, (SRVPRD) All Employees: Service-Providing Industries, (USTPU) All Employees: Trade, Transportation & Utilities, (USWTRADE) All Employees: Wholesale Trade, (USTTRADE) All Employees: Retail Trade, (USFIRE) All Employees: Financial Activities, (USGOVT) All Employees: Government.

In order to generate the principal components, we attempt to make the components as real time as possible. That is, beginning with $t = 1979:1$, we extract out a principal component using data that only spans the 1960:1 through $t$ time period. We construct the principal components time series by adding one quarter of data. That is, for the 1979:2 principal component observations, uses data that spans the 1960:1 though 1979:2 time period. We subsequently repeat that process through the end of our sample.

Recently, numerous articles examining the macroeconomic effects of macroeconomic uncertainty have developed. Specifically, Bloom’s (2009), Baker, Bloom, and Davis (2015) and Sim, Zakrjasjek, and Gilchrist (2010) develop models in which uncertainty shocks adversely affect output. Recently, Baker, Bloom, and Davis (2012) addressed this by developing a policy-related uncertainty index. We include this uncertainty index for two reasons. First, many market participants and academicians argued in favor of a “Greenspan put.” The “Greenspan put” was essentially a belief that if financial markets or macroeconomic conditions became volatile, Chairman Greenspan would lower interest rates to lower uncertainty and volatility. Thus, our aim is to include the new Baker, Bloom, and Davis (2012) uncertainty index in the Federal Reserve’s reaction function and include the uncertainty as a potential threshold variable. Baker, Bloom, and Davis (2012) spans the 1985Q1 – present time period. As such, all of our estimations span the 1985Q1–2008Q2 time period. We chose to end our sample period in the 2008Q2 due to the beginning of the financial crisis in the third quarter of 2008. Table 1 displays the summary statistics of the main variables that we are used in our analysis. The summary statistics for the variables displayed in Table 1 cover our sample period from (1985Q1–2008Q2). Also, it is important to note that all of the variables used in our analysis were pre-tested for unit roots to ensure stationarity; all unit root tests may be obtained upon request of the authors.
Table 1: Summary statistics (1985Q1–2008Q2).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>2.55</td>
<td>4.38</td>
<td>0.76</td>
<td>0.9</td>
</tr>
<tr>
<td>Output gap</td>
<td>0.14</td>
<td>2.46</td>
<td>−2.75</td>
<td>1.04</td>
</tr>
<tr>
<td>Federal funds rate</td>
<td>5.04</td>
<td>9.73</td>
<td>1.00</td>
<td>2.18</td>
</tr>
<tr>
<td>Uncertainty index</td>
<td>98.2</td>
<td>158.53</td>
<td>63.11</td>
<td>21.75</td>
</tr>
<tr>
<td>Output PC</td>
<td>0.002</td>
<td>0.83</td>
<td>−0.62</td>
<td>0.28</td>
</tr>
<tr>
<td>Labor PC</td>
<td>−0.05</td>
<td>0.45</td>
<td>−0.61</td>
<td>0.19</td>
</tr>
<tr>
<td>Capacity utilization</td>
<td>−0.01</td>
<td>0.62</td>
<td>−0.84</td>
<td>0.28</td>
</tr>
</tbody>
</table>

### 4 Results

#### 4.1 Linear Taylor rule estimates

All variables used to extract out principal components were transformed to ensure they were stationary before we produced the output and employment factors. The top panel of Figure 1 displays the output principal component measured on the left hand axis and the real-time output gap [as estimated in Bunzel and Enders (2010)] measured on the right-hand axis; likewise, the bottom panel of Figure 1 displays the employment principal component measured on the left-hand axis and the real-time output gap measured on the right hand side.

Figure 1: Output gap and output factor.

