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Teaching the Relationship Between Multiplication and Division Through Differentiated Instructional Strategies

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Honors Capstone

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Abstract

This capstone project reports the differentiated instructional strategies that were integrated throughout a math unit. The capstone will report the effectiveness of two instructional strategies that were integrated into a math unit that focused on students' understanding of the relationship between multiplication and division. The math unit had 12 lessons taught over three weeks between the pre-and post-assessment. I will report how the strategies worked and the effect the strategies had on students learning. I collected and analyzed the pre/post-test data and found that nonlinguistic representations and multi-level questioning are effective instructional strategies for increasing student learning.

Keywords: instructional strategies, nonlinguistic representations, multi-level questioning

Background

In collaboration with my mentor teacher, it was determined that for the capstone project I would plan, implement, and assess a math unit. The project was conducted in a third-grade classroom within a clinical practice placement at a suburban elementary school in Nebraska's Papillion LaVista Community School District. The elementary school is a Title One school as 50% of the students are eligible for Free and Reduced Lunch through the National School Lunch Program (NSLP). Of the 394 students, 14.8% of students are English Language Learners (ELL). The student-to-teacher ratio is 13:1 at this elementary school.

Introduction

For this project, the students participated in a pre-test prior to instruction. At the end of the unit a post-test was administered to assess their knowledge of the relationship between

multiplication and division. The pre/post-test was comprised of 10 questions. Throughout the unit, I integrated two instructional strategies to determine if the differentiated instructional strategies would increase student learning. The first instructional strategy integrated into the lessons was nonlinguistic representations, and the second was multi-level questioning.

A nonlinguistic representation is an expression of an idea that goes beyond the use of words: pictures, simulations, graphic organizers, drawings, models, diagrams, or demonstrations. Researchers Kelly (2020) and Rainyn (2016) discussed the importance and effect of nonlinguistic representations and how the strategy can be embedded to be used as an instructional tool to aid in student learning. Kelly (2020) noted that nonlinguistic representations are beneficial for students to retain the information they are learning. Kelly (2020) stated, "The more learners use both modes [nonlinguistic and linguistic] of storing knowledge, the better able they are to have sustained learning and quicker recall of information" (p. 1). It was also noted that having multiple representations of information addresses different learning styles that the students may have (Kelly, 2020).

Seeing and hearing information in multiple ways is beneficial for students to have a deeper understanding when learning new material. Teachers want students to be engaged in lessons and the learning process. When lessons include nonlinguistic representations, it engages them with visuals and hands-on activities that enrich student learning. Kelly (2020) stated, "explicitly engaging students in the creation of visual representations stimulates and increases attention to and interpretation of new knowledge" (p. 4). Rainyn (2016) conducted a study that looked at how visual learning increased students' higher-order thinking skills. Rainyn (2016)

concluded that nonlinguistic representations, specifically visual learning, enabled students to move into higher-order thinking.

I integrated nonlinguistic representations into my unit of instruction to meet the different learning styles of the third-grade students. I found that incorporating different teaching styles and instructional strategies into my unit expanded the student's understanding of the relationship between multiplication and division.

The second strategy I integrated into the unit of instruction was multi-level questioning. There is an abundance of research that supports 'questioning' as an effective strategy to assess student learning before, during, and after instruction. Gunn (2008) conducted research that reported how questioning was beneficial in increasing students' critical thinking. Gunn (2008) also noted that when teachers ask students to think critically about a question, the teacher can collect qualitative and quantitative data from their responses to assess their understanding.

Kaskens (2022) conducted a study that focused on using math interviews throughout a math unit to assess student learning. Kaskens found that during the students' interviews conducted, it was observed that students could identify the strategies they used to answer math problems. Through questioning, the students verbally explained how and why they solved the questions the way they did. The study found that utilizing questioning interviews was an effective strategy to assess the problem-solving process the students used to solve math problems which caught students' misconceptions of solving math problems.

Participants

This unit of study was taught in a third-grade classroom composed of 19 students. Of the 19 students, 11% qualify for special needs services, 5% are ELL students, and 11% of students qualify for Free and Reduced Lunch through NSLP. Of these students, 11% have Individualized Reading Improvement Plans (IRIP). Five percent are in an Intermediate Intervention Program (IIP) for reading, and 21% have Intermediate Intervention Program (IIP) plans for math. These interventions and placements were implemented following analysis of the student's Northwest Evaluation Association (NWEA) Measure of Academic Progress (MAP) scores.

