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Abstract

In the technology and design industries, one product builds on another: A smart television enhances a smart phone. However, due to complementary features, the utility that is gained by owning both products from the same firm is greater than the sum of the two products' utility if purchased from separate firms. Aftermarkets suggest that the margins of the second product would increase. Instead, we show that the firms' complementary utility offset each other, which results in reduced prices. Further, consumer purchase behavior is a function of the product release order; given a different release schedule, some consumers would purchase from a different company.

Keywords Aftermarkets · Bounded rationality · Complementary goods · Hotelling · Path dependence

JEL classification L11 · L13 · D82 · D43

1 Introduction

Many technology goods exhibit firm-specific complementarity behavior when purchased from the same manufacturer. As an example, a person can make calls with her phone and can watch TV with her smart TV; but initially only when both items are made by the same company can she stream a video from her phone to her TV.

When the smart phone was released, firms anticipated that there would be a firm-specific complementary television product, but consumers might not have anticipated this. Most features of both products do not require the complementary device, and firms compete on differentiated features. Moreover, the firm-specific complementary features that we model in this paper are concrete benefits for the consumer—e.g., streaming to the TV from the phone—and not a function of the psychological factors that are often associated with brand loyalty.[1] This 'bonus utility' is a special case of within-firm complementarity behavior and it exists for a large number of technological and design products.[2] Even if the feature set is not expanded, it is often less costly for a customer to learn about a new product from the same firm—especially since the interface is more likely to resemble that of the firm's previous products.

These firm-level, path-dependent, *partially* complementary goods are different from what we have seen in the previous literature. As they are largely independent goods that sometimes exhibit complementary behavior, they are different than aftermarket[3] goods or goods that exhibit switching costs. Because the consumer is aware of only the already-released products, the consumer does not take later products into account. The firms—recognizing that they are creating path dependence – reduce the price in the first market in an attempt to capture later sales. This is different from goods with switching costs and lock-ins (Klemperer, 1987a, 1987b, 1995; Farrell & Shapiro, 1988, 1989; Beggs & Klemperer, 1992), where the firms have increased market power for the stream of later purchases. Instead, these mostly independent goods have some complementary value when purchased from the same firm; but some consumers purchase from the competing firm because that product better matches their preferences. In our model, consumers are better off as a result of the bonus utility—in some cases even when they purchased the two goods from different firms.

The paper also relates to a strand of literature on product mix and multiproduct firms (Dixit & Stiglitz, 1977; Raubitschek, 1987; Shaked & Sutton, 1990; Anderson & de Palma, 1992; Bernard et al., 2010). The additional utility or satisfaction that arises from consumption of different products from the same brand —e.g., a brand-reputation effect—have been widely explored in the literature (Oliver, 1999). The bonus utility that is considered in our study varies from the brand effect as it

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[1] We use the term 'psychological factors' to describe purchase decisions that are driven by non-concrete benefits. We do not mean the term negatively. We simply mean that purchases that are driven by psychological factors might be more driven by the marketing department than by the engineering department.

[2] See Sect. 5 for more examples from different industries.

[3] For a literature review of aftermarkets, see Ellison (2005).

emerges from additional enhancements or conveniences that are provided solely by the joint consumption of the two stand-alone products of a firm, such as a smart phone and a smart TV. In the latter case, the firm can sometimes influence the level of bonus satisfaction by providing various tangible features that are available only when the two products are used together. However, when considering features such as a common user interface or design, this additional satisfaction is inherent to the products and cannot be influenced substantially by the firm.[4]

In this paper, we develop a simple two-period, two-product complete information game in the linear city model framework. The main result is that firms competing for market share end up charging a lower price for the initial good as they anticipate the bonus utility in the second good market for the consumers who buy from them initially. This can results in an overall decrease in prices. Given the same preferences, some consumers would have purchased from a different firm if the products were released in a different order or the bonus utility did not exist. When bonus utilities for firms are symmetric, the outcomes in this paper are comparable to those that are obtained from certain stylized switching-cost models.[5] However, when bonus utilities are asymmetric, the outcomes diverge from those in the switching-cost literature.

The remainder of the paper is organized as follows: In the next section, we present the baseline model, solve for the equilibrium, and provide analysis. In Sects. 3 and 4, we extend our model to two periods and sophisticated consumers, respectively. Section 5 discusses potential applications of our model, while Sect. 6 pro- vides concluding remarks.

2 The Baseline Model

Consider a duopoly model where two firms create two products: product A is sold in a first market and product B is sold in a second market.[6] There is an additional benefit from owning both products from the same firm. We will define this benefit—*bonus utility*— as μ and y for firm 1 and 2, respectively. For our purposes, we assume that the values of μ and y are governed by the nature of products and are not chosen by the firm. While consumers are incentivized to purchase both products from the same firm, it is possible to own one product from each of the two firms. At

[4] Some bonus utility would be costly for the firm to avoid even if the firms did not want to provide it. For instance, software firms could choose to have very different user interfaces across products instead of similar interfaces. The firms would likely always choose to have similar interfaces when targeting similar consumers, as this would: require less effort for the design team; result in fewer support calls as consumers would be more familiar with the product; and would be in their customers' best interest.

[5] In particular, a two-period model of switching cost with: (1) myopic consumers; (2) forward looking firms; and (3) full independence of product preferences (Villas-Boas, 2015) It is important to note, however, that even when there is a convergence in the outcomes of the models, the outcomes are driven by different underpinnings.

[6] Product A is simply the product that is released first, while product B is the product that is released second.

