Sensitivity of the Wolf’s and Rosenstein’s Algorithms to Evaluate Local Dynamic Stability from Small Gait Data Sets: Response to Commentaries by Bruijn et al.

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Sensitivity of the Wolf’s and Rosenstein’s Algorithms to Evaluate Local Dynamic Stability from Small Gait Data Sets: response to commentaries by Bruijn et al.

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Rebuttal letter:

Assessing gait stability using the Largest Lyapunov Exponent ($\lambda_1$) has become popular, especially because it may be a key measure in evaluating gait abnormalities in patient populations. However, clinical settings usually involve having small gait data sets and accurate determination of $\lambda_1$ estimates from such sets is difficult. In an effort to address this issue, Cignetti et al. recently identified that $\lambda_1$ estimates using the algorithm of Wolf et al. (W-algorithm) were more sensitive than those using the algorithm of Rosenstein et al. (R-algorithm) in order to capture age-related decline in gait stability from small data sets. Thus, they advocated the use of the former algorithm. Some concerns about the study were expressed afterwards by Bruijn et al. and we welcome the opportunity to discuss them in the present letter.

Bruijn et al. expressed four concerns about the validity of the methods used by Cignetti et al. that could have biased the results. First, they indicate that although speed difference between young adults (YA) and older adults (OA) was not significant, it does not exclude speed as a confounder of the aging effect on gait stability. Although we agree that a perfect matching of YA and OA with respect to speed would definitely avoid confounding, such matching is highly unlikely as YA walk usually faster than OA. Accordingly, matching statistically the two groups in terms of average speed appears to be the best compromise between ecological validity and methodological validity. However, a mean to further avoid the confounding of speed on $\lambda_1$ is to evaluate group difference by using analyses of covariance (ANCOVAs) instead of using analyses of variance (ANOVAs), thus controlling for speed effect. As reported in Table 1, results from ANCOVAs run on the data sets of Cignetti et al. confirmed previous results obtained using ANOVAs. In particular, with respect to the main effect of age, $\lambda_1$ remained significantly larger in OA as compared to YA regardless the size of the data set (i.e., 3600, 7200, and 10800 data points) when using the W-algorithm. Such result
was obtained only for the largest data set (10800 data points) when using the R-algorithm. Therefore, the difference in \( \lambda_1 \) between YA and OA reported by Cignetti et al.\(^2\) is not biased by the inter-group difference in walking speed.

Second, Bruijn et al.\(^1\) argued that the time series of YA might have counted more strides than those of OA due to shorter stride time, increasing artificially \( \lambda_1 \). However, the stride time was the same in YA and OA with mean \( \pm \) standard error values of 1.27 \( \pm \) 0.03 s and 1.26 \( \pm \) 0.05 s, respectively (Table 2). These data are in agreement with previous studies that reported similar values of stride time in both YA and OA populations\(^3,5,6\). Accordingly, \( \lambda_1 \) exponents were estimated in the study of Cignetti et al.\(^2\) from a similar number of strides for both groups. Specifically, the time series with the 3600 data points had 47 strides in both YA and OA, the time series with 7200 data points had 94 strides in YA and 95 strides in OA, and the time series with 10800 data points had 141 strides in YA and 143 strides in OA (Table 2). Hence, Bruijn et al.\(^1\) were mistaken in assuming that an inter-group difference in stride time could have biased the difference in \( \lambda_1 \) between YA and OA in Cignetti et al.\(^2\)'s study.

A third concern expressed by Bruijn et al.\(^1\), closely related to the previous one, relates with the fact that Cignetti et al.\(^2\) did not normalize time using average stride time when estimating \( \lambda_1 \) with the W-algorithm, expressing the exponential rate of divergence per second. In the case where stride time would have been different between YA and OA, such a procedure could have influenced the difference in \( \lambda_1 \) between YA and OA and could have biased comparisons with the R-algorithm, which normalized exponential rate of divergence to average stride time. However, as previously indicated, stride time was similar between the two groups so that the absence of time normalization in the W-algorithm procedure could not influence the difference in \( \lambda_1 \) between the two groups. An intuitive way to clarify this point is
to rearrange Eq. (3), as adjusted by Bruijn et al.\textsuperscript{1} to normalize time using average stride time ($t_{\text{stride}}$), as follows:

$$\lambda_1 = t_{\text{stride}} \times \left( \frac{1}{t_M - t_0} \sum_{k=1}^{M} \frac{\ln L'(t_k)}{L(t_{k-1})} \right)$$

It becomes evident that time normalization would have only consisted in multiplying $\lambda_1$ by average stride time for each subject, and thus would have not changed the group difference since YA and OA had similar stride time values. However, the adjustment proposed by Bruijn et al.\textsuperscript{1} to Eq. (3) is important to consider when stride time is different between groups.

Finally, Bruijn et al.\textsuperscript{1} questioned the use of the W-algorithm given the fact that could be more affected by changes in the embedding dimension and the reconstruction delay than the R-algorithm\textsuperscript{2}. Although there is evidence that these two reconstruction parameters can only be estimated with limited precision from short time series\textsuperscript{4,8}, which would make estimates from the W-algorithm less reliable than those from the R-algorithm, Cignetti et al.\textsuperscript{2} also demonstrated using Lorenz data that the R-algorithm underestimates $\lambda_1$, overlooking the expansion of the attractor trajectories. When considering attractors with convergent regions as the gait attractors, $\lambda_1$ estimates thus only reflect poorly the exponential rate of divergence of neighboring trajectories, especially when these attractors are reconstructed from small data sets that make the probability of finding close nearest neighbors that may diverge far apart low. Accordingly, there are advantages and disadvantages for using either the W-algorithm or the R-algorithm, and selecting one over the other must be data-driven.

In sum, contrary to what has been stated by Bruijn et al.\textsuperscript{1}, findings of Cignetti et al.\textsuperscript{2}'s study did not suffer from methodological bias. Therefore, Cignetti et al conclusions that the W-algorithm is more sensitive than the R-algorithm, to capture age-related decline regarding dynamic stability from small gait data sets, is strongly supported by our experimental results.
References:


