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Joint spacing in the Caples Lake granodiorite of the Sierra Nevada Batholith in Eldorado National Forest, California: A comparative analysis of joint sets and data resolution

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UNIVERSITY OF NEBRASKA - OMAHA COLLEGE OF ARTS AND SCIENCES

DEPARTMENT OF GEOGRAPHY/GEOLOGY

Joint spacing in the Caples Lake granodiorite of the Sierra Nevada Batholith in Eldorado National Forest, California: A comparative analysis of joint sets and data resolution

A Senior Thesis

by

Jimmy Wood

Submitted in partial fulfillment

of the requirements

for the degree of

Bachelor of Science

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ABSTRACT

Joints are the most common deformation structure in the Earth's upper crust and exert a significant influence on structural stability, landscape morphology, and fluid flow . Therefore, a greater understanding of fracture parameters (e.g., length, aperture, etc.) allows us to more accurately predict their presence, persistence, and prevalence, in the subsurface . We study the fracture spacing of two sub-orthogonal joint sets—66 NE-246 SW and 330 NW-150 SE—in the Caples Lake granodiorite of the Sierra Nevada Batholith, California. Specifically, we investigate 1) their spacing distributions with a keen interest in power-law (fractal) spacing, 2) distribution comparisons between master and cross joints, and 3) the usability of Google Earth datasets in joint spacing analyses. Spacing was calculated from position data obtained in the field and on Google Earth along one-dimensional traverses orthogonal to the mean joint strike of a set , with a target sample size of 100 for a stable fractal dimension. We tested fractal behavior through loglog cumulative frequency vs. spacing plots , determined the spacing distribution with the Chisquared (χ^2) goodness-of-fit test, and compared distributions with the Kolmogorov-Smirnov statistic and the Coefficient of Variation . All four datasets exhibit non-fractal behavior and can instead be better described by lognormal or gamma distributions. This may be the result of sampling biases such as truncation or censoring , which can be possibly overcome with greater sample sizes and extending our lower limit of measurement an order of magnitude into the millimeters. Master and cross joints have slightly different distributions as expected from joints of different age; however, this relationship is still unclear and should be further explored with a greater sample size and less opportunistic sampling scheme that encourages shorter traverses further upslope on an outcrop. Google Earth datasets were significantly inadequate for joint spacing analyses. As expected, they routinely underestimate smaller spacings and as a result generate larger, artificial spacings, and distributions of shifted form and position. Within fracture analysis, Google Earth should remain a tool for field site reconnaissance and mapping only.

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TABLE OF CONTENTS

INTRODUCTION

In virtually all rock types and all tectonic environments we find steeply dipping fractures (Pollard & Aydin, 1988). In general, rock fractures comprise one of the five primary categories of deformation structures that permeate the Earth's lithosphere (Pollard & Martel, 2020). Their often (and rightly) proclaimed ubiquity and variability present a near inexhaustible research focus for the structural geologist, or student, with even a slight interest in fracture analysis (Segall & Pollard, 1983). These features can be simply defined as any discontinuity within a geologic material that has developed in response to stress (Bonnet et al., 2001). Generally, all fractures share these three fundamental characteristics (Pollard & Aydin, 1988).

- 1. They are largely distinguished by two parallel surfaces called the fracture faces or walls.
- 2. These faces are approximately planar or sub-planar to each other.
- 3. The faces' displacement—separation perpendicular to the fracture face—is very narrow compared to the overall length of the fracture (Fossen, 2016; Pollard & Aydin, 1988) .

While fractures observed in nature largely adhere to these characteristics, their exact expression can be highly variable, both between adjacent fractures and along the length of an individual fracture. For example, one fracture face may only be locally approximated by a planar surface, while at a larger scale is better characterized as curvi-planar.

Fractures are the result of brittle deformation—a trademark of the upper crust environment—and form where stresses exceed the local rupture strength of a rock (Fossen, 2016). These stresses may be of local to regional extent and are characterized as either external or internal. Examples of external stresses include tectonism, overburden accumulation or removal, those related to topography, and changes in fluid pressures, while the latter may include thermal and residual stresses, as well as those related to diagenetic processes. The threshold to cause rupture depends on the rock's mineralogy and composition, its confining pressure—related to the depth of burial—and, perhaps most importantly, randomly distributed and oriented microdefects in the rock itself (Fossen, 2016). This last point may explain why fractures form where they do. Griffith (1924) suggested that natural rocks and the crystals that form them are naturally imperfect (Fossen, 2016; Pollard & Aydin, 1988). Thus, rocks likely contain an abundance of flaws and cracks on the micro-scale. Other features such as pore space, voids, and grain boundaries, can all be considered microcracks in this context. Their presence weakens rock and if oriented near the minimum stress direction, assist in fracturing as shown in **Figure 1** (Fossen, 2016).

Figure 1. Growth of a tensile fracture through the linking of microfractures. Adapted from Fossen, (2016).

The current classification scheme separates fractures into three primary modes, distinguished by the relative displacement of paired fracture faces (**Figure 2**) (Fossen, 2016).

Figure 2. Primary fracture Modes I-III, along with Mode IV. Adapted from Fossen, (2016).

Mode I, or opening mode, involves a displacement normal to the walls of the fracture. Mode II and III both describe wall parallel displacements—modeling faults—but with different relative motion (Bonnet et al., 2001; Fossen, 2016). A fourth, closing mode, encompasses contraction features such as styolites, but is not a primary focus of this work. These are merely end members of a fracture continuum as modes can also combine to create hybrid cracks (Fossen, 2016).

The focus of this work is opening mode or extension fractures (Fossen, 2016). The most common type has small to moderate displacements and are called joints—for the purpose of this work we will use fracture and joint interchangeably (Pollard & Aydin, 1988). As previously mentioned, almost all rocky outcrops exhibit joints (Fossen, 2016). They occur as a series of subparallel fractures that define a joint set, under the assumption that they have formed under the influence of the same stress conditions during the same fracture episode (Pollard $\&$ Aydin, 1988). All joints within a given set largely share the same morphology and orientation. Natural joint patterns are commonly comprised of more than one set—usually up to three or four, with various orientations relative to each other—which together define a joint system (Fossen, 2016). These may form many varying patterns, of which some examples are shown in in **Figure 3.** The physical interaction of these sets is valuable and may reveal important age relationships and

distinguish fracture forming episodes. If members of one set consistently terminate against members of another, we may reasonably assume the former to be younger than the latter (Pollard & Martel, 2020).

Figure 3. Examples of common joint patterns. Our field sets take the form of **a**. From Fossen, 2016.

Under the previous assumption, we infer that joint sets, having different orientations, formed under contrasting stress fields which evolve over geologic time (this may not always be the case, though, and should be corroborated with field observations) (Pollard & Martel, 2020). This record of rock deformation is of inherent interest to structural geologists (Segall & Pollard, 1983) and allows them to reconstruct paleostress conditions and unravel the history of an area (Ehlen, 2000). Jointing is also of special interest to the geomorphologist as they can greatly

influence the physiography of landforms in rock cut landscapes (Ericson et al., 2004) such as coastlines, stream networks, mountain peaks, and desert mesas (Pollard & Martel, 2020).

Fracture analysis is also of practical interest to many industries central to infrastructure and natural resources. By providing secondary permeability to generally impermeable materials such as crystalline basement rocks, compacted and cemented sedimentary beds, and shales, jointing can have a significant effect on subsurface fluid flow (Fossen, 2016) (Le Garzic et al., 2011). This has implications for not only water resource management but the mining and energy industries as ore forming fluids, oil, gas, and even geothermal energy, may all be transmitted through fractures (Pollard $\&$ Aydin, 1988). As the meso- and macroscopic expression of microcracks and other defects (Fossen, 2016), joints also play a crucial role in rock deformability by physically weakening the host rock (Fossen, 2016; Pollard & Aydin, 1988); therefore, joints must be carefully investigated when planning certain engineering and development projects. If building a highway or powerplant; installing a dam, bridge, or tunnel; or attempting to ascertain the least-risk setting for a nuclear waste repository (Pollard & Aydin, 1988); knowledge of the local fracture pattern is of paramount importance. To understand the influence of fracture systems in these settings their geometrical attributes such as length, aperture, density, etc., the spatial distribution of those values, and the spatial distribution of the joints themselves must be determined (Le Garzic et al., 2011; Segall & Pollard, 1983).

