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A robust technique for optimal fitting of roll-over shapes of human locomotor systems

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A B S T R A C T

The roll-over shape (ROS) effectively characterizes the lower limb's ability to roll forward during the single-limb support phase of human walking. ROS is modelled as an optimally fitted circular arc to the center of pressure (CoP) data transformed in the shank/leg-based local coordinate system. The commonly used method for optimal fitting of ROS is complex to implement and eliminates inherent individual variability in the ROS parameters during walking. We propose and validate a novel computerized method for optimal fitting of roll-over shapes of the lower limb during walking. Gait data of a healthy individual from Winter's book was used to generate ankle- foot and knee-ankle-foot roll-over shapes using the proposed method. The goodness of fit and form of both the roll-over shapes were validated with the literature. To test the robustness of the proposed technique, small random perturbations were introduced to the transformed CoP data and the effect of these small changes in the data on the ROS parameters was studied. The ROS parameters such as radius, arc length, subtended arc angle, and horizontal and vertical shift in the arc center did not change substantially with small changes in the CoP data. The proposed method is computationally efficient, and easy to implement for optimal fitting and characterization of ROS.

1. Introduction

The human ankle-foot complex demonstrates rocker-like qualities during the stance phase of walking [1], which could be effectively modelled using roll-over shape (ROS) [2, 3]. ROS characterizes the effective geometry traced by the lower limb systems during the single limb stance phase of walking. Literature shows that ROS in healthy young individuals is invariant to walking speed, shoe heel height, and additional weight carried on the torso [4–7]. Furthermore, prosthetic feet that replicate the physiological ROS result in more symmetric gait, and are highly preferred by the users [8, 9]. Hence, ROS is considered as an important parameter for the design, alignment and evaluation of lower extremity prostheses and orthoses. The ROS parameters (arc radius, arc length, position and arc angle) are functionally important from the biomechanics point of view and have physical significance when it comes to designing lower extremity assistive devices. The ROS essentially characterizes the deformation of prostheses/orthoses under the loading conditions experienced during walking. ROS arc parameters (radius and arc length) are primarily linked to the mechanical properties and the device or shoe design while the position and orientation of ROS are important for the precise alignment of lower limb prosthetic and orthotic devices. Furthermore, literature shows that increased rocker radius provides an enhanced ability to resist external perturbations implying improved stability in biped robots [10]. Similarly, roll-over radius can be used to evaluate stability in older individuals and people with locomotor disorders. Currently, the concept of ROS finds application in the design of footwear [11, 12], prosthetic feet [13–17], ana- lysing lower limb orthoses [18–20] and modeling foot-ground interaction during walking in biped robots and humans [21, 22].

ROS is generated by transforming the center of pressure (CoP) of the ground reaction force from heel contact to contralateral heel-contact into a shank/leg-based coordinate system [4]. These transformed data points are then fitted to a circular arc, which is a simple model that effectively characterizes the ROS [3]. Determining the best circular fit to a set of data points is usually formulated as a non-linear total least square fit problem and solved using the derivative-based Gauss-Newton minimization technique. A similar commonly used method of ROS generation uses a non-linear circular regression algorithm (steepest-descent method) with initial guesses of radius and arc center obtained from the quadratic Taylor series expansion of the equation of a circle [4]. However, this straightforward approach has been shown to be inefficient, and radii found in this way are sensitive to small changes in the data [3, 23] due to noise/outliers in the data, or inherent variability in each walking trial. It has been shown that these small changes in the CoP data cause large variations in roll-over parameters [3]. The optimal fitting of ROS and accurate evaluation of roll-over parameters is important for the design and alignment of prosthetic and orthotic ankle-foot systems, which may have further implications on kinematics, kinetics and metabolic cost in individuals walking with these assistive devices [2, 24, 25].