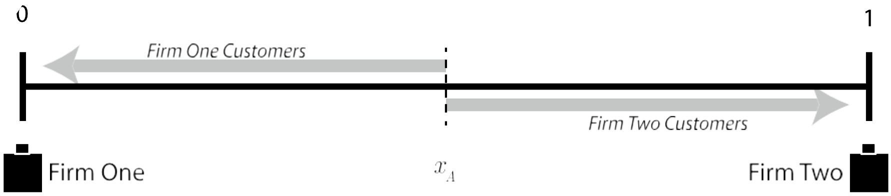

Fig. 1 Here we show a linear city from zero to one. Consumers are uniformly distributed in preferences between zero and one. The consumer who is indifferent between the two firms is referred to as the indifferent consumer (who is labeled $_A$). In the market for product A, any consumer located to the left of $_A$ will buy from firm 1, while any consumer to the right of $_A$ will purchase from firm 2. The firms are implicitly choosing $_A$ by choosing prices

the time of the first product's release, the firms know that the second product will exist, but the consumers do not. For concreteness, think about product A as a smart phone and product B as a smart TV.[7][8]

Each market is described by a linear city of length one (Hotelling, 1929). Consumers are distributed uniformly and independently, along each city, with a total population that equals one for each market. Each potential consumer eventually buys one unit of both products; both markets are covered. Firms 1 and 2 are located at positions zero and one, respectively, in both markets. We assume that both firms have constant identical marginal costs (Neven, 1985), so as to focus on the effect of the bonus utility.

2.1 The Equilibrium

A linear city model is solved by finding the consumer who is indifferent between the two firms' products, as any consumer to the left of that point (see Fig. 1) will obtain more utility from the firm that is located at zero. Consequently, if U_{1A} is the utility that the consumer receives from purchasing product A from firm 1 and U_{2A} is the utility that the consumer receives from purchasing the good from firm 2, then the following will determine the indifferent consumer for the first market:

[7] The assumption of myopic consumers, which is standard in the switching cost and path dependence literature (see, for instance, Cabral & Villas-Boas, 2005; Dubé et al., 2009), is reasonable, since many of the applications for which the paper is relevant are durable goods (see Sect. 5 for details), in which case consumers are not always aware of future releases of new products, developments of product categories, and, most importantly, the nature and extent of integration between a company's line of products (a source of bonus utility). Nonetheless, in Sect. 4 we explore the equilibrium behavior when consumers are sophisticated, and we show that our main results that are reported in this section remain qualitatively unaffected.

[8] Our model does not consider the potential for pirated goods or open-source alternatives (to be clear, we do not consider piracy and open-source alternatives to be the same thing only that they might act on the model in a similar way). Many of our example products are durable goods where piracy is difficult and open-source alternatives are scarce. Nonetheless, in software both do exist. For simplicity, we are ignoring these alternatives.

$$\underbrace{s_A - p_{1A} - t_A x_A}_{U_{1A}} = \underbrace{s_A - p_{2A} - t_A(1 - x_A)}_{U_{2A}}$$
$$x_A = \frac{-p_{1A} + p_{2A} + t_A}{2t_A} \tag{1}$$

where s_A is the gross consumer benefit from product *A*, t_A is the transportation cost of each unit of travel, and the prices charged by each firm for product *A* are p_{1A} and p_{2A} for firm 1 and 2, respectively.[9]

For product *B*, x_A consumers obtain bonus utility μ if they purchase good *B* from firm 1, while $1 - x_A$ obtain bonus utility y if they purchase from firm 2. In both cases, the bonus utility is a function of purchasing product *A* and *B* from the same firm. The position of an individual consumer in the market for product *B* is independent of his position in the market for product *A*. Further, the bonus utility (μ or y) is independent of the consumer's position as it is generated from the use of product *A* (either training or complementary features).

We first determine the indifferent consumer in the second market among those who purchased product *A* from firm 1. We set the utility of purchasing product *B* from firm 1 ($U_{1B\mu}$) equal to the utility of purchasing product *B* from firm 2 (U_{2B}):

$$\underbrace{s_B + \mu - t_B x_{B\mu} - p_{1B}}_{U_{1B\mu}} = \underbrace{s_B - t_B(1 - x_{B\mu}) - p_{2B}}_{U_{2B}}$$
$$x_{B\mu} = \frac{t_B + \mu - p_{1B} + p_{2B}}{2t_B} \tag{2}$$

Where: s_B is the gross consumer benefit from product *B*; t_B is the transportation cost for each unit of travel; and the prices charged by each firm for product *B* are p_{1B} and p_{2B} for firm 1 and 2, respectively. The solution in (2) – $x_{B\mu}$—describes the proportion of x_A consumers who will also purchase product *B* from firm 1. Alternatively,
$1 - x_{B\mu}$ represents the proportion of x_A consumers who purchase product *B* from firm 2 despite having purchased product *A* from firm 1 (see Fig. 2).

Similar to firm 1, $1 - x_A$ consumers, who purchase product A from firm 2, obtain bonus utility y if they purchase good *B* from the same firm. We can obtain the indifferent consumer by setting the utility of purchasing product *B* from firm 1 (U_{1B}) equal to the utility of purchasing product *B* from firm 2 (U_{2By}):

[9] Using quadratic transport costs, we find no qualitative difference in our main result: Similar to the model that is presented here, the final prices for the model with quadratic costs are $p^*_{1A} = p^*_{2A} = c_A + t_A - \mu$, and $p^*_{1} = p^*_{2} = c_B + t_B$. The indifferent consumer final solutions in Eqs. (1), (2), and (3) are identical regardless of whether the transport costs are specified linearly or quadratically.

Fig. 2 In the market for product A, $_A$ consumers purchased from firm 1. Therefore, $_A$ proportion of consumers in market B get an additional amount of utility by owning both A and B from the same company: μ. The indifferent consumer in the market for product B—given that the consumer purchased product A from firm 1—is represented by $_{B\mu}$

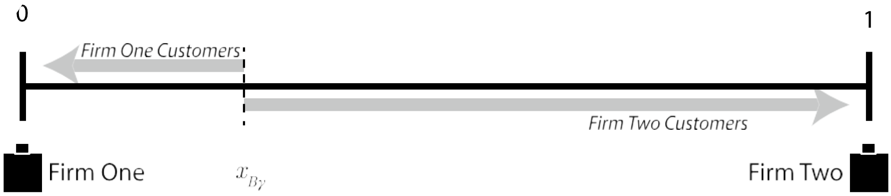

Fig. 3 In the market for product A, $1 - x_A$ consumers purchased from firm 2. Therefore, $1 - x_A$ proportion of consumers in market B get an additional amount of utility by owning both A and B from the same company (y). The indifferent consumer in the market for product B given the consumer purchased product A from firm 2 is represented by $_{B}$ $_{y}$

$$\underbrace{s_B - t_B x_{By} - p_{1B}}_{U_{1B}} = \underbrace{s_B + y - t_B(1 - x_{By}) - p_{2B}}_{U_{2By}}$$
$$x_{By} = \frac{t_B - y - p_{1B} + p_{2B}}{2t_B} \tag{3}$$

where $1 - x_{By}$ describes the proportion of the $1 - x_A$ consumers who will also purchase product B from firm 2, while x_{By} will purchase good B from firm 1 after purchasing good A from firm 2 (see Fig. 3).