Methods and Materials

The goal of the math unit was for students to understand the relationship between multiplication and division. The goal for the end of the unit was for students to be able to do the following tasks independently through:

- solving division story problems,
- reading a story problem and decipher between using multiplication or division,
- solving 1-step division and multiplication problems using a strategy [cubes, array, picture] efficiently, and
- writing the inverse division problem to a given multiplication problem.

The math unit aligned with the Nebraska state standard 3.1.2.f which states that students will use objects, drawings, arrays, and symbols to explain the relationship between multiplication and division (NDE, 2015, p. 11).

In the Papillion LaVista Community School District, the students are graded on a proficiency scale. Level 1: Beginning, Level 2: Progressing, Level 3: Proficient, Level 4:

Advanced. The target for students was to be Level 3 (Proficient). To assess each student's proficiency, I administered a pre-test. Using the pre-test data, I placed students in small groups for instruction specific to their needs. The lessons taught were based on the Papillion LaVista Community Schools Mathematic Curriculum. The third-grade math Unit 5 Clinical Skills Assessment [CSA] was the pre-test and post-test that were given.

I began the unit by administering a pre-test that included multi-level questioning. Following the analysis of the pretest results, I created small groups. The groups were created to either remediate or enrich instruction based on the student's pre-test scores. I intermittently administered assessments throughout the unit to assess their understanding. Based on the results I made changes in the small groups.

For the unit of instruction, I integrated two research-based instructional strategies to determine if the instructional strategies would increase student understanding. The first strategy I used to engage learners was nonlinguistic representations. I integrated this research-based strategy to provide support to the content I was teaching. Through integrating nonlinguistic representations (pictures, arrays, and snap cubes) students were provided with many ways to solve multiplication and division problems. The nonlinguistic representations provided the students with multiple strategies to demonstrate what they knew. This strategy was utilized in whole-group instruction as students learned different strategies to solve multiplication and division problems. This strategy was also used within small group instruction, focusing on specific nonlinguistic representations to solve problems based on students' needs.

The second research-based strategy I integrated into the lessons was multi-level questioning. This was incorporated by conducting 1-minute math interviews. Through

conducting math interviews, I was able to ask multi-level questions to check individual students' understanding. The 1-minute math interviews were integrated throughout the unit to support the content objectives of the lessons. Each time I implemented the 1-minute interviews, I recorded the strategies the students self-reported using to solve given math problems and then I analyzed their responses. This data assisted me in determining the skills they were using to solve the math problems. Multi-level questioning was incorporated into whole-group instruction as students would answer various leveled questions during partner work and independent work. This strategy was also utilized during small-group instruction. During small group time, the students would be asked various questions about the problems they were working on.

At the end of the unit, the students were given a post-test consisting of the same questions as the pre-test. I analyzed the data to determine students' growth and proficiency. I reviewed the data to determine if the two instructional strategies impacted the students learning.

Results

Appendix 1 includes the pre-assessment and post-assessment taken from Unit 5 Clinical Skills Assessment [CSA]. Tables 1 and 2 represent students' correct answers on each question of the pre/post-assessment which includes the question's difficulty level related to the proficiency scale. I chose an arbitrary symbol "1" to represent a correct answer. Following the review of the test results, I identified individual student levels. Figure 1 includes the students' pre-assessment and post-assessment scores based on the proficiency scale.

Pre-Assessment

Proficiency scale for questions	Level 2					Level 3				Level 4
Question #	1	2	3	4	5	6	7	8	9	10
Student 1	1	1		1						
Student 2	1	1		1	1					
Student 3	1	1		1						
Student 4				1				1	1	
Student 5		1		1	1					
Student 6		1		1	1					
Student 7				1	1					
Student 8		1		1				1	1	
Student 9				1	1					
Student 10	1		1	1	1					
Student 11	1	1		1	1			1	1	
Student 12		1		1	1		1			
Student 13		1	1	1	1		1	1	1	
Student 14		1		1	1	1		1	1	
Student 15		1		1	1		1	1	1	
Student 16		1		1			1	1	1	
Student 17		1		1						

Student 18	1	1		1	1		1	1	1	
Student 19					1					
Total	6/19	14/19	2/19	18/19	13/19	1/19	5/19	8/19	8/19	0/19

Table 1 reports how students scored on each question of the pre-assessment. It is divided into levels. The questions on the pre-assessment became increasingly advanced throughout the test to assess proficiency.

Post-Assessment

Proficiency scale for questions	Level 2					Level 3				Level 4
Question #	1	2	3	4	5	6	7	8	9	10
Student 1		1		1			1	1	1	
Student 2	1	1	1	1	1		1	1	1	
Student 3	1	1	1	1	1		1	1	1	
Student 4		1		1				1	1	
Student 5		1	1	1	1			1	1	
Student 6		1	1	1	1					
Student 7		1		1	1		1	1	1	
Student 8	1	1	1	1	1	1	1	1	1	

Student 9		1	1		1		1	1	1	
Student 10		1		1	1	1		1	1	
Student 11	1	1	1	1	1	1	1	1	1	
Student 12	1	1	1	1	1	1		1	1	1
Student 13		1	1	1	1			1	1	1
Student 14		1		1	1		1	1	1	
Student 15		1		1	1		1	1	1	
Student 16		1		1		1	1	1	1	
Student 17		1		1	1					
Student 18	1	1			1	1	1	1	1	
Student 19		1		1	1		1			
Total	6/19	19/19	9/19	17/19	16/19	6/19	12/19	16/19	16/19	2/19

Table 2 reports how students scored on each question of the post-assessment. The table is divided into levels to show how the assessment is set up. The questions became increasingly advanced throughout the test to assess proficiency.

