

3-2002

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### Recommended Citation

Kurza, Max J. and Stergiou, Nikolaos, "Effect of normalization and phase angle calculations on continuous relative phase" (2002). *Journal Articles*. 105.

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# Effect of normalization and phase angle calculations on continuous relative phase

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## Abstract

The purpose of this investigation was to determine if phase plot normalization and phase angle definitions would have an affect on continuous relative phase calculations. A subject ran on a treadmill while sagittal plane kinematic data were collected with a high-speed (180Hz) camera. Segmental angular displacements and velocities were used to create phase plots, and examine the coordination between the leg and thigh. Continuous relative phase was calculated with a combination of two different amplitude normalization techniques, and two different phase angle definitions. Differences between the techniques were noted with a root mean square (RMS) calculation. RMS values indicated that there were differences in the configuration of the non-normalized and normalized continuous relative phase curves. Graphically and numerically, it was noted that normalization tended to modify the continuous relative phase curve configuration. Differences in continuous relative phase curves were due to a loss in the aspect ratio of the phase plot during normalization. Normalization tended to neglect the nonlinear forces acting on the system since it did not maintain the aspect ratio of the phase plot. Normalization is not necessary because the arc tangent function accounts for differences in amplitudes between the segments. RMS values indicated that there were profound differences in the continuous relative phase curve when the phase angle was normalized and a phase angle was calculated relative to the right horizontal axis.

*Keywords:* Continuous relative phase; Phase angle; Normalization; Coordination

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## 1. Introduction

Mean continuous relative phase provides quantitative information about the spatial organization of segments during a given task (Scholz, 1990; Schaner et al., 1990). Continuous relative phase is derived from the phase plots of two segments during a movement pattern. Since various segments may have different amplitudes and velocities, it has been suggested that the components of the phase plot should be normalized to avoid one segment dominating the continuous relative phase pattern (Burgess-Limerick et al., 1993; Hamill et al., 1999; Li et al., 1999; van Emmerik and Wagenaar, 1996). However, other researchers have presented continuous relative phase measures with no normal-

ization applied to the phase plots (Clark and Phillips, 1993). Furthermore, various literature sources have presented different techniques for calculating phase angles that are used for these measures (Clark and Phillips, 1993; Hamill et al., 1999; Scholz, 1990). Based on the literature, it is unclear the effects of normalization and phase angle definitions on continuous relative phase measures. Thus, the purpose of this investigation was to determine if the various types of phase plot normalization and phase angle definitions presented in the literature would have an affect on continuous relative phase calculations for segments that share a common joint.

## 2. Methods

A healthy male subject (mass=72.5 kg, height=1.78 m) ran on a treadmill (Performance USA, Hauppauge, New York) at a self-selected pace ( $2.23\text{ms}^{-1}$ )

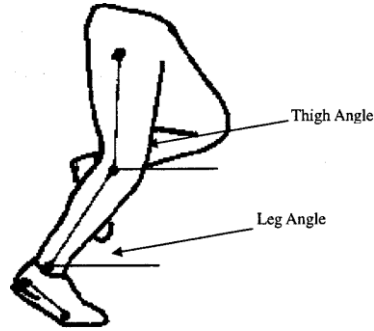


Fig. 1. Marker locations and angle definitions.

while kinematic data of the right sagittal lower extremity were collected using a high-speed (180Hz) camera (JC Labs, Mountain View, California). The subject read and signed an informed consent that was approved by the University Institutional Review Board. Prior to videotaping, reflective markers were positioned on the subject's right lower extremity. Marker placements were as follows: (a) greater trochanter, (b) axis of the knee joint as defined by the alignment of the lateral condyles of the femur and (c) lateral malleolus (Fig. 1). Joint markers were digitized using the Peak Motus system (Peak Performance Technologies, Inc., Englewood, Colorado) for 10 consecutive footfalls. The obtained positional coordinates of the markers were scaled and smoothed using a Butterworth low-pass filter with a selective cut-off algorithm based on Jackson (1979). The cut-off frequency values used were 13-16Hz.

