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Experience of a Noyce-Student Learning Assistant in an Inquiry-Based Learning Class

University of Nebraska - Omaha
Mathematics

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Outline of the Talk

How can an inquiry-based learning approach be applied to an Introductory Abstract Mathematics course?

Introduction

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Introduction

Introduction to Abstract Mathematics is an introductory proof writing course that provides a foundation in logic, set theory, and number systems.

The NLA assisted the CI with all aspects of the course including:

* material preparation
* class organization
* peer/teamwork facilitation
* class instruction
* presentation assistance and assessment
* individual tutoring

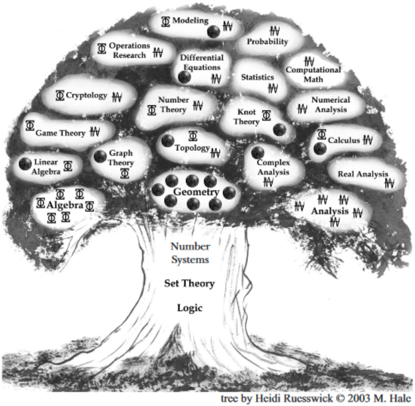


Figure: Tree of Mathematics [4]

Introduction

In conjunction with learning the techniques of proof, the IBL section of the course focuses on the process of *doing* mathematics rather than simply studying mathematics.

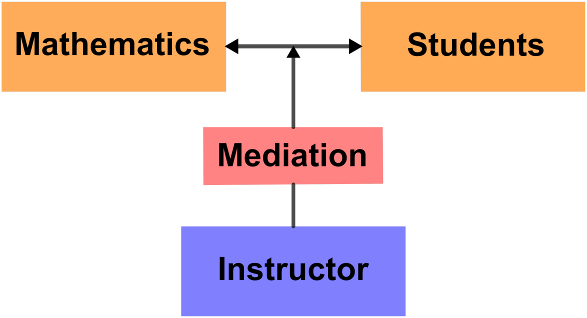


Figure: General IBL Structure

Materials - Course Access and Provided Materials

- **Canvas** - Learning Management System [5]
- **Syllabus** - adapted from Dana C. Ernst, Northern Arizona University [3]
- **Additional textbook**: *Sets, Functions, and Logic- An Introduction to Abstract Mathematics*, by Keith Devlin [6]
- **Portfolio Information**
- **Course notes** - adapted from Dana C. Ernst, Northern Arizona University [2]; supplemented by the CI [7]

  (includes proof with: number theory, set theory, functions, and relations)
- **Solutions manual** - CI [7]; supplemented by NLA

Methods - Structure & Tasks

- Presentations are the main method for instruction
- Students must absorb content prior to each class
- Presentation assignments are randomized

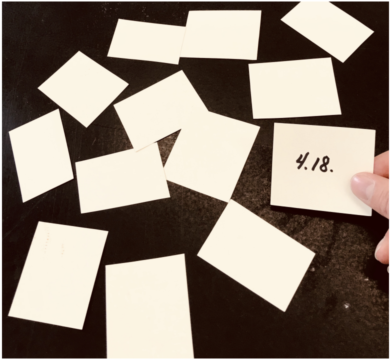
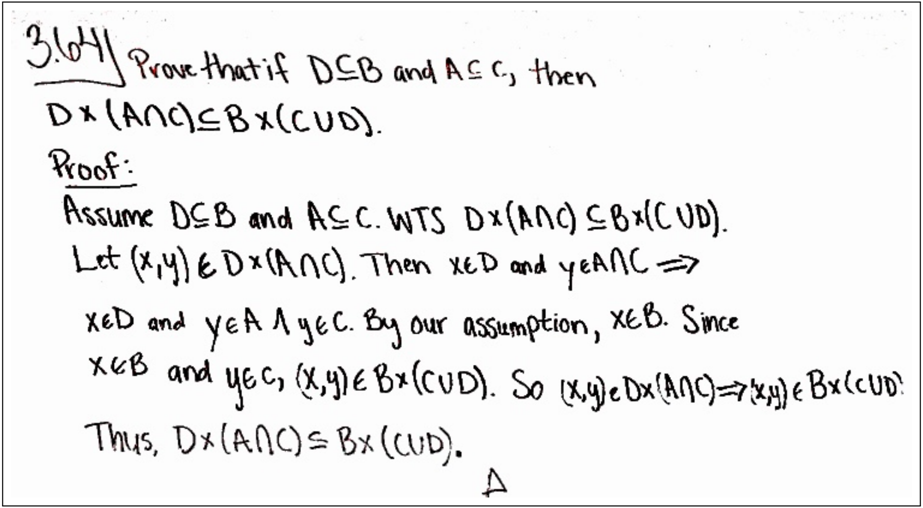