As a preliminary analysis and a baseline, we first estimate a traditional linear Taylor rule over the 1985Q1–2008Q2 time period. That is, we estimate the following:

\[
i_t = (\alpha_0 + \alpha_1 \pi_t + \alpha_2 y_t + \alpha_3 i_{t-1} + \alpha_4 i_{t-2} + \varepsilon_t)
\]

where \(\pi_t\) is the PCE inflation rate, \(y_t\) is the real time output gap as defined in Bunzel and Enders (2010). The coefficient estimates as well as the AIC are displayed in the top panel of Table 2. Note in the top panel of Table 2, that the coefficients on the output gap and inflation rate are positive and statistically significant at conventional levels. Moreover, note that the sums of \(\alpha_3 + \alpha_4\) is 0.95 which suggests a high degree of interest rate smoothing. However, as noted above, we also include the output principal component, labor market principal component, and the growth rate of the uncertainty index in the reaction function of the central bank. As such, we estimate

\[
i_t = (\alpha_0 + \alpha_1 \pi_t + \alpha_2 y_t + \alpha_3 i_{t-1} + \alpha_4 i_{t-2} + \alpha_5 \text{output}_{pc} + \alpha_6 \text{labor}_{pc} + \alpha_7 \text{uncertainty}_t + \varepsilon_t)
\]
Table 2: Linear Taylor rules.

<table>
<thead>
<tr>
<th>Start</th>
<th>End</th>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
<th>$\alpha_5$</th>
<th>$\alpha_6$</th>
<th>$\alpha_7$</th>
<th>Aic</th>
<th>Bic</th>
</tr>
</thead>
</table>
| Panel A: Baseline Taylor rule:  
$i_t = (\alpha_0 + \alpha_1 \pi_t + \alpha_2 y_t + \alpha_3 i_{t-1} + \alpha_4 i_{t-2} + \varepsilon_t)$ | 1985:01 | 2008:02 | 0.01 | 0.081 | 0.17 | 1.49 | -0.54 | -207.08 |
|       |       | 0.16 | 1.69 | 5.05 | 23.39 | -9.39 |           |           |           |
| Panel B: Taylor rule including principal components:  
$i_t = (\alpha_0 + \alpha_1 \pi_t + \alpha_2 y_t + \alpha_3 i_{t-1} + \alpha_4 i_{t-2} + \alpha_5 output_{pc} + \alpha_6 labor_{pc} + \alpha_7 uncertainty_t + \varepsilon_t)$ | 1985:01 | 2008:02 | -0.10 | 0.10 | 0.11 | 1.46 | -0.51 | 0.71 | 0.03 | -0.08 | -217.65 |
|       |       | -0.88 | 2.41 | 3.99 | 25.03 | -9.50 | 3.65 | 0.15 | -1.94 |           |         |
| Panel C: Taylor rule: capacity utilization:  
$i_t = (\alpha_0 + \alpha_1 \pi_t + \alpha_2 y_t + \alpha_3 i_{t-1} + \alpha_4 i_{t-2} + \alpha_5 cap_t + \alpha_7 uncertainty_t + \varepsilon_t)$ | 1985:01 | 2008:02 | -0.13 | 0.07 | 0.11 | 1.47 | -0.50 | 0.40 | -0.11 | -220.37 |
|       |       | -1.17 | 1.54 | 4.17 | 25.8 | -9.56 | 3.77 | -2.82 |           |         |

Each of the above panels display linear Taylor rules. The coefficient estimates are displayed in the second row of each panel and T-statistics are displayed in row 3.

Panel B of Table 2 displays the estimates of (2). A number of features of the estimates in (2) are worth noting. First, note that the AIC and BIC both suggest that the (2) is a better fit than the traditional Taylor rule. Again note that the coefficients on inflation ($\alpha_1$) and the real-time output gap ($\alpha_2$) are positive and significant at conventional levels. Second, $\alpha_3 + \alpha_4$ suggests a substantial degree of interest rate smoothing. Third, note that the coefficient on the output principal component ($\alpha_5$) is large and statistically significant; however, the coefficient on the labor market principal component is not statistically different from zero. Fourth, note that the coefficient on the uncertainty index is $-0.08$ and statistically significant suggesting that increases in uncertainty results decrease the Federal Funds rate over the Greenspan era.