Historical Perspective

The first major work on jointing (in the Geological Society of America *Bulletin*) was carried out by Becker in 1893. Observations that outcrops in the Sierra Nevada are always fractured led to the argument that orogenies could never be meaningfully discussed until the mechanical significance of jointing and faulting was understood, thus highlighting their intrinsic value; however, up until then joints were not simply ignored. Eleven years earlier, field notes left by Gilbert, conducting field work in the Great Salt Lake Desert of Utah on rectangular drainage systems in post-glacial sediments, sparked a lively debate on their origin. His claim that no "satisfactory explanation has ever been given of the origin of the jointed structure in rocks," lead to a number of contrasting suggestions from desiccation to magnetic forces. Unknown to them, this question had already been answered in Great Britain by Hopkins in 1835—almost half a century before. He interpreted joints as discontinuities caused by tensile stress (Pollard & Aydin, 1988), the general definition accepted today (Fossen, 2016). Later work by Becker (1893) and Van Hise (1896) would also yield the same conclusion (Pollard & Aydin, 1988).

Joint Spacing

The International Society of Rock Mechanics suggests eleven parameters to provide a quantitative description of fractures—spacing and orientation are the most commonly reported (Fatt, 1994). Joint spacing refers to the distance between two adjacent, sub-parallel fractures (**Figure 4**) (Ryan, 2000).

Figure 4. Joint spacing in the field (original photo, left, and interpreted, right). Black lines indicate joint traces—dotted where not certain, such as through regolith and vegetation, or for eroded faces. Red and blue lines indicate the spacing of the traced joint sets. Note how they vary, both between different fractures of the same set, and between the same two fractures along their lengths.

A large body of work has already been devoted to joint spacing in sedimentary rocks and has revealed a roughly linear relationship between spacing and layer thickness (Bai & Pollard, 1999; Ehlen, 2000), an idea which goes back to at least the late 1960s when Hobbs proposed one of the earliest theoretical models for describing joint spacing (Bai & Pollard, 1999). The linearity between bed thickness and joint spacing extends to effective layers as well—certain geometrical configurations which mechanically approximate the same response of true layers to jointing (Palmström, 1995). In this way a previously existing joint set may act as a mechanical layering who's spacing (i.e., thickness) constrains the spacing of subsequent joints (Ruf et al., 1997).

Of interest to us is the spacing distribution of an individual joint set. Various distribution patterns have been reported in the literature (National Research Council [NRC], 1996) including lognormal, gamma, and exponential , but over the past few decades it has become increasingly apparent that many fracture properties follow a power-law distribution and therefore exhibit

fractal or multi-fractal behavior (Ehlen, 2000). Fractals are generally characterized by the selfsimilarity of some characteristic at different observation scales (Velde et al., 1991). These kind of scaling laws are promising (Bonnet et al., 2001) as they offer the extrapolation from small samples to much larger ones—e.g., outcrop scale to regional scale (Le Garzic et al., 2011)—and could greatly simplify somewhat slow sampling methods and procedures (Velde et al., 1991). Added interest in fractal behavior stems from the practical uses of fracture analysis mentioned above. The underlying controls on fracture scaling and the spacing distribution are also likely related to both the nature of the host rocks and the conditions of deformation, and so may shed light on the intricacies of fracture forming history (Gillespie et al., 2000).

Though joints are found in igneous and metamorphic rocks (Fossen, 2016) they remain much less studied than the sedimentary variety (Wong et al., 2018). This may be due to the perceived convenience of working in the more well constrained geometries, age relationships, and burial histories, of soft rocks (Fossen, 2016). Spacing in intrusive igneous rocks has been reported as nonuniform (Pollard & Aydin, 1988), but a similarly consistent relationship between spacing and rock geometry—such as spacing to layer thickness—or other traits has not yet been established. Even so, intrusive igneous rocks are ideal research targets for their relatively simple mineralogy, their, to a first approximation (Pennacchioni & Zucchi, 2013), homogenous and isotropic structure, and because they can be assumed to have responded to deformation forces in the most direct manner (Velde et al., 1991).

A primary component of this work also takes place in Google Earth. Despite an extensive array of high-resolution imagery across the globe, Google Earth has largely been relegated in the research community to mainly educational and visualization purposes; Few have actually utilized that imagery to yield quantitative data from which they plan to draw conclusions. This notion is

not unprecedented. One of the first datasets believed to be produced from Google Earth imagery from Constantine and Dunne (2008) successfully predicted the size distribution of oxbow lakes along the Sacramento river (Fisher et al., 2012). Furthermore, not only does Google Earth deliver high-resolution imagery, but imagery that can be accessed on almost any computer, by almost anyone, and at the best price—free. It also opens up parts of the Earth that might preclude field work due to sheer remoteness, political conflicts, and, of course, the high cost of obtaining aerial imagery (Fisher et al., 2012) and travel. Therefore, its use as a primary dataset must be further investigated.

To further these endeavors, this work investigates joint patterns in the Sierra Nevada mountains of California, U.S.A. Here, granite plays a major role in geologic structure (Ericson et al., 2004) and we gather datasets from both the field and Google Earth. With these datasets we seek to answer three questions:

- 1. What are the spacing distributions of the two, dominant joint sets in our study area?
- 2. How do the spacing distributions of these two joint sets of apparent different age compare?
- 3. Do datasets collected at Google Earth scales yield the same distribution as datasets collected in the field at the outcrop scale.

BACKGROUND GEOLOGY

From the sweltering Mojave Desert to the rumbling Cascade Range (Bateman, 1968) the Sierra Nevada Mountains extend over 600 km (965 mi) along the eastern border of California following a NNE-SSW trend (Ericson et al., 2004). Together, these peaks comprise the tallest, longest, and youngest, mountain chain in the contiguous U.S (Busby et al., 2008). At the largest

scale the Sierras can be approximated by an asymmetric, rigid, tilted block, sloping relatively gently to the west and much more steeply to the east, effectively isolating the Central Valley of California from the Basin and Range province, respectively (Ericson et al., 2004) (**Figure 5**).

Figure 5. Large scale diagram of the Sierra Nevada Mountains highlighting fault block tilt. From "Sierra Nevada," 2012.

At the core of these mountains is the Sierra Nevada Batholith (SNB) (**Figure 6**). It is one of the largest in North America (Cecil et al., 2012) yet is more accurately described as a composite structure formed from hundreds of individual plutons (Ericson et al., 2004). These bodies—traced to the long-lived (70-120 Ma (Bateman, 1968)), island-arc (Ericson et al., 2004) subduction of the Pacific plate beneath the North American (Cecil et al., 2012)—intruded strongly deformed but weakly metamorphosed Paleozoic and Mesozoic sedimentary rocks and

volcanics (Bateman, 1968). The resulting average plutonic composition largely ranges between a quartz monzonite and granodiorite, in comparable abundance (Bateman, 1968).

Figure 6. Generalized geologic map of the SNB. Rectangle marks boundaries of **Figure 7A**. Adapted from Sendek,

(2016).

Along with rock type, other geologic and geographic factors appropriately match the SNB to our goals. During the Pleistocene Sierra Nevada peaks were extensively glaciated several times (Bateman, 1968; Ericson et al., 2004). This has left a legacy in the form of excellent outcrop exposures which not only assist with remote mapping and study site selection, but also the ease of travel, access, and the speed of data collection (Ericson et al., 2004). Excellent exposures such as these do come with drawbacks though. In sculping these outcrops glaciation and deglaciation have also led to the formation of new joints in the form of sheeting and have made all fractures more susceptible to exploitation, and subsequent alteration, via weathering and erosion. In other words, jointing in the SNB is plentiful but complicated. Steeply dipping joints (Pennacchioni & Zucchi, 2013) are also ubiquitous in the granitic part of the Sierras (Ericson et al., 2004), typically of regional extent (Segall & Pollard, 1983) with noticeably similar patterns from area to area (Ehlen, 1999). These allow for observation on a wide range of scales, from micro cracks on a thin section to lineaments extending for 10s of kilometers (Ericson et al., 2004). Thus, sample sizes can be appropriately large and an investigation of the area with aerial imagery is possible.

METHODOLOGY

Study Area & Site Description

Our field area consists of four sites in Eldorado National Forest, CA, southwest of Silver Lake, in the northern SNB. This was identified during a preliminary reconnaissance in Google Earth by Morse et al. (2020) (**Figure 7**).

Figure 7. Study area. **A.** General location relative to Lake Tahoe and Carson City, NV**. B.** Location of four field sites—1.3, 1.4, 1.6, 2.4—and camp relative to Silver Lake and Highway 88. Light blue markers indicate sites which yielded data used in analysis. From Google Earth.

Outcrops are composed of Cretaceous plutons, of which, there are at least three major separately emplaced bodies, accompanied by several smaller ones in the surrounding area (general overview shown in **Figure 8**) (McKee et al., 1982). Our sites were solely underlain by the Caples Lake Granodiorite—the second major pluton—described as "a predominantly medium-coarse grained porphyritic hornblende biotite granodiorite," by McKee et al. (1982) during a USGS mineral resource exploration of the area. K-Ar dating carried out by Evernden and Kistler (1970) on biotite yielded 91.7 Ma and 94.3 Ma, and on hornblende, 99.6 Ma (McKee et al., 1982).

Figure 8. Geologic map of the study area. Covering the majority of this snapshot is the Caples Lake Granodiorite (Kcl) and reworked volcanics (Ta). Other regional plutons crop out to the east, outside of our study area, outlined in maroon. Adapted from McKee et al., (1982).

Within a given outcrop joints were commonly associated with a consistent suite of other sub-parallel structures, also observed elsewhere in the batholith: mineralized joints—commonly filled with epidote and chlorite (Martel et al., 1988) with narrow, bleached alteration zones (Segall & Pollard, 1983); thin, ductile shear zones (Pennacchioni & Zucchi, 2013); aplite dikes and pegmatites; and a weak magmatic foliation.

Data Collection

Field

To characterize joint spacing, joint position data was collected along a one-dimensional traverse or scanline—in this case a long measuring tape. Along the traverse, each position of intersection with a fracture (p) was recorded from which spacing (s) could be calculated through the simple relation, $s = p_j - p_i$, where the indices j and i correspond to the jth and ith fractures, respectively (Fossen, 2016). The determination of joint spacing between adjacent fractures in a set will allow us to directly determine the overall spacing distribution of our two dominant sets.

As previously mentioned, most joint sets are a part of a greater joint system; therefore, any traverse along a natural outcrop will likely intersect with fractures outside of the set of interest. To ensure a spacing distribution is representative of that set alone—the general goal—a careful sampling procedure must be followed. Here, we simply set the traverse perpendicular to the mean joint strike of one set (Fossen, 2016) and ignore all joints that are not mutually subperpendicular to that line (in the field this cutoff was likely around 5° -10°, to account for individual variations in joint strike). Also, the most representative dataset of this type is captured with the longest sample line. This helps eliminate sampling bias as we are likely driven to place a traverse where we perceive there to be a greater number of datapoints (e.g., a heavily fractured cluster), and thus forgoing larger swathes of unbroken rock—carrying equally insightful information. To achieve this in the field we adopt an opportunistic sampling scheme (Palinkas et al., 2013). Still, this is often unattainable due to constraints placed upon us by our equipment and the outcrop itself. Our measuring tape allowed for a maximum traverse length of 30 m. This can be extended almost indefinitely by placing the start of one sample line at the end of the previous and along the same heading, but block geometry and the presence of regolith and vegetative

cover often precluded its full utilization (McCaffrey et al., 1993). Outcrops were instead sampled via a sequence of offset traverse segments—3.85 – 30 m long—forming a larger, discontinuous traverse. To eliminate confusion, we will refer to these individual sample lines as "segments" and their combined whole as the "traverse" throughout the rest of this paper. Datasets from each traverse yielded through this segmentation are considered accurate as the segments had little to no sampling overlap (McCaffrey et al., 1993). **Figure 9** shows the traverses from which data was analyzed.

Figure 9. All analyzed traverses. Joint sets recorded distinguished by color. Scales read, **A.** 100 m, **B.** 60 m, and **C.** 30 m. From

Google Earth.

Traverses were sampled until a target sample size of at least 100 spacings was achieved (Ehlen, 1999). This target represents the minimum number of measurements needed to obtain a stable fractal dimension (D)—one that does not change with an increased sample size. Until this threshold is met, the fractal dimension will vary markedly and our ability to accurately characterize a system significantly decreases (Ehlen, 2000). A similar sampling scheme was repeated for both dominant joint sets to capture the two patterns at all sites. At most, we were able to reach 93 measurements, which is considered close enough to the target value. For the 66°NE - 246° SW set (strike determined as the average of measurements along traverses), average joint spacing was sometimes large enough that only around half this number could be reached with even a 150 m traverse. Our sampling goal for that set—modified in the field, hence opportunistic—became at least 50 measurements. To overcome these limitations and more accurately capture spacing, datasets from each traverse were compiled according to joint strike into two composite datasets—one for each joint set (Gillespie et al., 2000). Lastly, joint data is only meaningful when considered with respect to joint orientation, especially when comparisons between sets are to be made (Ryan, 2000); therefore, fracture strikes were measured with a Brunton compass along with dip (Ehlen, 1999).

Google Earth imagery

To directly compare methods of data collection we laid out all segments of each traverse from the field in Google Earth. Generated traverses were calibrated with GPS coordinates and heading data taken from each segment to ensure accuracy. Sampling procedures from the field were repeated and adapted to the imagery. At the outcrop and sub-outcrop scales all linear features crossing the traverse at the appropriate angles were assumed to be structural discontinuities as done by Ericson et al. (2004). Still, due to resolution it is likely that some of

these assumed individual discontinuities instead represent narrow zones of dense jointing (Ericson et al., 2004). No later corrections were made to account for this discrepancy as a direct comparison of the methods is our primary concern. Resolution is further exacerbated by the above-mentioned outcrop constraints, namely erosive and vegetative cover. All relevant datasets collected in the field and Google Earth will be included in **Appendix A**. Our procedure, including data extraction from Google Earth and conversion to a workable format, will also be included in **Appendix B**.

Data Analysis

Joint spacing was first calculated via the above equation for all segments $(s = p_j - p_i)$. The following analyses—explained for field data first—were carried out for traverse data and then for composite datasets to capture any variation and discrepancies between the samples and the population. A more detailed description of the statistical analyses will follow in **Appendix C**.

Fractal analysis

Fractal behavior was investigated first as it follows a simple procedure. On a log-log plot of cumulative frequency vs. joint parameter—in this case spacing—fractal behavior will yield a straight line with a slope of −D (fractal dimension) (**Figure 10**) (Bonnet et al., 2001; Ehlen, 2000; Le Garzic et al., 2011; McCaffrey et al., 1993).

Figure 10. Power-law (fractal) distribution on a log-log cumulative frequency vs. vein/joint parameter, such as spacing. From McCaffrey et al., (1993).

Spacing distribution goodness-of-fit test

A power-law may not always be an appropriate model for fracture parameters (Bonnet et al., 2001) and so joint spacing distributions were also evaluated statistically—for composite datasets only—following a procedure adapted by Ehlen (1999). Stem-and-leaf plots were generated from the data which were then used to derive its frequency distribution. These distributions were then compared with lognormal, normal, gamma, and exponential distributions (see **Figure 11**) through the Chi-squared (χ^2) goodness-of-fit test at the 95% confidence level (Ehlen, 1999)(see **Appendix C**).

Figure 6.2: The normal curve.

The null hypothesis (H₀) was either rejected if the test value χ^2 was greater than some critical value—exact value dependent on dataset and distribution tested against, see **Appendix A** and **C**—or rejected if otherwise (Walpole et al., 2012).

Kolmogorov-Smirnov tests for statistical similarity

Observed frequencies were also compared with the Kolmogorov-Smirnov (K-S) test (Ehlen, 1999). Since we assume the distributions to be highly skewed and we cannot assume that samples have been taken from a normal population (Walpole et al., 2012), the nonparametric test was employed (Ehlen, 1999). Nonetheless, this test allows us to identify significant similarities and differences in the joint spacing between the two preferred orientations by examining their

Figure 11. General form of lognormal, gamma (and exponential, $\alpha = 1$), and normal distributions. The tests performed do not assume the parameters shown (Walpole et al., 2012).

relative cumulative frequency distributions under the null hypothesis that the datasets come from the same distribution (**Figure 12**).

Figure 12. Example cumulative frequency distributions compared through the K-S statistic. From Boadu & Long, (1994).

Coefficient of variation

A final analysis used to characterize spacing was the coefficient of variation (c_v) which expresses the degree of clustering along a sample line (Gillespie et al., 2000). It can be calculated with,

$$
c_v=\frac{\sigma}{\mu},
$$

where σ and μ are the standard deviation and mean spacing of a dataset, respectively. Certain values are characteristic of specific distributions— $c_v = 1$ points toward a Poisson distribution otherwise, $c_v > 1$ indicates clustering, and $c_v < 1$ indicates anti-clustering or regular spacing (Gillespie et al., 2000).

All the above analyses were then done for the Google Earth datasets with the exception that they were also compared to the corresponding field set through the K-S statistic.

RESULTS

General Statistics

Joint set	Mean strike (°)	Traverses	Segments	Data source	Ν	Spacing (cm)			
						Min	Max	Med.	
	150-330	4	17	F	292	1	1020	33	
1				GE	112	45	1073	183	
2	66-246	4	23	F	275	1	1426	14	
				GE	131	38	1887	210	

Table 1: Dataset Statistics

Table 1 displays the general characteristics of our four datasets, each labeled either F or GE—corresponding to data source, the field or Google Earth—and in later tables with a 1 or 2 corresponding to the particular joint set, 2 being the master joints, 1 being the cross joints (according to recorded truncation data (Morse et al., 2020)). Here we summarize these results. Sample sizes along the same joint set between data sources are over $2.0 - 2.6$ times higher in the field than Google Earth. Minimum calculated spacings were also smaller by 0.37 m to almost 0.5 m, while differences in maxima ranged almost an order of magnitude from 0.53 m to over 4.50 m. Measures of location such as median spacing were thus shifted to much larger values for GE datasets.

Calculated joint spacings in the field range over three orders of magnitude between 1 centimeter to 10s of meters within both sets. Those for the master set were notably larger than for the cross joints by about 4.0 m. Joint spacings from Google Earth ranged from 10s of centimeters

to 10s of meters—highlighting this data source's previously mentioned disparity in the smaller sub-half-meter spacings.

Figure 13. Joint spacing histograms of field data (A, joint set 1, B, joint set 2) and Google Earth data (C, joint set 1, B, joint set 2). C, and D, insets capture the structure of the Google Earth distributions hidden by the y-axis (frequency) range. (Each color used corresponds to that specific dataset in all of the following plots).

Frequency histograms of joint spacing indicate the data may be highly right skewed as expected for this kind of structure—though the inner structure of the first bin is mostly hidden from view and we cannot say for sure or discus modality. Still, bin 1 is much more pronounced for F data which again showcases the constraints of imagery resolution. Insets of **Figure 13C** and **D** show GE (Google Earth) histograms with a smaller y-axis range to show a more representative form of the distributions. Note the much higher frequencies in bins up to spacings of 500 to 600 cm. These are not encountered as often in the outcrop data where they are also especially dwarfed by measurements up to $60 - 86$ cm.

Fractal Analysis

Figure 14. Log-log cumulative frequency vs. joint spacing plots of each dataset. **A.** F1. **B.** F2. **C.** GE1. **D.** GE2.

Generated log-log plots in **Figure 15** yield spacing distributions that can be characterized by generally smooth to somewhat sharp concave down curves, rather than a linear relationship; this holds true for all traverses on an individual basis as well as for the composite datasets shown above. Thus, our data does not reveal fractal behavior by this test, again, revealed by linearity (Bonnet et al., 2001; Ehlen, 2000; Le Garzic et al., 2011; McCaffrey et al., 1993). These curves also conveniently visualize sampling disparities at both the very small and very large spacings as data point density decreases substantially at both extremes. Along those lines, visual comparison between F and GE further highlights the lower limit problem of the imagery.

Goodness-of-Fit Tests

Joint set	Data source	Lognormal		Normal			Gamma			Exponential			
		Critical	Test	p	Critical	Test		Critical	Test	p	Critical	Test	
÷		15.507	11.533	0.17	14.067	57.357	~1	15.507	3.715	0.88	14.067	53.726	\sim 0
	GE	15.507	3.068	0.93	15.507	42.281	~1	15.507	11.574	0.17	12.592	47.947	~ 0
$\overline{\mathbf{c}}$	ь.	19.675	15.957	0.14	21.026	84.534	~1	19.675	6.313	0.85	18.307	78.780	~1
	GE	19.675	22.199	0.02	19.675	126.594	~1	18.307	32.156	~1	18.307	32.652	~ 0

Table 2: Chi-Squared Goodness-of-fit Test Results (α = 0.05)

Chi-squared goodness-of-fit results in **Table 2** show both the critical and test values of each test along with their associated p-values. As a reminder, H₀ is rejected if the test value is greater than the critical and/or if the p-value is less than the level of significance, α . Our results indicate that the distributions for almost all datasets are either lognormal or gamma at the 0.05 confidence level—highlighted in green. Interestingly, we see that GE2 fails all tests. Note how close the critical and test values are for a lognormal distribution. Critical values vary naturally between test and dataset as a function of the given degrees of freedom of each frequency distributions (see **Appendix C1**). P-values show semi-consistent trends. All normal and

exponential tests yield values of approximately zero. Field p-values are also quite similar, 0.17 and 0.14 for lognormal, and 0.88 and 0.85 for gamma, for F1 and F2, respectively. The highest p-value is associated with the lognormal test for GE1, 0.93.

Kolmogorov-Smirnov Tests

Figure 14. Cumulative frequency distribution comparisons used in the K-S test. **A.** F1-F2. **B.** GE1-GE2. **C.** F1-GE1. **D.** F2-GE2. (again, red: F1, light red: F2, dark gray: GE1,

	Joint set Comparison	Critical	Test	р
1	$F1-F2$	0.114	0.127	0.02
	GE1-GE2	0.175	0.117	0.38
	$F1-GE1$	0.151	0.552	\sim 0
2	F2-GE2	0.145	0.406	\sim 0

Table 3: Kolmogorov - Smirnov Test Results (α = 0.05)

Figure 16 visually highlights the similarities between datasets at the same scale and the differences between those that were not. It should be noted that the *Cumulative frequency* plotted along the y-axis here is different than that in **Figure 15**. Here, the accumulation is through the relative frequency—taken as a portion or percentage of the sample size *N*—rather than whole number frequencies of encountering a certain value or less. **Table 3** displays our results for the K-S test itself. Again, we consider critical, test, and p-values.

Fig. 16A compares the master joints of set 2 and the cross joints of set 1. Set 2, wholly lying below set 1, can therefore be characterized by greater spacings, as discussed above. According to the K-S statistic we reject our null hypothesis that the datasets come from the same distribution. Still, note how close the test value is to the critical (0.127 vs. 0.114, respectively) and how close the p-value is to the level of significance (0.02 vs 0.05, respectively) (highlighted in yellow in **Table 3**). **Figure 16B** compares these two joint sets in Google Earth. Note their near identical behavior until about 150 cm (indicating the master joints to be less widely spaced than the cross joints) which then shifts to generally resemble the true relationship. This K-S test yields test and p-values (0.38) that indicate there is no statistical difference between the two datasets (highlighted in green in **Table 3**).

Figures 16C and **16D** compare data sources, F and GE, for joint sets 1 and 2,

respectively. Here are the key differences. As expected, GE curves exhibit a much more rapid rise in cumulative frequency with greater spacing and even greater spacings than were reported in the field. Each curve also exhibits an inflection point within the first 200 centimeters, yet again highlighting the little role smaller spacings paly in their distributions. On the other hand, field data completely lack an inflection point. Resulting p-values were virtually zero indicating these distributions were distinctly statistically different.

Coefficient of Variation

The coefficient of variation is a measure of clustering along sample lines (Gillespie et al., 2000). All datasets but GE1 were shown to exhibit clustering through its calculation (**Table 4**). In the field cross joints appeared clustered to a higher degree than the master joints by visual inspection. Their respective c^v appears to indicate that as well (1.54 vs. 1.43, respectively).

DISCUSSION

The range of our spacing data is somewhat close to that recorded by Martell et al. (1988) in the Mount Abbot Quadrangle in the central SNB. These went from a few 10s of centimeters to 10s of meters (Martel et al., 1988)—ours scaled an extra order of magnitude into the single digit

centimeters. Joint spacing at our sites is also non-uniform—observed in other granitic rocks such as the Florence Lake Mount Givens granodiorite, also of the central SNB (Segall & Pollard, 1983)—in contrast to the more regular spacing of cross joints in sedimentary layers (Ruf et al., 1997).

Measurement Uncertainty

As measurements were taken in the field it is appropriate to address their uncertainty. Each position recorded can be considered accurate with a standard uncertainty of 0.005 cm—half the smallest tick mark on our tape (one mm). Even if we were to determine the resulting uncertainty of our spacings through propagation of errors with standard deviations

$$
\left(\Delta s = \sqrt{\Delta p_j^2 + \Delta p_i^2}\right)
$$
 - the Δ here signifying uncertainty—the result would be similarly small;

however, our data is not error free. On one hand, the use of a one-dimensional traverse may introduce a sampling bias which can be significant where strikes vary from perfectly orthogonal to the traverse (Ryan, 2000; Sousa, 2010). As previously mentioned, we worked with a cutoff of around 5°-10° from perpendicular to allow for variations in individual joint strikes. This may be overcome with a simple geometric correction which considers the angle between a joint and the normal to the traverse enabling us to calculate the true spacing (Ryan, 2000; McCaffrey et al., 1993).

There were also many instances where measurements required a projection from the outcrop to the traverse. In one situation the measuring tape did not lay perfectly flat on the portion of the outcrop being sampled. This was largely due to the weathering and erosion of blocks downslope. We were then required to project our measurements from the traverse to the joints underneath. Similarly, if sampling through a section of regolith or vegetative cover, lateral projections were made in a similar manner. Measurements recorded in this fashion may have an uncertainty of up to a few centimeters, depending on the distance of the projection, and may have included significant errors, especially effecting our representation of the smaller spacings. Traverse locations were chosen in an attempt to maximize length and therefore did not wholly consider these limitations. A more refined criteria for traverse location that explicitly attempts to maximize length and minimize the need for projections could be employed to minimize these effects. Also, similar measurements can be made to establish a vertical correction to the horizontal to model "flatness" in relation to the outcrop.

Are the Spacing Distributions Fractal?

A power-law may be assumed to model the spacing distribution of a joint population when its trend on a log-log cumulative frequency vs. spacing plot can approximate a straight line (Bonnet et al., 2001; McCaffrey et al., 1993). Our datasets lack this behavior and instead resemble data collected by McCaffrey et al. (1993) in the County Galway Granite at Mace Head, Ireland (**Figure 17**).

Figure 15. Log-log cumulative frequency vs. vein spacing plots for individual datasets and a composite (McCaffrey et al., 1993). This has been shown to be typical of the negative exponential and lognormal distributions (McCaffrey et al., 1993). Further tests carried out by McCaffrey et al. (1993) agree and a linear regression analysis yielded an \mathbb{R}^2 of 96-99.1% for a lognormal distribution and only 60-82% for an exponential. Lognormal and exponential distributions (Clark et al., 1995; McCaffrey et al., 1993; Wong et al., 2018) are also commonly seen in the literature to describe joint spacing (Le Garzic et al., 2011; Sousa, 2010). Lognormal is perhaps the most frequently reported (Palmström, 1995) and has been observed in other granites such as in Stripa, Sweden by Roleau and Gale (1985). Our goodness-of-fit results seem to generally agree with these findings except that our distributions are best fit by a lognormal and gamma—exponential is in fact a subset of the gamma distribution (Walpole et al., 2012).

These results may represent the true distribution or simply further sampling biases at the small and large scales called truncation and censoring, respectively (Bertrand et al., 2015; Bonnet et al., 2001; Le Garzic et al., 2011). In most cases, these biases may cause deviations on the experimental log-log plot from linearity for power-law distributions (Bonnet et al., 2001). An under sampling of the smallest spacings is typically due to the resolution of the sampling technique used, while an underestimation of the largest may be due to the finite size of the sample domain and the lower probability of encountering larger size population values (Bonnet et al., 2001; Le Garzic et al., 2011). Most authors have simply truncated their own data to stem the former, effectively removing the part of their distribution believed to be biased. Unfortunately, there is not an established standard for this threshold and most researchers subjectively fix it below the point which they believe joints to be incompletely recorded. Spacing values less than the mode of a distribution are generally considered to be incompletely sampled so this could be an appropriate cutoff. Though sampling resolution is thought to be the primary cause of deviation from a power-law trend at small scales other mechanism have been suggested. One is the existence of a physical lower limit to power-law size populations such as 1 m for length distributions. All power laws in nature must have these lower and upper cutoffs. The generally accepted range of values for a robust characterization of a power-law distribution is 2-3 orders of magnitude (Bonnet et al., 2001). We achieved this with our opportunistic traverse locations and lengths but perhaps extending our measurements and measurement accuracy into the millimeters may yield more representative results (McCaffrey et al., 1993).

Given these, and other, sampling biases, many different distributions such as the powerlaw have been shown to produce samples that are approximately lognormal (Bonnet et al., 2001; Palmström, 1995) and even gamma under the appropriate conditions (Bonnet et al., 2001). Analog experiments by Reches (1986) of fracture system development have also shown that there exists a transition between power-law and lognormal size distributions with increased deformation indicating that distributions may change with time and that a power-laws could help identify young populations (Bonnet et al., 2001). Rives et al. (1992) even found that twodimensional trace lengths were best described as lognormal in sample but as a population were power-law (Ehlen, 2000). Clearly, a rigorous sampling scheme from the smallest to the largest scales is required to appropriately test fractal geometries (McCaffrey et al., 1993). An added suggestion is to apply the goodness-of-fit test directly to a power-law distribution. This was not considered for this paper as it requires parameter estimation procedures that are outside the present capabilities of the author. Still, a maximum likelihood estimation could be applied to not only determine power-law parameters but more explicitly those for the gamma, exponential, and even Weibull distributions (not tested in this paper). Similarly, the estimation of the probability of making Type I (rejecting H⁰ when true) and Type II (accepting H⁰ when false) errors and linear regression analyses for correlation should be considered (Walpole et al., 2012).

Lastly, Ehlen (1999) recommends a minimum number of spacing measurements to acquire a stable fractal dimension that is based on the nature of the sampled joint pattern itself. We chose a target of 100-150 per traverse, however, this is best applied to regularly spaced joints, while 200 or more spacings are required for a non-uniform or irregular spacing such as at our sites (Ehlen, 2000). As previously mentioned, no traverse surpassed these thresholds, but our composite datasets have substantially—292 and 275—and it remains unclear whether a fractal dimension may have stabilized or not. Therefore, as another check on fractal behavior, future studies should determine whether that stability was reached.

Now, the reason for such a wide variety of statistical joint spacing distribution models is still unclear (Wong et al., 2018); however, there are some suggestions. As with the power-law, an assortment of distributions may characterize different stages of fracture pattern development (Ehlen, 2000). This could be extremely useful in determining the progression of deformation

events related to fracturing and further corroborate age relationships between fracture patterns. Also, lognormal distributions may characterize an upper limit of joint density—saturation threshold—in granites where progressive joint development effectively ceases (Wong et al., 2018). It has also been suggested that distributions may be related to the mode of fracturing and the initial stress conditions (Boadu $& Long, 1994$). These suggestions largely pose questions about the evolution of a joint system which may be best explored through numerical modeling. Not a lot of fracture spacing work has been done with computational methods (Wong et al., 2018) but the ability to control material parameters, boundary conditions, and the laws of physics, may provide invaluable insight on the key elements of joint development (Fossen, 2016).

How do the Spacing Distributions Compare?

Preliminary results ignited an interest to directly compare the spacing distributions of our two joint sets statistically which could reveal something about the joined fracture forming histories of these two sets, and the nature of spacing in relation to that history. The K-S test between the master and cross joints led to a rejection of our null hypothesis that the distributions are the same (**Table 3**). This is to be expected for different joint sets—even within the same rock unit (Pollard & Aydin, 1988)—especially ones with such consistent truncation relationships (Morse et al., 2020). We also notice distinct differences between them such as generally larger spacings within the master set (**Figure 16A**) and a lesser degree of clustering (**Table 4**); however, the grounds for rejection are quite equivocal. The critical region was any value greater than 0.114. Our test value was only 0.127, only crossing that threshold by 0.013. Similarly, our p-value was quite close to the level of significance (0.02 vs. 0.05, respectively). Therefore, we are unable to infer the likely reality from these results (Walpole et al., 2012). It is likely that

many of the previous recommendations to achieve even more representative datasets could overcome this and provide a more distinct outcome. When comparing joint spacing distributions, it may also be of interest to plot their respective relative cumulative frequency distributions (**Figure** 16) whether or not the K-S test will be performed to effectively visualize their differences and similarities.