Given the firms' outputs in the two markets, the firms' profits are defined as follows:

$$\begin{aligned} n_1 &= x_A(p_{1A} - c_A) + (p_{1B} - c_B)[x_A x_{B\mu} + (1 - x_A)x_{By}] \\ n_2 &= (1 - x_A)(p_{2A} - c_A) + (p_{2B} - c_B)[x_A(1 - x_{B\mu}) + (1 - x_A)(1 - x_{By})] \end{aligned} \tag{4}$$

where c_A and c_B are the cost per unit of product A and B, respectively. Because the proportion of consumers who receive the bonus utility is a function of sales of product A, the indifferent consumer solutions—$x_{B\mu}$ and x_{By} – are weighted by the product A sales of the two firms: x_A and $1 - x_A$.

Each firm chooses prices for both products while taking the other firm's expected behavior as given.[10] We find the first order conditions using the equations in (4) and solve for the best response functions:

$$
\begin{aligned}
p_{1A}(p_{2A}, p_{1B}) &= \frac{2t_B(c_A + p_{2A} + t_A) - (y + \mu)(p_{1B} - c_B)}{4t_B} \\
p_{2A}(p_{1A}, p_{2B}) &= \frac{2t_B(c_A + p_{1A} + t_A) - (y + \mu)(p_{2B} - c_B)}{4t_B} \\
p_{1B}(p_{2B}, p_{2A}, p_{1A}) &= \frac{t_A(2c_B + 2t_B + 2p_{2B} + \mu - y) + (y + \mu)(p_{2A} - p_{1A})}{4t_A} \\
p_{2B}(p_{1B}, p_{1A}, p_{2A}) &= \frac{t_A(2c_B + 2t_B + 2p_{1B} + y - \mu) + (y + \mu)(p_{1A} - p_{2A})}{4t_A}
\end{aligned}
\tag{5}
$$

The price of product A that is charged by a given firm is increasing in the rival firm's price—$\frac{\partial p_{1A}}{\partial p_{2A}} > 0$ and $\frac{\partial p_{2A}}{\partial p_{1A}} > 0$—and is decreasing in the price of good B – $\frac{\partial p_{1A}}{\partial p_{1B}} < 0$ and $\frac{\partial p_{2A}}{\partial p_{2B}} < 0$. Similarly, the price of product B for either firm increases in the rival's price—$\frac{\partial p_{1B}}{\partial p_{2B}} > 0$ and $\frac{\partial p_{2B}}{\partial p_{1B}} > 0$—while it is decreasing in the firm's price for product A – $\frac{\partial p_{1B}}{\partial p_{1A}} < 0$ and $\frac{\partial p_{2B}}{\partial p_{2A}} < 0$. Intuitively, if the price of product B increases, the firm reduces the price that it charges for product A, so as to attract more customers and increase the sales of its complementary product.

Simultaneously solving the system of four equations in (5) yields Nash equilibrium price levels:

$$
\begin{aligned}
p^*_{1A} &= \frac{1}{2}\left(2c_A + t_A - \mu - y + \frac{\overbrace{2t_A(9t_At_B - \mu(\mu + y))}^{\Omega}}{18t_At_B - (\mu + y)^2}\right) \\
p^*_{2A} &= \frac{1}{2}\left(2c_A + t_A - \mu - y + \frac{\overbrace{2t_A(9t_At_B - y(\mu + y))}^{\Phi}}{18t_At_B - (\mu + y)^2}\right) \\
p^*_{1B} &= c_B + t_B + \frac{3t_At_B(\mu - y)}{18t_At_B - (y + \mu)^2} \\
p^*_{2B} &= c_B + t_B + \frac{3t_At_B(y - \mu)}{18t_At_B - (y + \mu)^2}
\end{aligned}
\tag{6}
$$

[10] In our baseline model, firms choose prices for both markets at the beginning of the game. In Sect. 3, we demonstrate the robustness of this assumption by considering a two-period market, whereby firms decide on prices in the market for product A in the first period and for product B in the second period. The results from the two-period analysis corroborate those that are presented in this section.

The firms charge lower prices for product *A* because they know that they will create bonus utility for the purchase of product *B*.[11] But, because both are creating additional utility, the utility difference is what determines the price of the second product. If the degree of bonus utility is identical ($\gamma = \mu$), the prices simplify to:

$$\begin{aligned} p^*_{1A} &= p^*_{2A} = c_A + t_A - \mu \\ p^*_{1B} &= p^*_{2B} = c_B + t_B \end{aligned} \qquad (7)$$

In essence, each firm anticipates the bonus utility and charges a lower price for the first product. However, because the bonus utility of each firm offsets each other, the two firms charge the same amount for product *B* as if there were no bonus utility. Clearly, consumers benefit because they receive the lower price for product *A*, but do not pay more for the second product. Prices are decreasing for the first product in the value of the bonus utility. This is similar in spirit to Fabra and Garcia (2015), where prices are decreasing in the value of the switching cost due to the poaching of customers.