Obviously, one issue that is not clear in (2) is how to interpret the coefficient on the output principal component. As such, in order to better understand the output principal component, we estimate the following:

$$y_i = \alpha_{output_{pc}} + \varepsilon_i$$  \hspace{1cm} (3)

where $y_i$ are the output series mentioned in Section 2 and the $output_{pc}$ is the output principal component. Table 3 displays the factor loadings from estimating (3). Note in Table 3 that the factor with the largest loading is CUMFNS which is the Capacity Utilization. As such, we replace the output principal component in (2) with capacity utilization. Then we re-estimate equation (2) including capacity utilization in place of the output principal component and we exclude the labor market principal component because it was not statistically different from zero. Equation (2) is now

$$i_t = (\alpha_0 + \alpha_1 \pi_t + \alpha_3 y_t + \alpha_3 i_{t-1} + \alpha_4 i_{t-2} + \alpha_5 cap_t + \alpha_7 uncertainty_t + \varepsilon_t) .$$  \hspace{1cm} (4)

Note that in Panel C of Table 2 that the AIC and BIC both suggest that the above Taylor rule is preferred to those in Panels B and C. In addition, note again, that the coefficients on inflation and output are positive. Interestingly, the coefficient on inflation is only marginally statistically significant whereas the coefficient on the output gap and the coefficient on capacity utilization are significant at the 1% level. Moreover, note again that the coefficient on the uncertainty index is negative (−0.11) and statistically different from zero suggesting that the Federal Reserve responds to increases in uncertainty by lowering the Federal Funds rate.

4.2 Non-linear Taylor rule estimates

As noted above, Qin and Enders (2008) and Bunzel and Enders (2010) find substantial evidence of nonlinearity in the Taylor rule over the Greenspan time period. Specifically, in order to estimate the nonlinear Taylor rule, Bunzel and Enders (2010) estimate the following threshold regression

\[
i_t = \left( \alpha_0 + \alpha_1 \tau_t + \alpha_2 y_t + \alpha_3 I_{t-1} + \alpha_4 I_{t-2} \right) I_t + \left( \beta_0 + \beta_1 \tau_t + \beta_2 y_t + \beta_3 I_{t-1} + \beta_4 I_{t-2} \right) (1 - I_t) + \epsilon_t
\]

where \(i_t\) is the quarterly Federal Funds rate, \(\pi_t\) is the chain weighted GDP deflator inflation rate over the past four quarters, \(y_t\) is the real-time output gap, \(x_{t-d}\) is the magnitude of the threshold variable in period \(t - d\) and the Heaviside indicator \(I_t = 1\) if \(x_{t-d} > \tau\) and \(I_t = 0\) otherwise. The threshold value \(\tau\) is obtained by a grid search over all possible values of \(x_{t-d}\) where each regime is forced to contain at minimum of 25% of the observations in the sample. Because \(\tau\) is a nuisance parameter, statistical significance between the coefficients in the two regimes are determined by Hansen’s (1997) bootstrapped \(F\)-statistic.

As noted above, we take the best fitting linear model (4) and use the Bunzel and Enders (2010) nonlinear methodology. Thus we estimate the following:

\[
i_t = \left( \alpha_0 + \alpha_1 \tau_t + \alpha_2 y_t + \alpha_3 I_{t-1} + \alpha_4 \text{uncertainty}_t + \epsilon_t \right) I_t + \left( \beta_1 \tau_t + \beta_2 y_t + \beta_3 I_{t-1} + \beta_4 \text{uncertainty}_t \right) (1 - I_t) + \epsilon_t
\]

where \(i_t\) is the quarterly Federal Funds rate, \(\pi_t\) is the PCE inflation rate, \(y_t\) is the real-time output gap, \(\text{uncertainty}_t\) is the change in capacity utilization, \(\text{uncertainty}_t\) is percentage change in the Baker, Bloom, and Davis (2012) uncertainty index, \(x_{t-d}\) is the magnitude of the threshold variable in period \(t - d\) and the Heaviside indicator \(I_t = 1\) if \(x_{t-d} > \tau\) and \(I_t = 0\) otherwise. We estimate (6) using three different threshold variables. First, we follow Bunzel and Enders (2010) and estimate their original model using \(\pi_{t-1}\); second, we estimate (6) using \(\pi_{t-1}\); third, we use capacity utilization \(\text{cap}_{t-1}\); fourth, we use uncertainty \(\text{uncertainty}_{t-1}\); and fifth, we use the labor \(\text{pc}_{t-1}\).