This investigation evaluated the coupling of the leg and thigh segments. The leg and thigh were modeled as pendula joined at the knee joint, and it was assumed that their individual angular displacements would represent a quasi-periodic motion. Angular displacements of the two segments were calculated relative to right horizontal (Fig. 1). Angular displacements and velocities were time normalized to 100 points for the stance period using a cubic spline routine to enable mean ensemble curves to be derived from the representative footfalls.

Subsequently, the phase portraits for the leg and thigh segments were generated, which is a plot of each segment's angular position versus its first derivative (Scholz, 1990). The amplitude of the respective components of the phase plots was normalized with two techniques (Table I). In addition, the components of the phase plots were not normalized at all. Both amplitude normalization techniques scale the angular displacement and velocity values to a range of  $\pm 1$ . Method A normalizes the angular displacement and velocity values based on the maximum absolute amplitudes of the

trajectory, such that zero angular displacement and zero angular velocity are maintained at the origin. Method B differs from Method A in that it normalizes the minimum angular displacement to -1 and maximum angular displacement to +1. With Method B, the zero point of the normalized angular displacement represents the midpoint of the given range of motion of the segment. As in Method A, Method B's normalized angular velocity maintains a zero velocity at the origin.

The phase plot trajectories were then transformed from Cartesian ( $z, p$ ) to polar coordinates, with a radius and phase angle  $c) = \tan^{-1}[p/z]$ . Phase angles were calculated with two different techniques: Reference Phase Angle, Standard Phase Angle (Fig. 2). The reference phase angle (Fig. 2a) was the acute angle formed by the terminal side of the radius and the horizontal axis. The reference phase angle had a range of 0-90°. The standard phase angle (Fig. 2b) was the angle formed by the terminal side of the radius and the right horizontal axis. The standard phase angle had a range of 0-180°. In both techniques, when the velocity of the trajectory was negative, the phase angle was also negative. Therefore, positive phase angles were calculated if the trajectory was within quadrants 1 and 2,

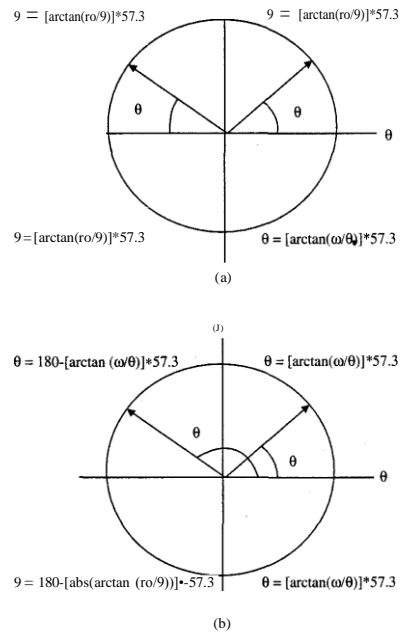


Fig. 2. Respective phase angle calculation methods evaluated are presented in two panels as follows: (a) reference phase angle, and (b) standard phase angle.

Table 1

Phase plot normalization methods, where  $\theta$  represents angular displacement,  $w$  angular velocity, and  $i$  represents each data point from heel contact to toe-off

Normalization	Angular displacement	Angular velocity
Method A	$\theta_i = \left( \frac{\theta_i}{\max\{ \theta_i \}} \right)$	$w_i = \frac{w_i}{\max\{ w_i \}}$
Method B	$O_i = \frac{2\pi / 8 - \min(O_i)}{\max(O_i) - \min(O_i)} - 1$	$w_i = \frac{w_i}{\max\{ w_i \}}$

and negative phase angles were calculated when the trajectory was within quadrants 3 and 4. To determine the effects of normalization and phase angle calculations on continuous relative phase, the following calculations were performed: non-normalized data with a standard phase angle, non-normalized data with a reference phase angle, normalization Method A with a standard phase angle, normalization Method B with a standard phase angle, normalization Method A with a reference phase angle, and normalization Method B with a reference phase angle.