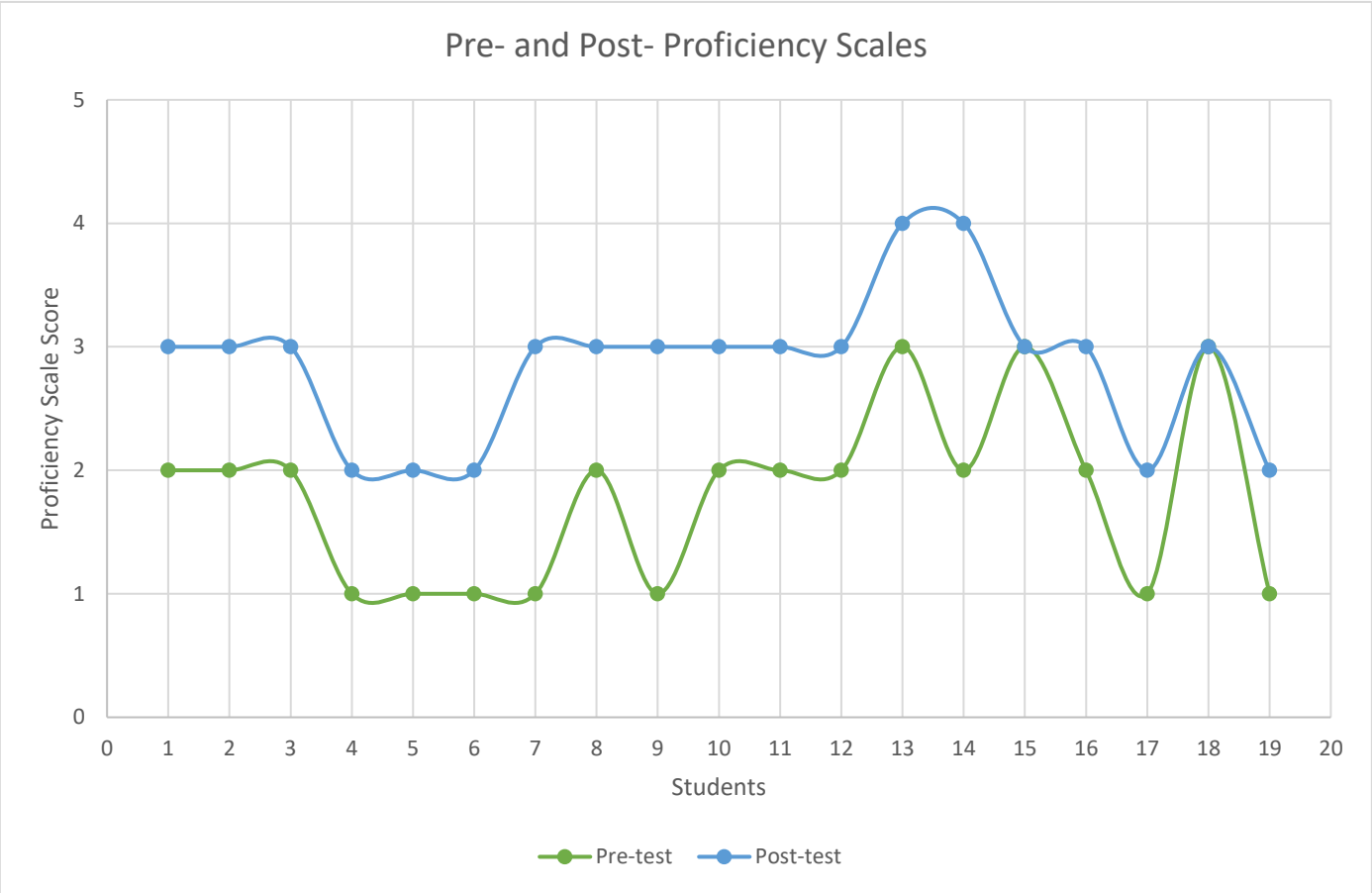

Figure 1. The graph reports the pre-test scores compared to the post-test scores for the students based on what they scored on their proficiency scale.

Students	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
1	NA	NA	NA	A	A	NS	NS	NS	NS	NA
2	NA	NA	NA	A	A	NS	NS	NS	NS	NA
3	NA	NA	NA	A	A	NS	NS	NS	NS	NA
4	NA	NA	NA	A	A	NS	NS	NS	NS	NA
5	NA	NA	NA	A	A	NS	NS	NS	NS	NA
6	NA	NA	NA	A	A	NS	NS	NS	NS	NA
7	NA	NA	NA	A	A	NS	NS	NS	NS	NA
8	NA	NA	NA	A	A	NS	NS	P	P	NA
9	NA	NA	NA	A	A	NS	NS	NS	NS	NA
10	NA	NA	NA	A	A	NS	NS	NS	NS	NA
11	NA	NA	NA	A	A	NS	NS	P	P	NA
12	NA	NA	NA	A	A	NS	P	NS	NS	NA
13	NA	NA	NA	A	A	NS	P	P	P	NA
14	NA	NA	NA	A	A	NS	NS	NS	NS	NA
15	NA	NA	NA	A	A	P	P	P	P	NA
16	NA	NA	NA	A	A	NS	P	P	P	NA
17	NA	NA	NA	A	A	NS	NS	NS	NS	NA
18	NA	NA	NA	A	A	NS	P	P	P	NA
19	NA	NA	NA	A	A	NS	NS	NS	NS	NA

CODES:
NA - NOT APPLICITABLE
A - ARRAY
P - PICTURE/DRAWING
C - CUBES
NS - NO STRATEGY SHOWN

Table 3. The table above reports the strategies each student used to achieve their results on each question of the pre-assessment.

Students	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
1	NA	NA	NA	A	A	P	P	NS	NS	NA
2	NA	NA	NA	A	A	P	P	NS	NS	NA
3	NA	NA	NA	A	A	P	P	NS	NS	NA
4	NA	NA	NA	A	A	P	P	P	P	NA
5	NA	NA	NA	A	A	C	C	A	A	NA
6	NA	NA	NA	A	A	P	P	A	A	NA
7	NA	NA	NA	A	A	P	P	P	P	NA
8	NA	NA	NA	A	A	P	P	C	C	NA
9	NA	NA	NA	A	A	P	P	P	P	NA
10	NA	NA	NA	A	A	P	P	NS	NS	NA
11	NA	NA	NA	A	A	A	P	NS	NS	NA
12	NA	NA	NA	A	A	P	P	NS	NS	NA
13	NA	NA	NA	A	A	A	A	P	P	NA
14	NA	NA	NA	A	A	P	A	NS	NS	NA
15	NA	NA	NA	A	A	NS	A	A	A	NA
16	NA	NA	NA	A	A	P	P	NS	NS	NA
17	NA	NA	NA	A	A	C	C	C	C	NA
18	NA	NA	NA	A	A	A	A	NS	NS	NA
19	NA	NA	NA	A	A	C	C	C	C	NA

CODES:
NA - NOT APPLICITABLE
A - ARRAY
P - PICTURE/DRAWING
C - CUBES
NS - NO STRATEGY SHOWN

Table 4 reports the strategies each student used to achieve their results on each question of the post-assessment.