Table 4 Panel A displays the results from estimating (5) using the threshold variable (one lag of inflation) as in Bunzel and Enders (2010); Table 4 Panel B displays the results from estimating (6) using one lag of inflation as the threshold variable; Table 4 Panel C displays the results from estimating (6) using one lag of the uncertainty index as the threshold variable; Table 4 Panel D displays the results from using one lag of capacity utilization as the threshold variable and Table 4 Panel E displays the results from using the labor market principal component as the threshold variable. The threshold value, \(\tau\), is listed along with the coefficient estimates and the AIC. The alpha (\(\alpha\)) coefficients display the Taylor rule coefficients when the model is above the threshold value (\(\tau\)) and the beta (\(\beta\)) display the results when the model is below the threshold value. Specifically, \(\alpha_1\) and \(\beta_1\) are the coefficients on inflation, \(\alpha_2\) and \(\beta_2\) are the measure on the real-time output gap, and \(\alpha_3\), \(\alpha_4\), \(\beta_3\), \(\beta_4\) are the coefficients on lags of the federal funds rate in the respective regimes, \(\alpha_5\) and \(\beta_5\) are the coefficients on capacity utilization and \(\alpha_7\) and \(\beta_7\) are the coefficients on the uncertainty index. As can be seen from Table 4, even though we use PCE inflation rather than the GDP deflator, our results are similar to those of Bunzel and Enders’s (2010) when estimating their model.
### Table 4: Nonlinear Taylor rules.

<table>
<thead>
<tr>
<th>Start</th>
<th>End</th>
<th>τ</th>
<th>$\alpha_1$</th>
<th>$\beta_1$</th>
<th>$\alpha_2$</th>
<th>$\beta_2$</th>
<th>$\alpha_3$</th>
<th>$\beta_3$</th>
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<th>$\beta_4$</th>
<th>$\alpha_5$</th>
<th>$\beta_5$</th>
<th>$\alpha_7$</th>
<th>$\beta_7$</th>
<th>Aic</th>
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<tbody>
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<td>0.27</td>
<td>0.16</td>
<td>-0.15</td>
<td>1.47</td>
<td>1.45</td>
<td>-0.51</td>
<td>-0.39</td>
<td>0.38</td>
<td>-0.05</td>
<td>-0.13</td>
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<td>-227.23</td>
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<td>0.07</td>
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<td>-222.21</td>
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<td>3.91</td>
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<td>1.41</td>
<td>-2.61</td>
<td>-1.72</td>
<td>-216.67</td>
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Each of the above panels display the results from estimating nonlinear Taylor rules using different threshold variables. The coefficient estimates are displayed in the second row of each panel and t-statistics are displayed in row 3. The bold row indicates the best fitting model.
The first thing to note in comparison of the AIC’s from Table 2 and Table 4 is that the linear model including capacity utilization and uncertainty has a better model fit than the nonlinear model in Panel A. Note that the AIC in Panel C of Table 2 is \(-220.37\) whereas the AIC of the nonlinear model in Panel A and Panel E of Table 4 is \(-217.83\) and \(-216.67\), respectively. However, all of the nonlinear models perform better than the linear traditional Taylor rule in Panel A of Table 2. Note also that the AIC in the nonlinear models in Panels A through D suggest that the nonlinear Taylor rule including capacity utilization and uncertainty perform better than the nonlinear model in which they are included. Using the labor market principal component as the threshold variable as displayed in Panel E resulted in the worst model fit. The best model according to the AIC is the model in Panel B when lagged inflation is used as the threshold variable. As such, we focus our discussion on the results from Panel B.

Somewhat surprisingly, note in Panel B that the threshold value is 2.02 very close to the explicit target noted by Bernanke in the introduction. Moreover, note that all of the coefficients are statistically significant at conventional levels with the exception of \(\beta_5\) which is the coefficient on capacity utilization when the model is in the low inflation regime. In the regime in which inflation is higher than 2.02%, the coefficient \((\alpha_1)\) on inflation is 0.18 (and significant at conventional levels), the coefficient \((\alpha_2)\) on the output gap is 0.16 and statistically significant. Note that \(\alpha_3 + \alpha_4\) is 0.96 which suggests a high degree of interest rate persistence. Interestingly, note that the coefficient on capacity utilization \((\alpha_5)\) is 0.38 and statistically significant. Moreover, note also that the coefficient on the uncertainty index is \(-0.13\) and statistically significant as well. In the low inflation regime, note that the coefficient on the inflation rate is 0.27 and statistically significant. Note that the Federal Funds rate is highly persistent in the low inflation regime as well. The starkest difference in the two regimes is the coefficient on capacity utilization in the low regime falls to \(-0.05\) and is not statistically different from zero suggesting that when inflation is low the Federal Reserve does not respond to increases in capacity utilization. Finally, note as well that the Federal Reserve responds to increases uncertainty by cutting interest rates as demonstrated by the \(-0.18\) coefficient.