Also, where consistent age relationships are observed, it would be of interest to measure the possible influence of the spacing of the arresting set on the spacing of the arrested set. It has been shown that discontinuities themselves affect the growth of other joints and that their spacing may act as an effective mechanical layering in sedimentary beds (Ruf et al., 1997). Investigations of this sort in igneous rocks may yield interesting relationships and assist with our characterization of joint patterns.

Can Google Earth Datasets Yield Representative Joint Spacing Distributions?

Lastly, we will discuss the utilization of Google Earth in fracture analysis—namely for joint spacing. Each analysis carried out in this paper highlights the underestimation of joint spacings in datasets collected through Google Earth Imagery (see **Figures 13, 14,** and **15**)—no spacings were recorded below 0.38 m. It is not surprising that K-S results show them to be statistically different than their counterpart field dataset. These results are unequivocal, unlike the previous, as test values were much larger than critical, and p-values are virtually zeros (Walpole et al., 2012). Still, distribution form seems somewhat preserved, through visual inspection of spacing histograms (**Figure** 13) and statistically, as GE1 was best fit by a lognormal distribution with the highest p-value of any test, 0.93. Position descriptors are not maintained however, as the underestimation of smaller spacings has led to the artificial generation of larger spacings and have shifted the distribution. Google Earth is still of great

scientific value and If used for spacing analyses the above constraints must be considered. The comparison of field datasets and those collected in Google Earth should be continued for refined quantitative measures of resolution bias and accuracy.

What is somewhat surprising is how similar Google Earth datasets 1 and 2 are to each other. They are the only datasets which pass the K-S test—and quite significantly with a p-value of 0.38. As, mentioned this is likely attributed to the significant overlap between the two destructions at the smaller spacings recorded. This may be an expression of the lower limits of the imagery's resolution. Both sets were sampled from the same images and we may therefore miss the same size joints. We cannot be exactly sure as Google Earth remains a black box with very limited public documentation on its inner workings (Fisher et al., 2012).

CONCLUSIONS

We investigate two sub-orthogonal joint sets in the Sierra Nevada Batholith, California, to, 1) characterize their spacing distributions with an interest in fractal behavior, 2) directly compare the spacing distributions of these sets of apparent different ages, and 3) determine whether Google Earth can yield representative datasets for joint spacing analyses.

The spacing distributions for master and cross joint datasets are best described by lognormal or gamma and not power-law (fractal) at the 0.05 significance level. This result may also reflect the true distribution or sampling biases such as truncation or censoring of which the fractal signature is sensitive too (Bertrand et al., 2015; Bonnet et al., 2001; Le Garzic et al., 2011). This may be overcome with even greater sample sizes and also by extending the lower limit of measurements another order of magnitude into the millimeters (Bonnet et al., 2001). Other analyses should also be performed alongside the log-log fractal test plot for a more robust determination of fractal behavior such as a fractal dimension stability analysis (Ehlen, 2000) and directly testing the fit of a power-law distribution.

As expected, the spacing of the older master joints and younger cross joints are different—the former exhibits a consistently greater frequency of wider spacings and lesser degree of clustering. The statistical difference is still unclear as K-S test and critical values are remarkably close. The previous recommendations related to sampling scheme would likely overcome this result. The relationship between the spacing of these two sets should also be explored further to determine whether arresting joints within igneous rocks may also define a mechanical layer thickness and therefore constrain the spacing of arrested joints (Ruf et al., 1997).

All analyses highlight departures from reality when comparing datasets collected in field outcrops and on Google Earth imagery. The latter consistently misses the smallest spacings and as a result generates artificially large spacings. They are shown to somewhat preserve distribution form (e.g., lognormality) but then shift their position towards larger values. Google Earth spacing distributions were then distinctly statistically different than field spacing distributions. Therefore, datasets from Google Earth cannot be analyzed under the assumption that they represent the true spacing distribution of a joint set. Ultimately, within fracture analysis, Google Earth can only be fully utilized as a field site reconnaissance and mapping tool, and any analyses done through it should be corroborated with field work.

APPENDIX

Appendix A – Data

Table A1: Field Dataset 1

Table A2: Field Dataset 2

Appendix B – Google Earth Dataset Collection

Traverse generation

Traverses in Google Earth are direct replications of their field counterpart and were generated with heading and coordinate info from the field (**Figure 16**). Once corrected for declination these coordinates were plotted with the *Add Placemark* tool (**Figure 16**). Once selected, a placemark is generated and its properties displayed where coordinates may be directly input. *Add Path* may then be selected to connect the endpoints of a given segment.

Figure 16. Google Earth window with traverse 1 at site 1.3 drawn. Closeup of toolbar indicates the tools used for data collection. From Google Earth.

These were then checked with the *Show Ruler* tool for length and heading.

Data collection

To collect the positions of intersection between joints and the traverse we again use the *Add Path* tool. A placemark may seem more intuitive, however, as shown in **Figure 17** the placemark itself makes accurate positioning difficult, no matter which style is used.

Figure 17. Issues related to using the placemark tool. Taken from Google Earth.

Google Earth paths are instead comprised of a sequence of points and each point is associated with a given set of coordinates. Therefore, each position of intersection can be recorded with one point of one path (it was later realized that an entire segment could then have been sampled by one singular path with multiple points) (**Figure 18**).

Figure 18. Zoomed in photo of **Figure 16** showing points assigned to intersections. The size of each point is exaggerated. Taken

from Google Earth.

If the beginning or end of the traverse is not intersected by a joint in the imagery its position needs to be recorded. The reasoning is illustrated in **Figure 19**. This figure considers two hypothetical traverse segments (blue) that sample a joint set (black). The position of intersection of each joint is designated by a green dot. If we were to calculate the spacing between each of these, we would also calculate the length of the diagonal between the segments (yellow, dashed) instead of spacings along their length. This sampled "spacing" is longer than the true spacing and can be avoided my recording the position of the end and beginning of the segments (blue dots).

Figure 19. Simple schematic showing the necessity of recording traverse endpoints.

Data conversion

Data related to each intersection was then exported as in .kml (Keyhole Markup Language) format—exporting in this fashion just saves the coordinate data in latitude and longitude. We cannot easily determine spacing in these units and so we convert them into

Universal Transverse Mercator (UTM) coordinates, northing and easting, with units of meters. Spreadsheets can be set up to do this, but we used online batch converters via [https://consult.hermes.com.np/batch-convert-lat-long-to-utm.](https://consult.hermes.com.np/batch-convert-lat-long-to-utm)

Spacing calculation

Spacing between each datapoint was determined through with algebraic distance equation between two points, $d = \sqrt{(n_f - n_i)^2 + (e_f - e_i)^2}$, where (n_f, e_f) are the coordinates of the jth joint and (n_i, e_i) those of the ith, in northing and easting respectively. For spacings that cover the end of a segment and the start of the next two of these calculations need to be made and then summed accordingly.

Appendix C – Statistical Methodology

Chi-squared (χ^2) *goodness-of-fit*

The Chi-squared test determines if a population has a certain theoretical distribution. This is based on how well the expected frequencies of the hypothesized distribution—in this case lognormal, normal, gamma, or exponential—fit the true, observed frequencies of the data. Goodness-of-fit is determined by the quantity,

$$
\chi^2=\textstyle\sum_{i=1}^k\frac{(o_i-e_i)^2}{e_i},
$$

where o_i and e_i are the observed and expected frequencies of the ith to the kth bin, respectively. Through this value we are then equipped to accept or reject the null hypothesis (H_0 : the spacing distribution is the one being tested against). A poor fit is indicated by a large χ^2 —rejection of the null—a good fit is indicated by a small value—therefore leading to acceptance (Walpole et al., 2012).