2.2 Unequal Bonus Utility

While offsetting bonus utilities result in a benefit to consumers, a firm can still benefit from increased bonus utility over its competitor: $\mu \lesseqgtr \gamma$. Consider the case when $\gamma = 0$ – no bonus utility from consumption of both products from firm 2 – and $\mu > 0$. If we use the equations from (6), prices can be restated as:

$$\begin{aligned} p^*_{1A} &= c_A - \frac{9t_Bt_A^2}{18t_Bt_A - \mu^2} + \frac{3t_A}{2} \\ p^*_{2A} &= c_A + \frac{9t_Bt_A^2}{18t_Bt_A - \mu^2} + \frac{t_A}{2} \\ p^*_{1B} &= c_B + \frac{3t_B\mu t_A}{18t_Bt_A - \mu^2} + t_B \\ p^*_{2B} &= c_B - \frac{3t_B\mu t_A}{18t_Bt_A - \mu^2} + t_B \end{aligned} \qquad (8)$$

We can see from (8) that $p^*_{1A} - p^*_{2A} = -\frac{\mu^2 t_A}{18t_Bt_A-\mu^2}$. This difference is always negative when $t_B > \frac{\mu^2}{18t_A}$; this is always true as indicated in the next paragraph. Similarly, from (8) we see that $p^*_{1B} - p^*_{2B} = \frac{6t_B\mu t_A}{18t_Bt_A-\mu^2}$. This difference is always positive when $t_B > \frac{\mu^2}{18t_A}$. Therefore, firm 1 will always charge less than firm 2 for product A and more than firm 2 for product B. From (8), we can derive the profit difference:

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[11] The first-period price reduction is not obvious given Ω and Φ in Eq. (6). However, $\Omega + \Phi$ simplifies to $2t_A$. Therefore, given the remainder of the price equation, the firms are net charging a lower price in market *A*. This holds true both for a simple average of market *A* prices and for a market-share-weighted average of prices as more consumers will purchase from the less expensive firm.

$$Jr_1 - Jr_2 = \frac{t_B\left(18 t_B t_A - \mu^2 + 18 t_A^2 - 3\mu t_A\right)}{36 t_B t_A - 2\mu^2} - \frac{1}{2}\left(\frac{3 t_B \mu t_A}{18 t_B t_A - \mu^2} + t_B - \mu + t_A\right)$$

$$= \frac{\mu\left(12 t_B t_A + \mu t_A - \mu^2\right)}{2\left(18 t_B t_A - \mu^2\right)} \qquad (9)$$

The sign of the profit difference can be recovered as follows: Substituting (8) into (1) results in $x_A^* = \frac{9t_B t_A}{18 t_B t_A - \mu^2}$, which is bounded between zero and one. This constraint indicates that $t_B \geq \frac{\mu}{9t_A}$ or, rearranged, $9t_A t_B \geq \mu^2$. Therefore, ¢ ≥ (f) and $\omega > 0$, and hence $\pi_1 - \pi_2 > 0$. Intuitively, this implies that an increase in firm 1's bonus utility μ results in an increased profit difference between firm 1 and firm 2, with firm 1 reaping greater profits compared to firm 2: $\pi_1 > \pi_2$.

For concreteness, we provide numerical results in Table 1. In this example, $t_A = t_B = 2$ and $c_A = c_B = c$. As indicated by Eq. (9), when $\mu > \gamma$, firm 1 experiences higher profit. Further, while firm 1 reduces its price for product A more when $\mu > \gamma$, firm 2 must reduce its price of product A due to the competition. In market B, firm 1 charges slightly more than does firm 2 when $\mu > \gamma$; but despite charging more, firm 1 serves more customers than firm 2. This difference is neutralized once $y = \mu$ (Scenario 3). Note that total producer surplus – $n_1 + n_2$ – is identical in all three scenarios. Scenario 4, where there is no bonus utility, is provided for comparison purposes.

It is notable that Table 1 highlights a weakness of location models in general: Location models assume that the market is fixed in size—regardless of the prices in that market. In practice, the markets in scenarios 1-3 would be larger than in Scenario 4 as the prices are lower.

2.3 Path Dependence and Complementary Goods

While the prices for product B may be identical when the two products (A and B) are sold independently, the customer base is not. Despite independent preferences for products A and B, purchasing product A from a given firm makes the consumer more likely to purchase product B from that same firm.

Consider the difference between the indifferent consumer for product B given that she purchased product A from firm 1 ($x_{B\mu}$) and the indifferent consumer for product B given that she purchased product A from firm 2 (x_{By}):

$$\begin{aligned} \Delta x_B &= x_{B\mu} - x_{By} \\ \Delta x_B &= \frac{t_B + \mu - p_{1B} + p_{2B}}{2t_B} - \frac{t_B - y - p_{1B} + p_{2B}}{2t_B} \\ \Delta x_B &= \frac{\mu + y}{2t_B} \end{aligned} \qquad (10)$$

Table 1 Numerical example for symmetric and asymmetric bonus utilities

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
μ	0.4000	0.3000	0.2000	0.0000
y	0.0000	0.1000	0.2000	0.0000
Firm 1 product A overall market share	50.11%	50.06%	50.00%	50.00%
Firm 1 product B overall market share	51.67%	50.84%	50.00%	50.00%
Firm 1's B share purchased A from firm 1	56.66%	55.83%	55.00%	50.00%
Firm 1's B share purchased A from firm 2	46.66%	45.83%	45.00%	50.00%
$_{1A}$	c + 1.7978	c + 1.7989	c + 1.8000	c + 2.0000
$_{2A}$	c + 1.8022	c + 1.8011	c + 1.8000	c + 2.0000
$_{1B}$	c + 2.0668	c + 2.0334	c + 2.0000	c + 2.0000
$_{2B}$	c + 1.9332	c + 1.9666	c + 2.0000	c + 2.0000
n_1	1.9688	1.9341	1.9000	2.0000
n_2	1.8334	1.8664	1.9000	2.0000

This suggests that as the indifferent consumers separate, the choice of firm in the market for product B becomes increasingly dependent on the outcome in the first market (see Fig. 4). This separation is order dependent: A given consumer might not make the same choices if product B was released before product A.