Continuous relative phase was calculated for each of the respective calculations by subtracting the phase angles of the leg and thigh segments throughout the stance period for each data point ( $\text{REL.PHASE} = \text{8LEG} - \text{8THIGH}$ ). The continuous relative phase curves for each segmental relationship were averaged across footfalls ( $n=10$ ), and mean ensemble curves were generated for each respective continuous relative phase calculation. Mean ensemble values were constructed by determining the mean value at each  $i$ th point of the continuous relative phase curve. Curve differences were noted by calculating the root mean square (RMS). A lower RMS value indicated greater similarity between the curves.

### 3. Results

Graphically, Method A (Figs. 3b and 4b) was better at maintaining the configuration of the phase plot trajectory. Normalization Method B (Figs. 3c and 4c) spanned the angular displacement across the two quadrants to fit the  $\pm 1$  range. Evaluation of the effect of the normalization on continuous relative phase measures suggested that both normalization techniques resulted in continuous relative phase scalar multiples that represented similar coupling patterns (Fig. 3a). However, the continuous relative phase curves had slightly different configurations that resulted in different values for the local minimum and maximum (critical) points of these curves.

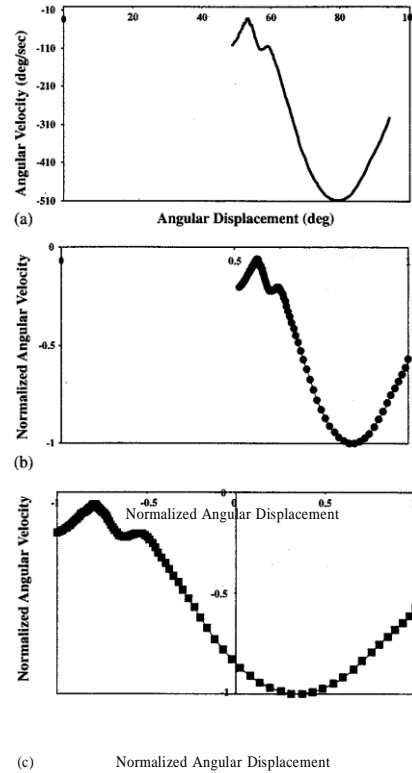


Fig. 3. Non-normalized and normalized leg phase plots during the stance period for one typical footfall. The phase plots are presented in three panels as follows: (a) Non-normalized, (b) Method A Normalized Phase Plot, and (c) Method B Normalized Phase Plot. It should be noted that Method B tended to stretch the trajectory to fit the  $\pm 1$  range.

Compared to the non-normalized data, smaller differences ( $22.7^\circ$ ) in the RMS values were noted when the phase angle was calculated as a reference phase angle and normalized with Method A (Table 2). The same observations (Table 2) were noted in the RMS when the phase angle was calculated as a standard phase angle and normalized with Method A ( $22.7^\circ$ ).

RMS values (Table 2) indicated that when the phase angle was calculated as a reference phase angle, normalization Method B's continuous relative phase curve had smaller differences compared to the other continuous relative phase curves ( $27.6^\circ$  and  $16.8^\circ$ ). However, RMS values indicated that the standard phase angle had profound effects on the calculated continuous relative phase for normalization Method B ( $70.6^\circ$ ). These results are also supported by the graphical observations of the curves for Method B (Figs. 3a and b). Normalization with Method B and using a reference phase angle (Fig. 3a; triangles) produced