To reduce this dependency of solution parameters (such as radius, arc length and arc center) on small changes in the data, a second fitting iteration was proposed [3]. The group median radius is obtained from all the walking trials for all study subjects processed in the first iteration. Then, the second iteration of circular arc fitting is carried out using the known median radius as an additional input for each individual walking trial. Although this approach removes the natural coupling of the radius with other parameters of a circular arc and minimizes sensitivity to small changes in the CoP data, it also eliminates the inherent individual variability in the ROS parameters during walking. A few other methods for modeling ROS as a circular arc have also been used [12, 26], however, to the best of our knowledge sufficient details and robust validation of these methods have not been reported in the literature. In this work, we propose a robust, computationally efficient, and easy to implement method for the optimal fitting of ROS. The proposed method combines an alternative robust approach for circular arc fitting proposed by Coope, (1993) [23] with an unconstrained multivariable optimization technique, thus eliminating the need for a second iteration with group median radius as an additional input. This study will generate ankle-foot roll-over shape and knee-ankle-foot roll-over shape using the proposed optimal fitting technique and validate this method with the literature. Another objective is to evaluate the computational efficiency of this new technique and robustness against variability in the CoP data.

2. Material and methods

2.1. Representation of roll-over shape

The published gait data (filtered marker and the CoP data) of a healthy individual was used to test the proposed technique [27]. The ankle-foot and knee-ankle-foot ROS were obtained by transforming the CoP in the direction of forward progression (CoP_x) from the lab-based global coordinate system into shank- and leg-based local coordinate systems, respectively [4]. The ankle-foot and knee-ankle-foot ROS characterize the effective rocker that the ankle-foot and the knee-ankle-foot systems conform to during the single limb support phase of gait (i.e. from heel contact to contralateral heel contact). The coordinate systems used to define these ROS are shown in Fig. 1.

2.2. Algorithm development for optimal fitting of roll-over shape

The optimization procedure to obtain the best fit circular arc for modeling ROS was implemented using custom-written computer pro- gram in MATLAB (R2020b, Mathworks Inc, Natick, USA). The detailed operational flowchart of the proposed technique compared to the previous method in the literature is shown in Fig. 2 and the mathematical formulation is provided in Appendix A.

The optimization was carried out in a single execution of the computer program and the optimization technique is described below:

1) The first approximation for the best fit circular arc parameters (center and radius) was obtained by trivially solving the reduced order problem of linear least squares.
2) The coordinates of the circular arc center and radius obtained from the first approximation were provided as an initial guess to the MATLAB based optimization program to find the best total least squares fit solution. The inbuilt function for the unconstrained minimization of multivariable nonlinear data (‘fminunc’) was used to obtain the best fit circular arc. In case of the total least squares solution, the objective is to minimize the squared sum of the Euclidean distances between the transformed CoP data point and the corresponding point on the fitted circular arc. The total least square technique accounts for errors in both the dependent (Shape Y) and independent variables (Shape X). The objective function (f) for the total least squares fit is defined as follows:

$$f = \sum_{i=1}^{n} \left(r - \sqrt{(shape_{xi} - center_x)^2 + (shape_{yi} - center_y)^2} \right)^2 \quad (1)$$

where,

r = radius of the best fit circular arc

($center_x$, $center_y$) = coordinates of the circular arc center.

($shape_x$, $shape_y$) = coordinates of transformed CoP data in the shank/leg-based local coordinate system.

n = total number of CoP data points from heel contact to contralateral heel contact

Roll-over shape is characterized by roll-over radius (r), arc length, subtended arc angle (θ) and coordinates of the circular arc center as shown in Fig. 3. The $center_x$ and $center_y$ coordinates of the circular arc center represent horizontal and vertical shift in the ROS location relative to the origin of the coordinate system which is fixed at the ankle. The arc length was calculated as the product of roll-over radius and the angle (θ in radians) subtended by the arc. The goodness of fit of a circular arc was quantified using the coefficient of determination, also known as "R- squared (R^2)", for the optimized best fit circular arc [28].