Reflection

The data collection provided me with great insight into what the students knew and what they needed instruction on. I was able to use this information to plan for whole-group and small-group instruction. For the students that had a good understanding of the relationship between multiplication and division, I pulled small groups to focus on more advanced problems. In Figure 1, students 13, 14, and 15 scored at Level 3 on the pre-assessment. Level 3 students were pulled for more advanced instruction occasionally throughout the unit. I would use larger equations or more difficult word problems with the Level 3 students. I also integrated nonlinguistic

representations as an instructional strategy to provide the students with an efficient strategy for solving problems. I also used multi-level questioning as an assessment strategy. I asked the students to think critically about how they were solving the problem and why it was solved that way for a deeper understanding of the concepts.

For the students that struggled with certain concepts of this unit, I created small groups to focus on specific strategies that they needed extra support on. In Figure 1, students 4,5,6,7,9,17,19 scored at Level 1 on the pre-assessment. These students were pulled every day during math time for small group instruction. During these small groups, I used a variety of nonlinguistic representations to support their understanding. This instructional strategy supported the Level 1 students' learning by providing them with visuals and manipulatives to assist them in deepening their understanding of how multiplication and division work. I used multi-level questioning as an assessment strategy. I asked students to think critically about how they were solving the problems and how multiplication and division related to each other. Students 4,5,6,7,17, and 19 were pulled for an additional 20 minutes of math intervention daily, due to other factors such as special needs services or low achievement Measures of Academic Progress [MAP] scores.

The remaining students scored at Level 2 on the pre-assessment. I checked the Level 2 students' understanding throughout the unit through their assignments and through questioning utilizing math interviews. Level 2 students were pulled into small groups occasionally throughout the unit to focus on specific skills or to address any misconceptions.

Following analysis of the data, I found that integrating nonlinguistic representations and multi-level questioning strategies within the math lessons was an effective instructional strategy.

Tables 3 and 4 show that prior to instruction, 36.8% of students used a nonlinguistic strategy [picture] for solving questions 6-9. After instruction, 100% of the students used a nonlinguistic strategy [array, picture/drawing, or cubes] for solving questions 6-9. This increase reflected the increase in student growth in understanding the relationship between multiplication and division. Ninety percent of students' scores increased from the pre-assessment to the post-assessment. Before instruction of the unit, 16% of students were on target (Level 3). On the post-assessment, 74% of students were on target.

This data supports the idea that implementing differentiated instructional strategies into the lessons increased students' understanding. When I compared the pre-assessments to the post-assessments, there was a significant increase in the students' learning. After instruction, with the implemented differentiated instructional strategies, 17/19 students grew, and 14/19 students scored at Level 3 [on target] for this learning standard. From the pre-assessment to the post-assessment, there was a significant increase in the use of various nonlinguistic representations to answer the test problems.

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Appendix

1. CSA [Pre- and Post-Test]

Name: _____

Indicator(s): 3.1.2.f: Use objects, drawings, arrays, words, and symbols to explain the relationship between multiplication and division (e.g., if 3 x 4= 12 then 12 ÷ 3= 4).

Level 2

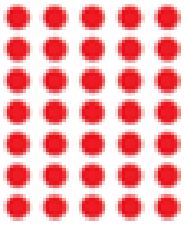
Directions: Choose the word from the box that will make each sentence true.

array	quotient	product	dividend	factor

1. The answer when two or more numbers are multiplied together is called a/an _____.
2. Items arranged in rows and columns is called a/an _____.
3. The answer after you divide one number by another number is called a/an _____.

Directions: For each problem, circle the correct expression to match the array.

4.



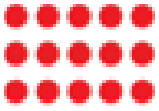
7 x 5

7 x 4

6 x 7

5 x 5

5.



15 ÷ 5

12 ÷ 3

15 ÷ 3

12 ÷ 3

10/24/2022

Level 3

Directions: Use any of the following numbers to create an equation that represents the problem. Numbers may be used more than once or not at all.

6.

Darius has 64 oranges. He places them in bags.
He puts 8 oranges in each bag.
How many bags does Darius need?

64 40 8 6 5 2

_____ ÷ _____ = _____

7.

Casey has 30 seashells.
She puts them in equal piles.
Each pile has 5 seashells.
How many piles does Casey make?

_____ piles

Directions: Write a related multiplication equation to match the division equations.

8.

18 ÷ 3 = 6

___ x ___ = ___

9.

45 ÷ 9 = 5

___ x ___ = ___

Level 4

Directions: Explain how these four representations are related.

10.

	30 ÷ 3 = 10
10 x 3 = 30	Mrs. Smith has a sheet of stickers. There are 3 rows of 10 stickers on her sheet.