Example Assume $y = \mu = 1$ and the travel cost for both markets is $t_A = t_B = 2$. Therefore, the indifferent consumers are defined as follows: $x_A = 1/2$, $x_{B\mu} = 3/4$, and $x_{By} = 1/4$. Suppose there is a consumer who is positioned between 1/4 and 1/2 for product A and 1/2 and 3/4 for product B. Given the current product release order, our consumer would purchase product A and B from firm 1. However, if product B came first, the bonus utility would be actualized with the purchase of product A. Therefore, the new indifferent consumers would be as follows: $x_B = 1/2$, $x_{A\mu} = 3/4$, and $x_{Ay} = 1/4$. Thus, if product B was released before product A, our example consumer would purchase both products from firm 2.

2.4 Leadership in Market *B*

The primary result of this paper suggests that the lack of a premium for product B is a function of the two firms' competing for customers who purchased good A from their competitor. However, while consumers are myopic, both firms are fully informed about the future existence of product B. In this section, we show that if one firm is aware of product B at the beginning of time, but the other is not, a premium for the second product will be maintained. Specifically, the firm with an informational advantage will charge more for the second product and this additional rent will be a function of the bonus utility.

Consider two "high tech" firms that produce product A in the first time period. Firm 1 is aware that it will release product B in the next period. Further, firm 1 also

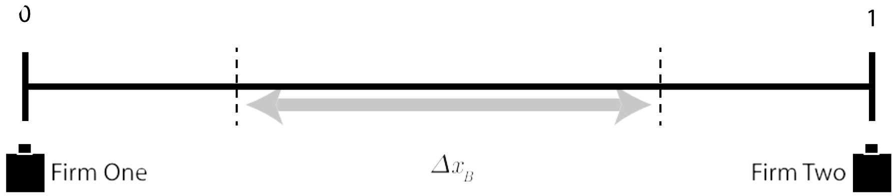

Fig. 4 As the bonus utility increases the difference between the quantity sold to 'existing customers' (customers who purchased product A) and new customers increases

knows that there is a bonus utility that is associated with owning products A and B from the same firm. Firm 2 does not know that product B exists in the first period, but firm 2 will produce a competing product after firm 1 releases its version of product B. Firm 1 knows that firm 2 will release a version of product B and thus takes its rival's action into account. Further, we assume the bonus utility is equal for both firms, for simplicity. We will describe this additional utility as μ.

This alteration to our multistage game can be solved by considering the last stage and migrating backwards in time. In the last stage, firm 2 maximizes its profits from product B given both firm 1's pricing and the result of the first stage of the game. In particular, firm 2's second stage profit can be described as follows:

$$\pi_{2B} = (p_{2B} - c_B)[x_A(1 - x_{B\mu}) + (1 - x_A)(1 - x_{By})] \tag{11}$$

where the solutions for x_A, $x_{B\mu}$, and x_{By} are the previously defined indifferent consumers (Eqs. (1), (2), and (3)). In this stage, firm 2 chooses p_{2B}. For succinctness, we will skip the first-order condition and move on to the best response function:

$$p_{2B}(p_{1B}, p_{1A}, p_{2A}) = \frac{t_A(c_B + t_B + p_{1B}) + \mu(p_{1A} - p_{2A})}{2t_A} \tag{12}$$

In the first stage, firm 2 is myopic and is unaware of the second stage. Therefore, firm 2 constructs its best response function based on the actions of firm 1 but without consideration of the second market. This response function is based on profit maximization of the firm's first-stage profit:

$$\pi_{2A} = (1 - x_A)(p_{2A} - c_A) \tag{13}$$

where the solutions for x_A, $x_{B\mu}$, and x_{By} are the previously defined indifferent consumers. Firm 2 chooses p_{2A} given firm 1's choice of p_{1A}. For succinctness, we will skip the first-order condition and present the firm's best response function:

$$p_{2A}(p_{1A}) = \frac{1}{2}(c_A + p_{1A} + t_A) \tag{14}$$

Finally, firm 1 maximizes its original profit equation with the exception that, given that firm 1 knows the actions of firm 2, the firm replaces all instances of p_{2B} with Eq. (12). We denote this new profit equation as $\pi_1 | p_{2B}$ (i.e., π_1 given p_{2B}), which is specified as follows:

$$n_1|p_{2B} = x_A(p_{1A} - c_A) + (p_{1B} - c_B)[x_A x_{B\mu} + (1 - x_A)x_{By}] \tag{15}$$

With this setup, firm 1 maximizes Eq. (15) by simultaneously choosing p_{1A} and p_{1B}. This results in the following best response functions:

$$\begin{aligned} p_{1A}(p_{2A}) &= \frac{4t_B t_A(c_A + p_{2A} + t_A) - 3t_B\mu t_A + \mu^2(-p_{2A})}{8t_B t_A - \mu^2} \\ p_{1B}(p_{2A}) &= \frac{2t_B[6t_B t_A - \mu(c_A - p_{2A} + t_A)]}{8t_B t_A - \mu^2} + c_B \end{aligned} \tag{16}$$

Using the best response functions from Eqs. (14) and (16), we can solve for the final prices in the model:

$$\begin{aligned} p_1^* &= c_A + \frac{6t_B\mu t_A}{\mu^2 - 12t_B t_A} + t_A \\ p_2^* &= c_A + \frac{3t_B\mu t_A}{\mu^2 - 12t_B t_A} + t_A \\ p_1^* &= c_B - \frac{18t_B^2 t_A}{\mu^2 - 12t_B t_A} \\ p_{2B}^* &= c_B + t_B\left(\frac{9t_B t_A}{\mu^2 - 12t_B t_A} + 2\right) \end{aligned} \tag{17}$$

Notably, $x_A^* = \frac{3t_B\mu}{2(12t_B t_A - \mu^2)} + \frac{1}{2}$, is bounded between zero and one. This constraint indicates that as long as $\mu < 4t_A$ then $\mu(3t_B + \mu) < 12t_A t_B$. As $3t_B\mu$ is always positive, $12t_B t_A > \mu^2$. Therefore, both firms' product A prices are lower than when the products are profit maximized independently: $c_A + t_A$; but firm 1's price is twice as sensitive to the bonus utility consequences than is firm 2's price. For product B, firm 1 always charges more than if the products were profit-maximized independently, but this is not always true of firm 2. Subtracting $c_B + t_B$ (the pricing of product B when the products are profit maximized independently) from p_{1B} results in $\frac{t_B(6t_B t_A + \mu^2)}{12t_B t_A - \mu^2}$, which is always positive. Subtracting $c_B + t_B$ from p_{2B} results in $\frac{9t_B^2 t_A}{\mu^2 - 12t_A t_B} + t_B$, which is positive only when $3t_A t_B > \mu^2$.