4.3 Forecast performance

While the AICs in Table 2 and Table 4 suggests that the model in Panel B of Table 4 is the best model, we also chose to evaluate four representative models by evaluating their out of sample forecasting capabilities. As such, we estimate recursive out-of-sample forecasts to compare the these four models. Our out-of-sample forecasts began in 2000Q1 and ended in 2008Q2 time period. Again, similar to Bunzel and Enders, given that we use the contemporaneous values of inflation, the output gap, capacity utilization, and the uncertainty index it is not possible to obtain a forecast for the federal funds rate in \(t+1\) without forecasting the other contemporaneous variables as well. As such, evaluating forecasts become problematic because any differences in the forecasting performance of the various functional forms of the Taylor rule might be due to the method used to forecast the contemporaneous variables. As such, we follow Bunzel and Enders and adopt “backward-looking” variants of the Taylor rule to circumvent this problem.

We compare the following four different variants of the backwards looking Taylor rule. First, we consider a simple linear Taylor rule:

\[ i_t = (\alpha_0 + \alpha_1 \pi_{t-1} + \alpha_2 y_{t-1} + \alpha_3 \delta_{t-1} + \alpha_4 \delta_{t-2} + \epsilon_t) \]  

Second, we consider an augmented linear Taylor rule which simply includes capacity utilization and the uncertainty index

\[ i_t = (\alpha_0 + \alpha_1 \pi_{t-1} + \alpha_2 y_{t-1} + \alpha_3 \delta_{t-1} + \alpha_4 \delta_{t-2} + \alpha_5 \text{cap}_{t-1} + \alpha_6 \text{uncertainty}_{t-1} + \epsilon_t) \]  

Third, we estimate the following simple nonlinear Taylor rule

\[ i_t = (\alpha_0 + \alpha_1 \pi_{t-1} + \alpha_2 y_{t-1} + \alpha_3 \delta_{t-1} + \alpha_4 \delta_{t-2} + \epsilon_t)i_t + (\beta_1 \pi_{t-1} + \beta_2 y_{t-1} + \beta_3 \delta_{t-1} + \beta_4 \delta_{t-2} + \beta_5 \text{cap}_{t-1} + \beta_6 \text{uncertainty}_{t-1} + \epsilon_t) (1 - i_t) + \epsilon_t \]  

and finally we estimate an augmented nonlinear Taylor rule

\[ i_t = (\alpha_0 + \alpha_1 \pi_{t-1} + \alpha_2 y_{t-1} + \alpha_3 \delta_{t-1} + \alpha_4 \delta_{t-2} + \alpha_5 \text{cap}_{t-1} + \alpha_6 \text{uncertainty}_{t-1} + \epsilon_t)i_t + (\beta_1 \pi_{t-1} + \beta_2 y_{t-1} + \beta_3 \delta_{t-1} + \beta_4 \delta_{t-2} + \beta_5 \text{cap}_{t-1} + \beta_6 \text{uncertainty}_{t-1} + \epsilon_t) (1 - i_t) + \epsilon_t \]
We compare the 1-step-ahead forecasts for the above four models over the 2000Q2–2008Q2 time period. That is, in order to obtain our first 1-step-ahead forecast, we estimate each of the above four models using data spanning the 1985Q1–2000Q1 time period; we subsequently obtain 1-step-ahead forecasts and then repeat the procedure expanding the estimation window by one quarter to obtain the next 1-step-ahead forecasts.