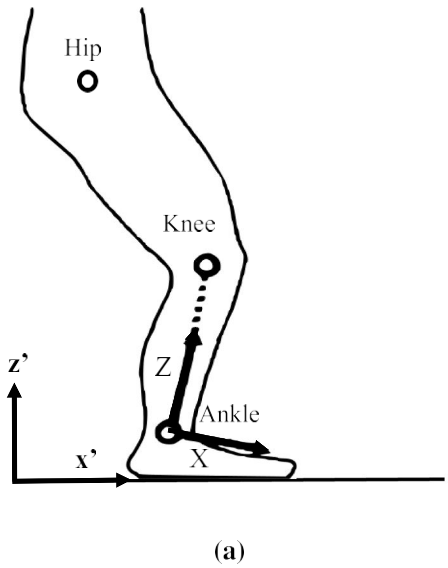
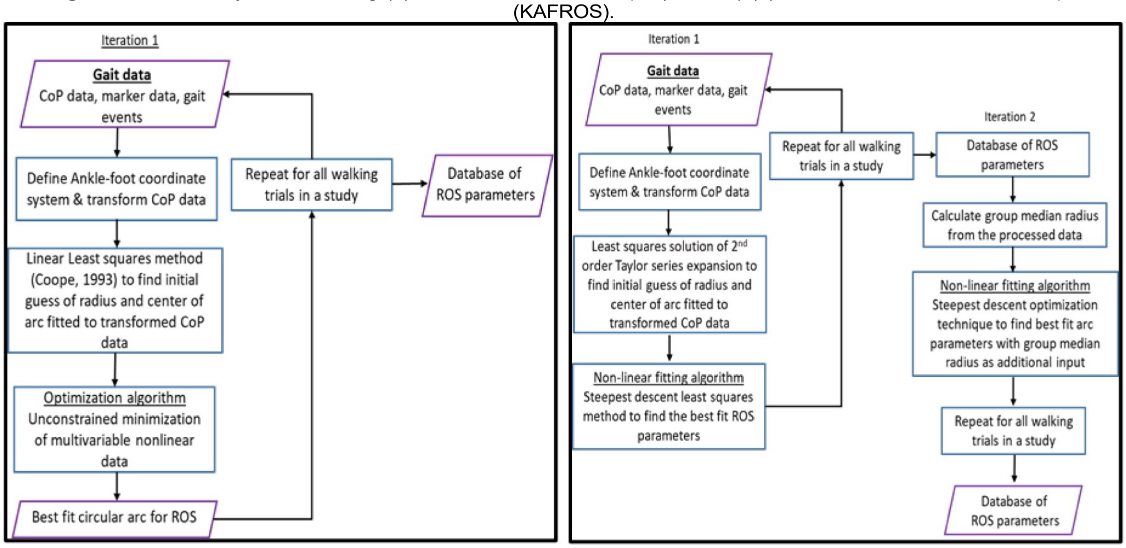


Fig. 1. Coordinate systems defining (a) Ankle-foot roll-over shape (AFROS) (b) Knee-Ankle-Foot roll-over shape (KAFROS).



(a) Operational flow chart of proposed method

(b) Operational flow chart of Hansen's method [3-4]

Fig. 2. Operational flowchart of proposed and Hansen's method used for generating roll-over shape.