The price difference between the firms can be expressed as follows:

$$\begin{aligned} \Delta_{pA} &= p_{1A} - p_{2A} = \frac{3t_B\mu t_A}{\mu^2 - 12t_B t_A} \\ \Delta_{pB} &= p_{1B} - p_{2B} = t_B\left(\frac{27t_B t_A}{12t_A t_B - \mu^2} - 2\right) \end{aligned} \tag{18}$$

Equation (18) indicates that the different product B pricing is a function of the bonus utility (μ). In essence, the leader's knowledge of the connected products allows them to set higher prices for product B. This is similar to the result in a standard Stackelberg model (Tremblay and Tremblay (2012), Section 11.1.2) where the leader has an advantage over the follower. The pattern of behavior that we see in the high-tech industry, where strategically complementary goods are often more expensive from the firm that first releases the product, is consistent with the analysis in this section.

2.5 Non-Covered Market

Section 2.3 suggests that customers are more likely than not to purchase product B from the same firm as product A. However, we can show that our main result is driven by the competition amongst firms in the second good's market.

Suppose that product B was purchased by only some customers such that the two firms did not directly compete in the second market. In this case, we would redefine Eq. (2) as follows:

$$\begin{aligned} \underbrace{s_B - t_B x_{B\mu} + \mu - p_{1B}}_{U_{1B\mu}} &= 0 \\ x_{1B\mu} &= \frac{\mu - p_{1B} + s_B}{t_B} \\ x_{1By} = \underbrace{x_{1B\mu}}_{\text{where } \mu=0} &= \frac{-p_{1B} + s_B}{t_B} \end{aligned} \tag{19}$$

Similarly, Eq. (3) would be expressed as:

$$\begin{aligned} 0 &= \underbrace{s_B + y - t_B(1 - x_{2By}) - p_{2B}}_{U_{2By}} \\ x_{2By} &= \frac{-y + t_B + p_{2B} - s_B}{t_B} \\ x_{2B} = \underbrace{x_{2By}}_{\text{where } y=0} &= \frac{t_B + p_{2B} - s_B}{t_B} \end{aligned} \tag{20}$$

where $x_{1B\mu}$ is the proportion of firm 1 product A consumers who purchase product B from firm 1. Similarly, x_{1By} is the proportion of firm 2 product A customers who purchase product B from firm 1. x_{2By} and $x_{2B\mu}$ are defined in a corresponding manner. Since consumers are purchasing products A and B from different firms, then $\mu = y = 0$ in x_{1By} and $x_{2B\mu}$.

In this setup, the firm finds the customer who is indifferent between purchasing product B from them or not at all. Therefore, the firm does not take into account the behavior of the other firm and effectively behaves as a monopolist in the second market. Using the indifferent consumer solution for product A from Eq. (1) and our solutions for $x_{1B\mu}$, x_{1By}, $x_{2B\mu}$ and x_{2By} from (19) and (20), we can define the profit equations as:

$$
\begin{aligned}
\pi_1 &= x_A(p_{1A} - c_A) + (p_{1B} - c_B)[x_A x_{1B\mu} + (1 - x_A)x_{1By}] \\
\pi_2 &= (1 - x_A)(p_{2A} - c_A) + (p_{2B} - c_B)[x_A(1 - x_{2B\mu}) + (1 - x_A)(1 - x_{2By})]
\end{aligned}
\tag{21}
$$

Similar to profit Equations in (4), the indifferent consumers for product B are weighted by the sales of product A. For simplicity, we will assume that $y = \mu$. We obtain first order conditions from Eq. (21) and solve for the Nash equilibrium prices:

$$
\begin{aligned}
p^*_{1A} &= p^*_{2A} = c_A + t_A - \frac{\mu}{2t_B}\left(s_B + \frac{\mu}{2} - c_B\right) \\
p^*_{1B} &= p^*_{2B} = \frac{c_B}{2} + \frac{\mu}{4} + \frac{s_B}{2}
\end{aligned}
\tag{22}
$$

Unlike in Eq. (7), prices for product B increase in μ—much like an add-on product. Intuitively, this suggests that the competition between the two firms in the second market is a necessary condition for the market behavior that is discussed in this article.

3 Two-Period Market

In the previous sections, we assumed that firms commit to prices in the first (product A) and second (product B) market at the beginning of the game. While firms might not have perfect knowledge of market B prices, they would still have an expectation about prices, and thus margins, based on their experience, which is a reasonable assumption. In this section, we formulate a two-period market to illustrate how equilibrium behavior changes when the firms' decisions about product A and product B are made sequentially. The formulation of our two-period model is similar in spirit to Klemperer (1987b); but, unlike Klemperer (1987b), we consider product complementarity and bonus utility. By backward induction, we start with the second period decisions.