Figure 2 displays the actual federal funds rate as well as the 1-step-ahead forecasts for each model. Table 5 displays the mean square error (MSE) as well as the root mean square error (RMSE) for the four models. First, note that both nonlinear models have lower MSE and RMSE than either of the linear models, which suggests that the nonlinear models produce superior forecasts. Additionally, note that the augmented nonlinear Taylor rule has a MSE of 0.05 and RMSE of 0.22 compared to the 0.06 and 0.24 of the nonlinear Taylor rule. While the nonlinear models appear to perform better, we also implement the Diebold-Mariano test to compare the forecasts of the four different models. First, we compare the forecasts of the Linear Taylor rule with the forecasts of the augmented linear Taylor rule. The Diebold-Mariano test results in a t-statistic of 1.09 and a p-value of 0.13 which mean that we cannot reject the linear Taylor rule forecasts in favor of the augmented Taylor rule forecasts. Second, given that we do not find a preference in the forecasts between the two linear models, we compare the results of the linear Taylor rule with those of the nonlinear and augmented nonlinear Taylor rules. Comparison of the linear Taylor rule forecasts and nonlinear Taylor rule forecasts results in a Diebold Mariano statistic of 2.60 which is significant at the 99% level; the Diebold-Mariano statistic comparing the linear Taylor rule with the augmented nonlinear Taylor rule was 2.31 which is also statistically significant at the 99% level which suggests that the forecasts of the nonlinear Taylor rule models are preferred to those of the linear model. Finally, we compare the forecasts from the nonlinear Taylor rule with those of the augmented nonlinear Taylor rule; the Diebold-Mariano statistic is 0.87 which suggests that we cannot reject the forecasts of the nonlinear Taylor rule model in favor of the forecasts of the augmented nonlinear Taylor rule.

![Figure 2: Output gap and labor factor.](image)

<table>
<thead>
<tr>
<th>Start</th>
<th>End</th>
<th>MSE</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Linear Taylor rule</td>
<td>2008:02</td>
<td>0.09</td>
<td>0.30</td>
</tr>
<tr>
<td>Panel B: Augmented linear Taylor rule 1985:01</td>
<td>2008:02</td>
<td>0.07</td>
<td>0.28</td>
</tr>
<tr>
<td>Panel C: Nonlinear Taylor rule 1985:01</td>
<td>2008:02</td>
<td>0.06</td>
<td>0.24</td>
</tr>
<tr>
<td>Panel D: Augmented nonlinear Taylor rule 1985:01</td>
<td>2008:02</td>
<td>0.05</td>
<td>0.22</td>
</tr>
</tbody>
</table>
Each of the above panels display the forecasting statistics from the above models. The mean square error (MSE) and root mean squared error (RMSE) are displayed in columns 3 and 4.

5 Conclusion

Given the recent clarifications of the Federal Reserve regarding their policy goals and targets, we re-estimated Taylor rules over the 1985–2008 time period. We find substantial evidence that the Federal Reserve responded to increases in macroeconomic uncertainty by cutting the Federal Funds rate by approximately 10 basis points. Moreover, we also found evidence that the Federal Reserve responded aggressively to increases in capacity utilization, especially when the inflation rate was above 2%. Similar to Bunzel and Enders (2010) we find substantial evidence suggesting that the Federal Reserve follows a nonlinear Taylor rule and that lagged inflation served as the best threshold variable. Our estimates of the inflation threshold were 2.02% when using PCE inflation in the estimated Taylor rules. Moreover, similar to Bunzel and Enders (2010) the nonlinear models we estimated result in better in-sample fit statistics as well as lower forecast errors. However, we were not able to draw a statistical difference between the forecasts of the nonlinear Taylor rule and our augmented nonlinear Taylor rule which included capacity utilization as well as the uncertainty index.

Our findings bear a number of important policy implications. For example, we find in Panel B of Table 4 that the responses of the Federal Reserve to the output gap and capacity utilization are both insignificant (with even negative point estimates) when the inflation rate is below the threshold value (2.02%). This seems to suggest that Federal Reserve does not respond as aggressively to the output target variable when the inflation rate is low as when the inflation rate is high. Second, the responses of the policy rate to the uncertainty index are negative in both the high and low inflation rate regimes, although this response is barely significant during the low inflation regime. This result lends support to the belief of “Greenspan put.” Such an implicit policy insurance to the financial market might have led to the potential moral hazard and possible excessive risk-taking behaviours.

Notes

1. Lehman Brothers collapsed in September 15, 2008 and Fannie Mae and Freddie Mac were taken into conservatorship in the third quarter of 2008.
2. All regressions are estimated using (Eicker-White) robust standard errors.

References


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