Frequencies (and bins) were derived through a small series of steps. First, we generate stem-and-leaf plots from the data. These plots constitute a tabular and graphic way of presenting large masses of statistical data and can be quite useful for simple characterizations (Walpole et al., 2012). As an example, consider the sample dataset in **Figure 20** below. Here we have 40 arbitrary values arranged by sampling order. To gather these into a more approachable format we may split each quantity into a stem and leaf. For example, 4.1 below can be separated into a stem of 4 and a leaf of 1, 3.5 into a stem of 3 and leaf of 5, and so on. In this way stems effectively function as bins and we may associate a frequency to each. Below, the "1" stem has 2 leaves; therefore, it has a frequency of 2. Now, spreadsheet commands can acquire this information much more conveniently whereas the above method is essentially determining frequency by hand. In this case, the latter was chosen to provide a more comprehensive first approach to the statistical analysis of geologic data. If the reader is especially familiar with statistical methods, they may easily determine frequencies via more computational methods.

${\bf Stem}$	Leaf	Frequency		
	69			
	25669			
3	0011112223334445567778899	25		
	11234577			

Figure 20. Example stem-and-leaf plot generated from arbitrary data. Adapted from Walpole et al., (2012).

Stems are chosen are somewhat arbitrarily, but it is generally accepted that a greater number will provides a more representative picture of the distribution and, usually, between 5-20 are set (Walpole et al., 2012). Our choice for stems is not as straightforward given the magnitude and range of our values, but since we solely seek their frequencies, we are satisfied with a much greater number. Datasets 1 and 2 were initially separated into 103 and 143 stems, respectively (the first stem is "0" to account for all spacings less than 10 cm). Leaves can be conveniently generated with Excel functions and we used the *REPT* as well as *COUNTIF*. The first will repeat a specified string of text a specified number of times and the second will tell you the number of times a specified string is found within a specified array. For a single cell this must be done for each leaf you wish to display. For example,

> $=$ REPT("0",COUNTIF(if in data, there exists 1*10+0)) & REPT("1", COUNTIF(if in data, there exists $1*10+1$)) & REPT("2", COUNTIF(if in data, there exists $1*10+2$)) &

> > …

&

REPT("9",COUNTIF(if in data, there exists 1*10+9)),

…

will display all the leaves of the first stem, 1, for all values in your dataset (spacings) from 10-19 cm. **Tables C1** and **C2** show the stem-and-leaf plots for field datasets 1 and 2.

Table C1: Field dataset 1 stem-and-leaf plot

Stem	Leaf	Stem Leaf	
0		52	
$\mathbf{1}$	01111122222334444445555667888888999999	53	
$\overline{2}$	001111111244456677789	54	
3	0012333567889	55	
4	00115779	56	
5	01233344	57	459
6	01122333489	58	
$\overline{7}$	25567788899	59	
8	23446778999	60	
9	000479	61	
10	049	62	
11	02379	63	
12	337	64	
13	0335777	65	
14	0056	66	
15	\vert 24	67	
16	167	68	17
17	12279	69	
18	$\sqrt{248}$	70	
19	6	71	
20	5678	72	
21	1669	73	
22	05	74 13	
23	078	75	
24	356	76	
25		77 \vert 7	
26	11	78	
27	2789	79	
28	5	80	
29	lз	81	
30	07	82	
31	89	83	
32	1	84	
33	007	85	
34		86	
35	9	87	
36	1	88	
37	17	89	
38		90	
39		91	
40		92	
41		93 3	
42		94	
43		95	
44		96	
45	336	97	
46		98	
47		99	
48		100	
49	14	101	
50		102 0	
51			

From there we calculate the frequency of each stem to build the frequency distribution. Here, we now consolidate frequencies into bins and separate each dataset into around 20, as generally accepted (Walpole et al., 2012). The expected frequencies of each bin are then calculated with the distribution function. For example, for the lognormal test of dataset 1 we would execute this command,

 $=$ LOGNORM.DIST(49, mean, standard deviation, TRUE)* sample size.

This yields the projected lognormal distribution of our spacing data from 0-49 cm based off of the required parameters—mean and standard deviation. Here, TRUE is a conditional statement that indicates we desire the cumulative distribution. For subsequent calculations TRUE would recalculate the frequencies from each previous bin so we must modify our command to,

 $=$ LOGNORM.DIST(99, mean, standard deviation, TRUE) –

LOGNORM.DIST(50, mean, standard deviation ,TRUE))* sample size.

The subtraction negates this resampling and only leaves behind the distribution between 50-99 cm, or the next bin. See **Tables C3** and **C2**.

Table C4: Field dataset 2 frequency distribution

Table C3: Field dataset 1 frequency distribution

161

47

22

14 16

> 8 8

> 3 $\mathbf 0$

4

 $\mathsf{o}\xspace$

3 $\mathbf 0$

 $\overline{2}$

 $\mathbf 1$

 $\mathbf 1$ $\mathbf 0$

0

 $\mathbf 1$

0 $\mathbf 1$

Class interval

 $0 - 49$ 50-99

100-149

150-199

200-249 250-299

300-349 350-399

400-449 450-499

500-549

550-599

600-649 650-699

700-749

750-799

800-849

850-899

900-949 950-999

1000-1049

In some cases, bins must be further consolidated so that they have a frequency of at least 5. With less, the criterion which ultimately leads to rejection or acceptance of the null may be inaccurate. The remaining number of bins will define our degrees of freedom, $v = k - 1$, where k is the number of bins (Walpole et al., 2012). With this we may finally determine the critical value of our test. We use the table from Walpole et al. (2012), attached, at a level of significance of 0.05.

If the calculated test value (χ^2) was greater than the critical, we are led to rejection, if less, we fail to reject. P-values were also calculated to corroborate our findings and can simply be defined as the probability of obtaining that Chi-squared value. In general, if greater than the level of significance we accept, if less we reject.

Table $\mathbf{A.5}$ Critical Values of the Chi-Squared Distribution

${\bf Table\ A.5\ (continued)\ Critical\ Values\ of\ the\ Chi-Squared\ Distribution}$

Kolmogorov-Smirnov

The K-S test fundamentally works the same way as the Chi-squared when the goal is to accept or reject a null hypothesis—that both sample distributions come from the same population. In this case we compare the fit between two cumulative distributions as explained in the methodology. The test value is their maximum difference between the bins of their cumulative frequencies. The critical value is can be calculated via,

 $=$ SQRT (- LN(alpha / 2)*0.5)* SQRT((sample size of F1 + sample size of F2)/(sample size of F1 * sample size of F2).

Table C5: F1-F2 cumulative frequency distributions								
Class interval	F1	F ₂	CF ₁	CF ₂	Difference			
$0 - 49$	161	120	0.5514	0.4364	0.1150			
50-99	47	41	0.7123	0.5855	0.1269			
100-149	22	25	0.7877	0.6764	0.1113			
150-199	14	11	0.8356	0.7164	0.1193			
200-249	16	15	0.8904	0.7709	0.1195			
250-299	8	10	0.9178	0.8073	0.1105			
300-349	8	8	0.9452	0.8364	0.1088			
350-399	3	8	0.9555	0.8655	0.0900			
400-449	0	2	0.9555	0.8727	0.0828			
450-499	4	7	0.9692	0.8982	0.0710			
500-549	0	6	0.9692	0.9200	0.0492			
550-599	3	5	0.9795	0.9382	0.0413			
600-649	0	1	0.9795	0.9418	0.0376			
650-699	2	2	0.9863	0.9491	0.0372			
700-749	1	$\mathbf{1}$	0.9897	0.9527	0.0370			
750-799	1	2	0.9932	0.9600	0.0332			
800-849	0	1	0.9932	0.9636	0.0295			
850-899	0	2	0.9932	0.9709	0.0222			
900-949	1	0	0.9966	0.9709	0.0257			
950-999	0	2	0.9966	0.9782	0.0184			
1000-1049	1	1	1.0000	0.9818	0.0182			
1050-1099	0	1	1.0000	0.9855	0.0145			
1100-1149	0	1	1.0000	0.9891	0.0109			
1150-1199	0	1	1.0000	0.9927	0.0073			
1200-1249	0	1	1.0000	0.9964	0.0036			
1250-1299	0	0	1.0000	0.9964	0.0036			
1300-1349	0	0	1.0000	0.9964	0.0036			
1350-1399	0	0	1.0000	0.9964	0.0036			
1400-1449	0	1	1.0000	1.0000	0.0000			
Total	292	275						