3.1 The Second Period: Product *B* Market

In the second period, we compute the indifferent consumer and the firms' optimal prices for product B, treating the first-period product A market shares (x_A and $1 - x_A$) of the two firms as given. The indifferent consumer and firms' output (as function of prices) of product B are determined based on Eqs. (2) and (3). Given the firms' outputs of product B, firms' second-period profits are specified as follows:

$$
\begin{aligned}
\pi_1 &= (p_{1B} - c_B)[x_A x_{B\mu} + (1 - x_A)x_{By}] \\
\pi_2 &= (p_{2B} - c_B)[x_A(1 - x_{B\mu}) + (1 - x_A)(1 - x_{By})]
\end{aligned}
\tag{23}
$$

Firms choose prices for product B taking as given the other firm's decision and also the firms' outputs of product A as given. Solving the first-order conditions we obtain the firms' second-period best response functions:

$$p_{1B}(p_{2B}, x_A) = \frac{c_B + p_{2B} + t_B + x_A\mu - (1 - x_A)y}{2}$$
$$p_{2B}(p_{1B}, x_A) = \frac{c_B + p_{1B} + t_B - x_A\mu + (1 - x_A)y}{2} \tag{24}$$

Simultaneously solving the system of two equations in (24) produces second-period subgame perfect Nash equilibrium levels:

$$p_{1B}(x_A) = c_B + t_B + \frac{x_A\mu - (1 - x_A)y}{3}$$
$$p_{2B}(x_A) = c_B + t_B + \frac{(1 - x_A)y - x_A\mu}{3} \tag{25}$$

It is apparent that pricing in the market for product *B* is determined by firms' market share in the market for product *A* as well as the difference in bonus utility.

The equilibrium output levels in the market for product *B* can be recovered by plugging (25) into Eqs. (2) and (3):

$$x_{B\mu}(x_A) = \frac{3t_B + \mu + 2(1 - x_A)(\mu + y)}{6t_B}$$
$$x_{By}(x_A) = \frac{3t_B - y - 2x_A(\mu + y)}{6t_B} \tag{26}$$

where recall that $x_{B\mu}(x_A)$ describes the proportion of x_A consumers—those who purchase product *A* from firm 1—who also purchase product *B* from firm 1, while $x_{By}(x_A)$ represents the proportion of $1 - x_A$ consumers—those who purchase product *A* from firm 2—who end up purchasing product *B* from firm 1. Subtracting equations in (26) from 1 yields analogous equilibrium output levels for firm 2.

3.2 The First Period: Product *A* Market

In the first period, we compute the indifferent consumer and the firms' optimal prices for product *A*. The indifferent consumer and the firms' outputs (as functions of prices) in the market for product *B* are determined based on Eq. (1). Similar to Klemperer (1987a), each firm selects its price while considering not only the effect on its first-period profits, but also the effect on its first-period market share and thus its second-period profits. In particular, firms maximize their total discounted profits given by:

$$\begin{aligned}
\pi_1 &= \pi_{1A} + \delta\pi_{1B}(x_A)\\
&= x_A(p_{1A} - c_A) + \delta\{[p_{1B}(x_A) - c_B][x_A x_{B\mu}(x_A) + (1 - x_A)x_{By}(x_A)]\}\\
\pi_2 &= \pi_{2A} + \delta\pi_{2B}(x_A)\\
&= (1 - x_A)(p_{2A} - c_A) + \delta\{[p_{2B}(x_A) - c_B][x_A(1 - x_{B\mu}(x_A)) + (1 - x_A)(1 - x_{By}(x_A))]\}
\end{aligned} \tag{27}$$

where π_{iA} and π_{iB} are firm *i*'s profits, for $i = \{1, 2\}$, in product *A* and product *B* markets, respectively; and $\delta \in [0, 1]$ is the common discount factor. Note that π_{iB} is the equilibrium profit (as functions of x_A) obtained for the second period. Substituting into $p_{1B}(x_A)$, $p_{2B}(x_A)$, $x_{B\mu}(x_A)$, $x_{By}(x_A)$, and x_A in (27) with (25), (26), and (1); taking first order conditions with respect to first-period prices; and then simultaneously solving the resulting system of two equations yield first-period subgame perfect Nash equilibrium levels:

$$p_1^* = c_A + t_A - \frac{8(\mu + y)}{3} + \frac{8t_A(\mu^2 - y^2)}{2[8(\mu + y)^2 - 27t_At_B]}$$
$$p_2^* = c_A + t_A - \frac{8(\mu + y)}{3} + \frac{8t_A(y^2 - \mu^2)}{2[8(\mu + y)^2 - 27t_At_B]} \quad (28)$$

Assuming identical bonus utility $(y = \mu)$, we can further simplify the prices to:

$$p_{1A}^* = p_{2A}^* = c_A + t_A - \frac{28\mu}{3} \quad (29)$$

Similar to our equilibrium findings in Sect. 2.1, firms charge a lower price for product A in order to entice consumers so they are more likely to purchase product B from the same firm. Unlike in Sect. 2.1, however, with sequential decisions about product A and product B the extent of the price reduction is weighted by the discount factor 1.

4 Sophisticated Consumer

Our analysis heretofore has focused on myopic consumers, whereby consumers were not aware, ex ante, of the existence of the market for product B in the future. We next consider equilibrium behavior when consumers are forward-looking/sophisticated, whereby they adapt their first period decision to their second period output choice. Towards this end, we extend our two-period model from the previous section by allowing consumers to maximize the discounted sum of their utilities over two periods in the first period decision-making process. Following the literature (see, for instance, Fudenberg & Tirole, 2000), we assume a common discount factor (1) for the consumers and the firms. Using backward induction, we begin with the second- period decisions.

4.1 The Second Period: Product *B* Market

The second-period consumer and firm decisions, and the resulting subgame perfect Nash equilibrium price and output levels, remain the same as in Sect. 3.1. Using the second-period equilibrium prices and output levels in (25) and (26), respectively, we can recover the second-period utility levels:

$$U_{1B\mu} = U_{2B} = s_B - c_B - \frac{3t_B}{2} + \frac{\mu}{2}$$
$$U_{2By} = U_{1B} = s_B - c_B - \frac{3t_B}{2} + \frac{y}{2} \quad (30)$$

The first line in (30) represents the utility of consumers who purchased product A from firm 1, while the second line corresponds to those who bought product A from firm 2. It is clear that the ranking of second-period consumer utilities depends on the difference in bonus utilities: $U_{1B\mu} - U_{2By} = \mu - y$.