Figure: Exercise Cards



3.64 Prove that if $D \subseteq B$ and $A \subseteq C$, then $D \times (A \cap C) \subseteq B \times (C \cup D)$.

Proof:

Assume $D \subseteq B$ and $A \subseteq C$. WTS $D \times (A \cap C) \subseteq B \times (C \cup D)$.

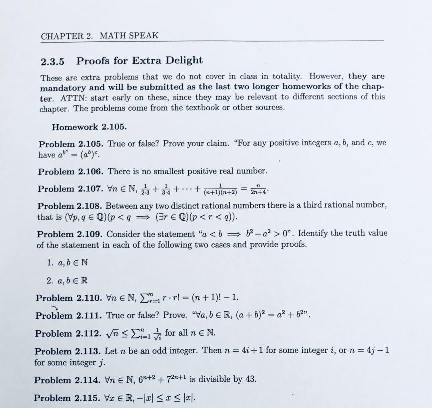
Let $(x,y) \in D \times (A \cap C)$. Then $x \in D$ and $y \in A \cap C \Rightarrow$ $x \in D$ and $y \in A \wedge y \in C$. By our assumption, $x \in B$. Since $x \in B$ and $y \in C$, $(x,y) \in B \times (C \cup D)$. So $(x,y) \in D \times (A \cap C) \Rightarrow (x,y) \in B \times (C \cup D)$.

Thus, $D \times (A \cap C) \subseteq B \times (C \cup D)$.

$\triangle$

Figure: Sample of Student Work

Methods - Structure & Tasks



CHAPTER 2. MATH SPEAK

2.3.5 Proofs for Extra Delight

These are extra problems that we do not cover in class in totality. However, **they are mandatory and will be submitted as the last two longer homeworks of the chapter.** ATTN: start early on these, since they may be relevant to different sections of this chapter. The problems come from the textbook or other sources.

Homework 2.105.

Problem 2.105. True or false? Prove your claim. "For any positive integers a, b, and c, we have $a^{b^c} = (a^b)^c$."

Problem 2.106. There is no smallest positive real number.

Problem 2.107. $\forall n \in \mathbb{N}$, $\frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{(n+1)(n+2)} = \frac{n}{2n+4}$.

Problem 2.108. Between any two distinct rational numbers there is a third rational number, that is $(\forall p, q \in \mathbb{Q})(p < q \implies (\exists r \in \mathbb{Q})(p < r < q))$.

Problem 2.109. Consider the statement "$a < b \implies b^2 - a^2 > 0$". Identify the truth value of the statement in each of the following two cases and provide proofs.

1. $a, b \in \mathbb{N}$
2. $a, b \in \mathbb{R}$

Problem 2.110. $\forall n \in \mathbb{N}$, $\sum_{r=1}^{n} r \cdot r! = (n+1)! - 1$.

Problem 2.111. True or false? Prove. "$\forall a, b \in \mathbb{R}$, $(a+b)^2 = a^2 + b^2$".

Problem 2.112. $\sqrt{n} \leq \sum_{i=1}^{n} \frac{1}{\sqrt{i}}$ for all $n \in \mathbb{N}$.

Problem 2.113. Let n be an odd integer. Then $n = 4i+1$ for some integer i, or $n = 4j-1$ for some integer j.