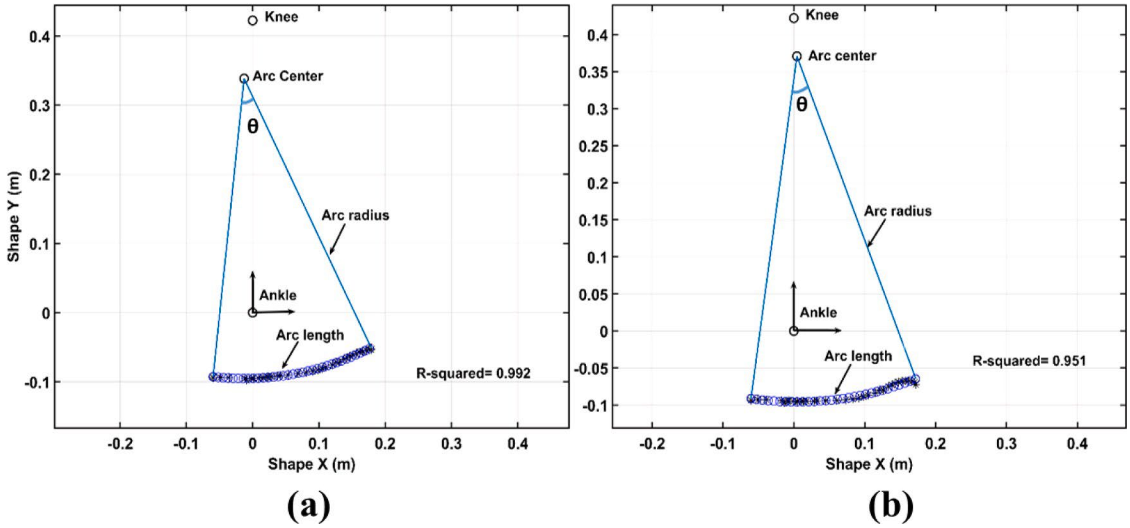


Fig. 3. (a) Ankle-foot roll-over shape (ROS) (b) Knee-ankle-foot ROS, generated using the proposed method. Typical ROS is characterized using arc radius, arc length, subtended arc angle (θ) and coordinates of the arc center. R-squared value provides the measure of goodness of fit of the circular arc to the transformed CoP data points for both shapes. Black stars indicate transformed CoP data points, blue circles represent the best fit circular arc points.

2.3. Testing the robustness and computational efficiency of the proposed method

Robustness of the proposed technique against small changes in the CoP data was tested by perturbing transformed CoP data points and evaluating the percentage change in the corresponding ROS parameters. The small change in the data was restricted to a maximum 15% change in the overall CoP data, which corresponded to a maximum of 5 points being systematically perturbed out of 35 CoP data points for the present gait data. The analysis was performed using MATLAB and ankle-foot ROS was used as an example for this case study.

Five CoP data points out of 35 total CoP data points for the gait trial under consideration, were randomly selected and perturbed. The amount of vertical shift in these data points was also randomly selected between ±10 mm. The mean ankle-foot ROS parameters for 50 such perturbed CoP data sets were evaluated and compared with the best fit circular arc parameters of the original CoP data set. The percentage change in the mean ROS parameters obtained from the perturbed data compared to ROS parameters from original data was calculated to evaluate their sensitivity to small changes in the CoP data. The per- centage change in the data was calculated using following formula:

$$\%\ Change\ in\ the\ data = \frac{(Mean\ value\ of\ perturbed\ data - Original\ data\ value)}{Original\ data\ value} \times 100$$

A lesser/similar percentage change in the ROS parameters compared to the induced 15% change in the CoP data will imply that the technique used for circular arc fitting is robust against variability in the data. The computational time for the proposed method was calculated using MATLAB (R2020b version) on a computer with the following specifications: Processor- Intel(R) Core(TM) i7–8550 U CPU @ 1.80 GHz - 1.99 GHz, Graphics card- NVIDIA GeForce MX150 2GB, RAM 16GB and storage drive 1TB + 128GB SSD.

3. Results

The best fit ankle-foot and knee-ankle-foot ROS generated using the proposed method are shown in Fig. 3. The R^2 value obtained in this study for ankle-foot ROS is 0.992 and knee-ankle-foot ROS is 0.951. The goodness of fit values between the data and the best fit circular arc for both the roll-over shapes are equivalent to those previously reported (ankle-foot ROS: R^2 = 0.99 and knee-ankle-foot ROS: R^2 = 0.95) in the literature [3].

The effect of variation in ankle-foot ROS parameters with small changes in the transformed CoP data is summarized in Table 1. Percent change for ROS parameters (arc radius, arc length, subtended arc angle, and vertical shift in the arc center) with respect to perturbation in the CoP data was minimal. However, the percent change in horizontal shift in ROS center was 11.2% with induced perturbations in the CoP data. The mean of goodness of fit value for 50 perturbed data sets was 0.974. The computational time for the proposed method was found to be 0.5119 s as shown in Table 2.