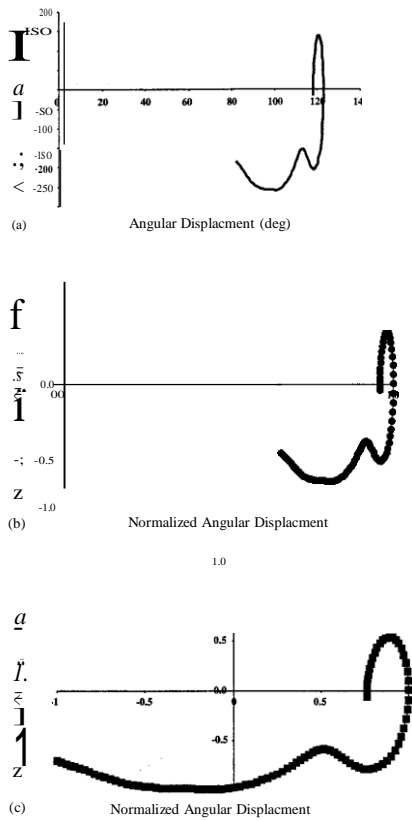


Fig. 4. Non-normalized and normalized thigh phase plots during the stance period for one typical footfall. The phase plots are presented in three panels as follows: (a) Non-normalized, (b) Method A Normalized Phase Plot, and (c) Method B Normalized Phase Plot. It should be noted that Method B tended to stretch the trajectory to fit the  $\pm 1$  range.

a continuous relative phase curve that had a similar configuration with the other curves. However, when the continuous relative phase curve was developed with normalization Method B and using a standard phase angle (Fig. 5b), large differences were noted between the

curves.

#### 4. Discussion

The goal of this investigation was to determine if the various types of normalization and phase angle definitions presented in the literature would have an effect

on continuous relative phase calculations for segments that share a common joint. Our results indicated that there were differences in configuration between the non-normalized and normalized continuous relative phase curves. Especially, using the standard phase

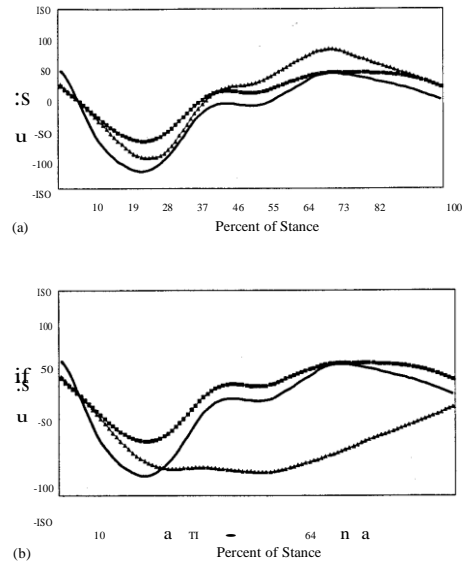


Fig. 5. Continuous relative phase graphs for all normalization-phase angle calculations evaluated. Each curve is an ensemble average over all trials ( $n=10$ ). In both panels, the bold line represents the non-normalized data, squares represents normalization Method A, and triangles represents normalization Method B. Continuous relative phase calculations are presented in the following panels as follows: (a) reference phase angles used to calculate continuous relative phase and (b) standard phase angles used to calculate continuous relative phase. It should be noted that normalization of the phase plot resulted in modifications of the critical points of the continuous relative phase curve. Furthermore, it should be noted that the phase plot normalized

with Method B and calculated with a standard phase angle resulted in large differences in the continuous relative phase configuration.

Table 2

Root Mean Square values (deg) for the respective continuous relative phase curves. It should be noted that the phase plot normalized with Method B and calculated with a standard phase angle resulted in large differences in the continuous relative phase curve configuration

Method	Root Mean Square value (degrees)
Reference phase angle	
Non-normalized vs. Method A	22.7
Non-normalized vs. Method B	27.6
Method A vs. Method B	16.8
Standard phase angle	
Non-normalized vs. Method A	22.7
Non-normalized vs. Method B	70.6
Method A vs. Method B	81.5

angle, calculated from a phase plot normalized with Method B.