Problem 2.114. $\forall n \in \mathbb{N}$, $6^{n+2} + 7^{2n+1}$ is divisible by 43.

Problem 2.115. $\forall x \in \mathbb{R}$, $-|x| \leq x \leq |x|$.

Figure: Supplementary Section [7]

- Pace of presentation delivery determines homework
- CI and NLA deliver presentations on chosen exercises
- NLA generated additional review exercises found at the end of each chapter
- NLA supervised these supplementary sections independent from the CI

Portfolio

- Purpose is to provide an individualized collection of the content
- Portfolio is comprised of course notes and all student work
- Student progress on portfolios is checked periodically
- The final portfolio is due on the day of the final exam
- Students are encouraged to use portfolios in future courses

Methods - Assessment & Exams

Student assessment is formative and summative

Formative

- Presentations (25%)
- Homework (25%)

Summative

- Portfolio (10%)
- Midterm exam (20%)
- Final exam (20%)

Practice exams and reviews occur one week prior to exam dates

The midterm and final are broken down as follows:

Take-home Exam

- Tests ability to independently learn a concept not discussed in class (8%)

In-class Exam

- Assesses retention of concepts and the ability to implement skills discussed in class (12%)

Results

- Students partake in a traditional, and optional, course evaluation from the University
- Additionally, students participate in a tailored questionnaire, also optional, regarding their IBL experiences
  - Composed of several multiple choice and free-response questions
  - Results are based on 56 responses over four semesters from Fall 2015 through Spring 2018

Results

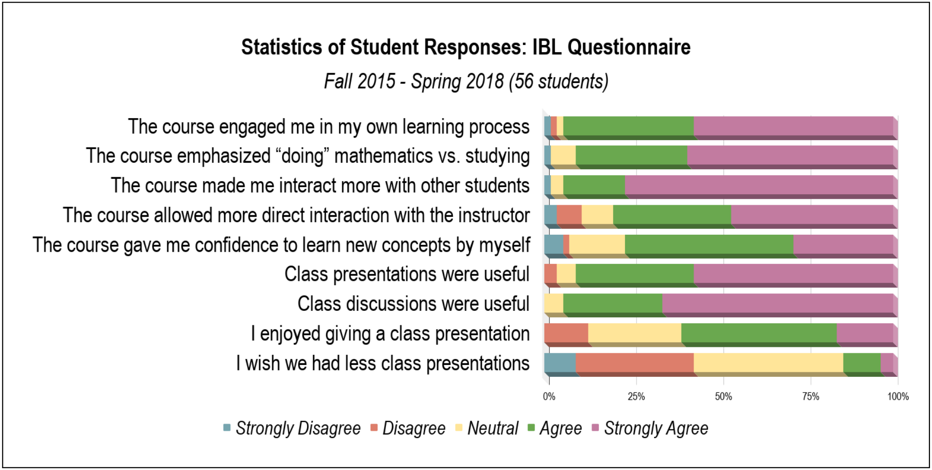


Figure: Student IBL Questionnaire Responses : Questions 1-9

Results

Selected Free-Responses Questions: 18-32

- *How, if at all, did this course make you a more independent learner/thinker?*

  “Working examples before going over any material in class by having only the text and course notes helped make me more independent. Sometimes it was great, other times it was quite frustrating.”

- *Would you recommend this IBL course to other students? Why or why not.*

  “Yes, more engaging classtime, better grasp of concepts.”

- *What are the positive aspects of the IBL experience in this course?*

  “IBL really teaches you to think hard on something and there is definitely that sense of pride from learning something new on your own.”

Discussion

Suggestions prompted from CI's and NLA's experiences as well as frequent proposals from students:

1. Offer a tentative homework list
2. Increase in-class and out-of-class teamwork via the discussion board and peer-review feature of Canvas [5]
3. Require students to check proofs/solutions prior to presentations
4. Limit the amount of time for open-class discussions during presentations
5. Ensure all students present an equal number of times and that there is a fair distribution of difficulty among problems

Introduction

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