Table 1
Variation in ankle-foot roll-over shape parameters with small perturbations in the transformed CoP data. Percent change from the original data for ROS parameters with respect to perturbation in the CoP data was very minimal except for the horizontal shift in the arc center.

ROS Parameters	Original COP data set	Fifty Perturbed CoP data sets (Mean ± SD)	% Change
Radius (m)	0.4336	0.4412 ± 0.0315	1.7
Arc length (m)	0.2453	0.2452 ± 0.0004	-0.02
Subtended Angle (deg)	32.4	32 ± 2.2691	-1.2
Horizontal shift in ROS center (m)	-0.0129	-0.0144 ± 0.0072	11.3
Vertical shift in ROS center (m)	0.3383	0.3458 ± 0.0311	2.2

Table 2
Comparison of the proposed ROS fitting method with previous method (proposed by Hansen [3, 4]).

Comparison Parameters	Proposed Method	Previous Method	Comments
Number of iterations required per gait cycle	1	2	Hansen's method needs second iteration with median value of the complete data set as initial guess.
Execution time for ankle-foot ROS generation (seconds)	0.5119	>0.5119	Execution time for Hansen's method is not computed but will be more than the proposed method as it involves computation of median value and then proceeds to the 2nd iteration.

4. Discussion

Literature shows that small differences in the CoP data result in large differences in the roll-over parameters and these differences are sensitive to the curve fitting method used [3, 29]. This sensitivity necessitates the use of a robust optimal fitting method to accurately generate ROS. The goal of this work was to present a simple, computationally efficient method to generate ROS and validate its robustness against slight variations in the CoP data. Both the ankle-foot and knee-ankle foot ROS appear similar to those presented in the literature [3, 4] for the same healthy subject's published gait data [27]. Furthermore, the R^2 values for both the ankle-foot and knee-ankle-foot ROS obtained using the proposed method match those previously reported [3]. The reproducibility and similarity of ankle-foot and knee-ankle-foot ROS parameters with the literature establishes validity of this method. The ease of implementation comes from the single execution of the algorithm to obtain roll-over parameters, and from not having to calculate median parameters for each subject and trial to use in a second iteration (Fig. 2). Apart from simplicity, this method is computationally more efficient than the previously used method. The time required for calculation of ankle-foot ROS parameters for one gait cycle is 0.5119 s, which is apparently lesser than the previous method [3] as it works in single-pass and does not require a second iteration with the median radius calculated from the entire data set as an additional input.

R-squared explains the percentage variability in Shape Y that can be explained using

Shape X in the fitted circular equation. In this respect, the circular arc seems to be a good model for the ankle-foot ROS as R^2 for ankle-foot ROS was close to 0.99 for the original CoP data and it did not change substantially with perturbations in the CoP data. With small systematic perturbations in randomly selected CoP data points, the percentage change in radius, arc length, subtended arc angle, and ver- tical shift in the arc center was minimal compared to percentage change induced in the CoP data. With increased perturbations in the CoP data, the horizontal location of the arc center shifted considerably, implying sensitivity of this parameter to variations in the CoP data. However, the percent change in this parameter (i.e. 11.2%) is less than the induced 15% change in the CoP data.

The robustness of this technique lies in the first approximate solution obtained by trivially solving the reduced order problem of linear least squares [23]. The optimization method in the second step further improves this robust initial guess to provide the best fit circular arc parameters. Furthermore, the objective function (Eq. (1)) is formulated such that it minimizes the Euclidean distance between the data points and fitted circle points. This geometric fitting approach is regarded as a highly accurate method for circle fitting [29].