Normalization of the phase plot should produce a scalar multiple of the original phase plot trajectory, and maintain the dynamic qualities of the segment

(Burgess-Limerick et al., 1993; Hamill et al., 1999; Li et al., 1999; van Emmerik and Wagenaar, 1996). However, our results suggested that the normalization techniques presented in this investigation modify the dynamic qualities of the oscillating segment (Figs. 3 and 4). This is due to the fact that these techniques normalize the phase plot coordinates (velocity and displacement) with different scale factors. By normalizing the data with different scale factors, the aspect ratio of the dynamics of the segment can be lost. Aspect ratio is the ratio of the velocity and displacement coordinates that define the trajectory configuration. A loss of the aspect ratio of the phase plot changes the non-linear behavior of the segment. These modifications were apparent in the normalized phase plot configurations (Figs. 3 and 4), where the trajectories are not scalar multiples. The aspect ratio of the phase plot defines the dynamic qualities of the segment. The loss of the aspect ratio resulted in different continuous relative phase curve values at the critical points (Fig. 5a). Changing the dynamic qualities of the oscillating segment is not the purpose of normalizing the phase plot. Rather, as suggested by the literature (Burgess-Limerick et al., 1993; Hamill et al., 1999; Li et al., 1999; van Emmerik and Wagenaar, 1996), the purpose of phase plot normalization is to produce a scalar multiple of the original trajectory such that amplitude differences between the oscillating segments do not affect coupling measures. Based on our data, it can be stated that current normalization techniques do not produce a scalar multiple of the original dynamics of the segment.

We suggest that amplitude differences between oscillators may not actually be a problem when calculating continuous relative phase as previously suspected. Since the arc tangent function is based on a ratio (velocity/displacement), differences in amplitude are removed with the phase angle calculation. The arc tangent function "normalizes" differences in amplitude between the segments based on this ratio. Therefore, it can be argued that amplitude normalization of the phase plots is not necessary due to the properties of the arc tangent function. Based on this fact, the notion that normalization is necessary to prevent a segment with a larger amplitude from dominating the continuous relative phase pattern would be incorrect. Continuous relative phase is not affected by differences in amplitude between segments due to the properties of the arc tangent function.

Our results suggest that the coupling of the two segments was inappropriately represented when the continuous relative phase was calculated with normalization Method B and a standard phase angle. This statement is based on reports from the literature (Li et al., 1999; Bates et al., 1978) and our data, where an examination of the thigh and the leg angular displace-

ments in the time domain revealed that the two segments should have an out-of-phase relationship in early stance because they move opposite to each other. During mid-stance the two segments move in a similar fashion or an in-phase relationship ( $0^\circ$ ). Later in stance, their relationship is more out-of-phase but not as much as in early stance. Continuous relative phase calculated via a standard angle using normalization Method B did not indicate such relationships between the two interacting segments. The noted differences in continuous relative phase appear to be due to the fact that normalization Method B modifies the dynamics of the oscillating segment. As stated previously, the dynamic qualities of the oscillator are contained in the aspect ratio of the phase plot trajectory. Normalization Method B tended to distort the dynamics of the oscillator by forcing the displacement coordinate of the trajectory to fit the  $\pm 1$  range, and scaling the velocity coordinate by its absolute maximum. This normalization routine uses two widely different scale factors, which changes the aspect ratio of the phase plot (Figs. 3c and 4c). Calculating the phase angle with the standard phase angle tends to exacerbate these modifications of the phase plot aspect ratio. Modifications in the aspect ratio are not as evident when the phase angle was calculated with a reference phase angle because it has an angle range from 0 to 90 which tends to minimize the effect of altering the aspect ratio of the dynamics of the oscillating segment.

In conclusion, this investigation detailed the effect of various normalization techniques and phase angle definitions on continuous relative phase measures for segments that share a common joint. Based on our results, the following criteria are proposed. Current normalization techniques may not be necessary. Amplitude differences between oscillators do not affect continuous relative phase measures. This is due to the fact that the arc tangent function is based on a ratio (velocity/displacement) that accounts for differences in segmental amplitudes. Either a standard or relative phase angle can be used in the calculation of continuous relative phase. Both phase angle measures provide the same detailed information about the continuous relative phase of the coupled segments. However, a standard angle should not be calculated if the phase plot has been normalized with Method B.

#### Acknowledgements

Gratitude is expressed to the University Committee on Research at the University of Nebraska at Omaha for funding this research project. We would like to thank Dr. Jody L. Jensen, Dr. Alan Hreljac, and Dr. Randy Tagg for their input in this study.

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