Table C5: E1-E2 cumulative frequency distribution

REFERENCES

- Bai, T., & Pollard, D. D. (2000). Fracture spacing in layered rocks: a new explanation based on the stress transition. *Journal of Structural Geology, 22*, 43-57.
- Bateman, P. C. (1968). Geologic Structure and History of the Sierra Nevada. *UMR Journal – VH McNutt Colloquium Series, 1*(8), 121-131.
- Bertrand, L., Géraud, Y., Le Garzic, E., Place, J., Diraison, M., Walter, B., & Haffen, S. (2015). A multiscale analysis of a fracture pattern in granite: A case study of the Tamariu granite, Catalunya, Spain. *Journal of Structural Geology, 78*, 52-66.
- Boadu, F. K., & Long, L. T. (1994). Statistical Distribution of Natural Fractures and the Possible Physical Generating Mechanism. *Pure and Applied Geophysics, 142*(2), 273-293.
- Bonnet, E., Bour, O., Odling, N. E., Davy, P., Main, I., Cowie, P. & Berkowitz B. (2001). Scaling of Fracture Systems in Geological Media. *Reviews of Geophysics, 39*(3), 347- 383.
- Busby, C. J, DeOreo, S. B., Skilling, I., Gans, P. B., & Hagan, J. C. (2008). Carson Pass— Kirkwood paleocanyon system: Paleogeography of the ancestral Cascades arc and implications for landscape evolution of the Sierra Nevada (California). *GSA Bulletin, 120*(3/4), 274-299.
- Cecil, M. R, Rotberg, G. L., Ducea, M. N., Saleeby, J. B., & Gehrels, G. E. (2012). Magmatic growth and batholitic root development in the Northern Sierra Nevada, California. *Geosphere, 8*(2), 592-606.
- Clark, M. B., Brantley, S. L., & Fisher, D. M. (1995). Power-law vein-thickness distributions and positive feedback in vein growth. *Geology, 23*(11), 975-978.

Ehlen, J. (1999). Fracture characteristics in weathered granites. *Geomorphology, 31*, 29-45.

Ehlen, J. (2000). Fractal analysis of joint patterns in granite. *International Journal of rock Mechanics and Mining Sciences, 37,* 909-922.

- Ericson, K, Migon, P., & Olvmo, M. (2004). Fractures and drainage in the granite mountainous area A study from Sierra Nevada, USA. *Geomorphology, 64*, 97-116.
- Fatt, N. T. (1994). Joint spacings in granitic rocks of eastern Kuala Lumpur area, Peninsular Malaysia. *Geol. Soc. Malaysia, Bulletin, 35*, 157-168.
- Fisher, G. B., Amos, C. B., Bookhagen, B., Burbank, D. W., & Godard, V. (2012). Channel widths, landslides, faults, and beyond: The new world order of high-spatial resolution Google Earth imagery in the study of earth surface processes. *Google Earth and Virtual Visualizations in Geoscience Education and Research: Geological Society of America Special Paper 492*, 1-22.
- Fossen, H. (2016). *Structural Geology.* Cambridge University Press.
- Gillespie, P. A., Walsh, J. J., Watterson, J., Bonson, C. G., & Manzocchi, T. (2000). Scaling relationships of joint and vein arrays from the Burren, Co. Clare, Ireland. *Journal of Structural Geology, 23*, 183-201.
- Le Garzic, E., de L'Hamaide, T., Diraison, M., Géraud, Y., Sausse, J., de Urreiztieta, M., Hauville, B., Champanhet, J.-M. (2011). Scaling and geometric properties of extensional fracture systems in the Proterozoic basement of Yemen. Tectonic interpretation and fluid flow implications. *Journal of Structural Geology, 33*, 519-536.
- Martel, S. J., Pollard, D. D., & Segall, P. (1988). Development of simple strike-slip fault zones, Mount Abbot quadrangle, Sierra Nevada, California. *Geological Society of America Bulletin, 100*, 1451-1465.
- McCaffrey, K, Johnson, J. D., & Feely, M. (1993). Use of Fractal Statistics in the Analysis of Mo-Cu Mineralization at Mace Head, County Galway. *Irish Journal of Earth Sciences, 12*, 139-148.
- McKee, E. H., Chaffee, M. A., Federspiel, F. E, McHugh, E. L., Cather, E. E., Scott, D. F., & Rumsey, C. M. (1982). Mineral Resource Potential of the Mokelumne Wilderness and Contiguous Roadless Areas, Central Sierra Nevada, California. *Department of the Interior United States Geological Survey.*
- Morse, S., **Wood, J.**, & Maher, H. (2020). *Observing Fracture Patterns at 3 Scales in the Sierra Nevada Batholith, Mokelumne Wilderness, California.* Poster presentation, Geological Society of America Annual Meeting, 26-30 Oct., Online.
- National Research Council. (1996). *Rock Fractures and Fluid Flow: Contemporary Understanding and Applications.* The National Academies Press.
- Olson, J. E. (2004). Predicting fracture swarms the influence of subcritical crack growth and the crack-tip process zone on joint spacing in rock. *Geological Society, London: Special Publications, 231*, 73-88.
- Palinkas, L. A., Horwitz, S. M., Green, C. A., Wisdom, J. P., Duan, N., & Hoagwood, K. (2013). Purposeful sampling for qualitative data collection and analysis in mixed method implementation research. *Administration and Policy in Mental Health and Mental Health Services Research, 42*(5), 533-544.
- Palmström, A. (1995). *RMi – a rock mass characterization system for rock engineering purposes* (400)*.* [PhD Thesis, Oslo University].
- Pennacchioni, G., & Zucchi, E. (2013). High temperature fracturing and ductile deformation during cooling of a pluton: The Lake Edison granodiorite (Sierra Nevada batholith, California). *Journal of Structural Geology, 50*, 54-81.
- Pollard, D. D., & Aydin, A. (1988). Progress in understanding jointing over the past century. *Geological Society of America Bulletin, 100*, 1181-1204.
- Pollard, D. D., & Martel, S. J. (2020). *Structural Geology: A Quantitative Introduction.* Cambridge University Press.
- Reches, Z. (1986). Network of shear faults in the field and in experiment. *Annals of the Israel Physical Society: Fragmentation, Form, and Flow in Fractured Media, 8*, 42-51.
- Rouleau, A., & Gale, J. E. (1985). Statistical characterization of the fracture system in the Stripa Granite, Sweden. *International Journal of rock Mechanics and Mining Sciences, 22*, 353- 367.
- Ruf, J. C., Rust, K. A., & Engelder, T. (1997). Investigating the effect of mechanical discontinuities on joint spacing. *Tectonophysics, 295,* 245-257.
- Ryan, J. (2000). Fracture spacing and orientation distributions for two-dimensional data sets. *Journal of Geophysical Research, 105*(B8), 19,305-19,320.
- Segall, P., & Pollard D. D. (1983). Joint formation in granitic rock of the Sierra Nevada. *Geological Society of America Bulletin, 94*, 563-575.
- *Sierra Nevada Mountain Range Geomorphology.* (2012, August). Geocaching. Retrieved February 2, 2021, from https://www.geocaching.com/geocache/GC3RQEW_sierranevada-mountain-range-geomorphology?guid=a86a1cb7-fd0a-4d08-813a-9fc3928d4ebb
- Sendek, C. (2016). *Zircon Geochemical and Isotopic Constraints on the Evolution of the Mount Givens Pluton, Central Sierra Nevada Batholith* (4777). [Master's thesis, San Jose State University]. SJSU ScholarWorks.
- Sousa, L. M. O. (2010). Evaluation of joints in granitic outcrops for dimension stone exploitation. *Quarterly Journal of Engineering Geology and Hydrology, 43*, 85-94.
- Velde, B., Dubois, J., Moore, D., & Touchard, G. (1991). Fractal patterns of fractures in granites. *Earth and Planetary Science Letters, 104*, 25-35.
- Walpole, R. E., Myers, R. H., Myers, S. L., & Ye. Keying. (2012). *Probability & Statistics for Engineers & Scientists.* Prentice Hall.
- Wong, L. N. Y., Lai, V. S. K., Tam, T. P. Y. (2018) Joint spacing distribution of granites in Hong Kong. *Engineering Geology, 245*, 120-129.