4.2 The First Period: Product *A* Market

In the first period, the indifferent consumer takes into account how its first-period decision affects his total discounted utilities for two periods. Hence, we solve for the indifferent consumer as:

$$U_{1A} + 8U_{1B\mu} = U_{2A} + 8U_{2By}$$

$$s_A - p_{1A} - t_A x + 8\left(s_B - c_B - \frac{3t_B}{2} + \frac{\mu}{2}\right) = s_A - p_{2A} - t_A(1 - x_A) + 8\left(s_B - c_B - \frac{3t_B}{2} + \frac{y}{2}\right)$$

$$x_A = \frac{-p_{1A} + p_{2A} + t_A}{2t_A} + \frac{8(\mu - y)}{4t_A} \tag{31}$$

In other words, the position of the indifferent consumer in the market for product A now also depends on the difference in bonus utilities and the discount factor, with $1 = 0$ producing the myopic consumer's position from Eq. (1).

Facing the first-period product A market shares (x_A and $1 - x_A$), each firm maximizes its total discounted profits as in (27). Specifically, substituting into $p_{1B}(x_A)$, $p_{2B}(x_A)$, $x_{B\mu}(x_A)$, $x_{By}(x_A)$, and x_A in (27) with (25), (26), and (31); taking first-order conditions with respect to first-period prices; and then simultaneously solving the resulting system of two equations produces first-period subgame perfect Nash equilibrium levels:

$$\begin{aligned} p_1^* &= c_A + t_A - \frac{8(\mu + 3y)}{6} + \frac{8(\mu^2 - y^2)\left(6t_A + 8(\mu + y)\right)}{12\left(8(\mu + y)^2 - 27t_At_B\right)} \\ p_{2A}^* &= c_A + t_A - \frac{8(3\mu + y)}{6} + \frac{8(y^2 - \mu^2)\left(6t_A + 8(\mu + y)\right)}{12\left(8(\mu + y)^2 - 27t_At_B\right)} \end{aligned} \tag{32}$$

Setting bonus utilities to be identical $(y = \mu)$, we can simplify the above expression to a familiar form:

$$p_{1A}^* = {}_{2A}^* = c_A + t_A - \frac{28\mu}{3} \tag{33}$$

5 Applications

In this section, we briefly discuss several real-life examples where the model that is analyzed in this study, along with its findings, can be particularly applicable.

5.1 Technology

When the iPhone was introduced in early 2007, there was no indication that the product would become a complementary good to tablets, television products, smart devices, and wearables. Similarly, Google Android's 2008 release suggested nothing more than a phone operating system and an initial reference device. Today, both firms are major participants in all of the aforementioned product areas.

Yet, while we may not always know the future path, we know there is a future path of product categories. Samsung, an existing phone manufacturer, has preferentially created "hooks" to their own new home automation tools.[12] Amazon's phone and tablets preferentially create bonus utility for their online services and new products, such as the Amazon Echo. As long as new product categories exist, firms might prefer (through complementary features) their own product within that new category. Even if they did not, consumers would benefit from the firms' common user interface across products.

5.2 Design

Dish-ware designers such as Lenox and Noritake commonly release new styles of place settings in phases. Place settings are enhanced by new dishes about once a year. While additions are initially predictable, the additions become increasingly exotic as the years proceed. This means that the firms are offering divergent features where a consumer may prefer a competitor's product; but consumers obtain additional utility because the new product matches the style of their existing place-settings.

Path dependent book releases are another example that is captured by our model. Other factors—e.g., critical acclaim—equal, when selecting between two authors' first books, a reader will likely purchase the book with the most interesting topic. When the authors release second books, the reader might not switch to the other author even if that book's topic is of great interest, due to the bonus utility of being familiar with the first author's vocabulary, writing style or plot of the series.[13]

5.3 Recreational Education and Professional Development

Recreational education courses such as amateur painting, sculpting, and woodworking classes feature instructors with a particular personality and teaching style. While the painting (or other creative work) is known to the customer when they sign up for the class, future classes are only known a few months in advance. A customer might

[12] For more information, see https://www.cnet.com/news/smartthings-to-control-samsungs-smart-appliances/.

[13] Sampling plays a role in this behavior as well. The purchaser gains more information as a result of the first purchase (e.g. book) and will be more informed when purchasing the second good. Bonus utility increases the likelihood the purchaser will buy the second good from the same producer. However, if the purchaser did not like the first good, this may encourage them to purchase the second good from a different producer.

select a pottery class based on the design of the work. However, once the student wishes to sign-up for another class, they are already accustomed to the teaching style of their former pottery instructor. This creates bonus utility if they take a class from the same instructor. The additional utility will result in some path dependent customers along with some who will switch based on the designs offered by other studios.

Similarly, corporate customers experience bonus utility as they repeatedly purchase services from the same professional education firm. Firms often invest in education services for their labor force (either for individual employees or groups). Once the corporate learners are accustomed to the instructional style of the education company, the firm has a higher propensity to purchase future services. However, it is not known what classes the professional development company might offer in the future.

6 Conclusion

In this article, we discuss the implications of a special case of within-firm complementarity behavior. Whether by technological interoperability, a common user interface or a matching design pattern, some goods produce additional utility when purchased from the same firm due to tangible features when the products are used together. While the firms know that these products will be released in the future, the customers might not be aware of this. This results in a price reduction in the first market.

Within-firm complementarity behavior exists for many reasons. Often times, this is a function of brand loyalty driven by non-tangible factors. What we show in this paper is that similar purchasing behavior can be driven by tangible features. This suggests that some repeated purchases are not driven by brand loyalty (i.e., psychological factors) but by tangible features that the consumer gains by purchasing multiple products from the same firm. This special case of within-firm complementarity behavior (which we refer to as 'bonus utility') is particularly prevalent in the technology industry where user interfaces and design languages are common to a firm's products. This common design produces a bonus utility that the customers enjoy since they are already used to the interface.

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Declarations

Conflict of interest The authors have no conflicts of interest to disclose.

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