Roll-over shape technique is primarily used by the assistive device and shoe designers. It plays an important role in the design of footwear, prosthetic foot and alignment of lower limb prostheses and orthoses. The detailed algorithm of the proposed technique is shown in Fig. 2 and the MATLAB source codes are made openly available through supplementary materials to facilitate other researchers' use of this technique in ROS studies.

The use of single healthy subject walking data for the demonstration and validation of the proposed method is one of the limitations of this study. However, using the same data set allowed the proposed method to be validated against previously published work. Future studies need to be carried out in healthy and various pathological populations to further investigate the utility of ROS technique from the clinical point of view. To the best of our knowledge, currently there are no studies that directly link assistive devices designed using optimally fitted ROS to improved quality of life in these device users which also needs to be investigated in future studies. The circular arc fitting technique used to characterize ROS is meant for the lower arc of a circle and works well for concave-up ROS found in able-bodied people. As reported previously, this technique has some limitations when flat or concave-down ROS are encountered in pathological populations [18, 30]. In such cases, instantaneous radius of curvature approach is used [13]. Furthermore, the problem of circle fitting to noisy kinematic data to determine the hip joint center of rotation has been extensively studied in biomechanics [31, 32]. The application of the proposed circular-arc fitting technique to determine the hip joint center location can be explored in future studies.

5. Conclusions

In conclusion, the proposed technique for the optimal fitting of ROS is robust to inherent individual variability in the CoP data, computationally efficient, and easy for other researchers to implement. This method could be applied in future studies that make use of ROS to design lower limb prostheses and orthoses, footwear and to model the foot- ground interaction in biped robots and human walking.

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Ethics approval

Not required as the gait data from Winter's book is used.

Declaration of Competing Interest

The authors would like to declare out of abundant caution that ankle- foot orthoses (AFO) were donated to the University of Nebraska at Omaha by Ottobock, for another related project. No data/AFO devices from those studies were used in this work. There are no other competing or financial interests with regard to this work.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.medengphy.2022.103756.

Appendix A. mathematical formulation

The problem of determining the circle of best fit to a set of data points (say $a_j \in R^n$, $j = 1, 2, .., m$ and $n = 2$) is typically solved using nonlinear total least squares approach, which is sensitive to small changes in the data points. Hence, we propose to formulate and solve this problem using the linear least squares method, illustrated by Coope (1993) [23].

The alternative problem is formulated as:

$$\min_{(x,r)} \sum_{j=1}^{m} \left\{ f_j(x,r)^2 \right\}$$

where $f_j(x,r)$ is residual evaluated as

$$f_j\,(x,r) = ||x - a_j\ \ ||_2^2 - r^2$$

The nonlinearity in the above problem is removed by making a nonlinear transformation of variables as below:

$$f_j\,(x,r) = x^T x - 2x^T a_j + a_j^T a_j - r^2$$

$$y_i = 2x_i \ and\ i = 1, 2, \ldots, n$$

$$y_{n+1} = r^2 - x^T x$$

$$f_j\,(x,r) = y_i a_j + a_j^T a_j + y_{n+1}$$

Let $b_j = \begin{bmatrix} a_j \\ 1 \end{bmatrix}$, and j = 1, 2, ..., m which is number of data points to be fit. Then the problem can be simplified as linear least square problem as follows:

$$\min_{y \in R^{n+1}} \sum_{j=1}^{m} \left\{ a_j^T a_j - b_j^T y \right\}^2$$

Alternatively, in a more compact form

$$\min_{y} ||By - d||_2^2$$

Where, $B^T = [b_1, b_2, \ldots, b_m]\ \ and\ d_j = ||aj||_2^2\ ;\ j = 1, 2, \ldots, m$

The optimal values for coordinates of the center and radius can then be obtained as:

$$x_i = \tfrac{1}{2} y_i,\ i = 1, 2, \ldots,\ m$$

$$r = \sqrt[2]{y_{n+1} + x^T x}$$

These values of the center and radius are then used as initial guess for MATLAB based inbuilt function for the unconstrained minimization of multivariable nonlinear data ('fminunc') to obtain the best fit circular arc.

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