TIME EVOLUTION IS A SOURCE OF BIAS IN THE WOLF ALGORITHM FOR LARGEST LYAPUNOV EXPONENTS

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INTRODUCTION

Human movement is inherently variable by nature. However, the term *variability* is often interpreted in various ways in movement science. Here, we recognize variability as the fluctuations in repeated processes. Observing variability through this lens reveals that natural processes and human movement alike share a rich complexity. One of the most common analytical tools for assessing movement complexity is the largest Lyapunov exponent (LyE) which quantifies the rate of trajectory divergence or convergence in an *n*-dimensional state space. Previous studies have demonstrated differences in LyE in kinematic signals during walking between healthy controls and ACL-deficient patients, peripheral artery disease patients, unilateral transtibial amputees and more[1].

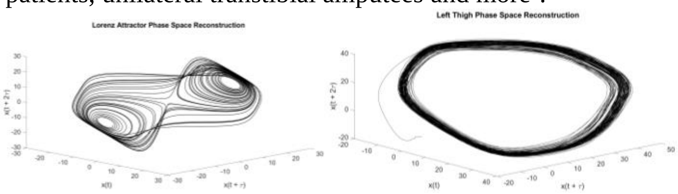

Figure 1. Phase space reconstruction of (left) simulated data (i.e., Lorenz attractor) and (right) experimental data (i.e., left thigh pitch).

One popular method for assessing LyE is the Wolf algorithm[2]. Many studies have investigated how Wolf's calculation of the LyE changes due to sampling frequency, filtering, data normalization, and stride normalization. However, a surprisingly understudied parameter needed for LyE computation is evolution time. Evolution time is the number of sample intervals by which each pair of neighboring points is followed before a new neighboring pair is chosen. For instance, chaotic systems require an evolution time that is not too large for convergence to theoretical values. The purpose of this study is to investigate how the LyE changes as a function of evolution time in both simulated data and experimental data (Figure 1).

METHODS

Subjects. The data from 36 healthy subjects (M/F = 18/18, age = 24.6 ± 2.66 yrs, BW = 71.9 ± 7.99 kg, HT = 1.73 ± 0.08 m) were extracted from an ongoing study, investigating whether individuals possess a unique, self-identifying, gait.

Experimental Procedure. Kinematic data during self-paced overground walking was collected on a ~200m indoor track. The data was captured at 200 Hz using Noraxon Ultium Motion™ inertial measurement unit (IMU) sensors (Noraxon, Inc., Scottsdale, AZ). Sensors were placed on the head, upper thoracic, lower thoracic, pelvis, upper arm, forearm, hand, thigh, shank, and foot. The subjects performed nine four-minute overground walking trials at their self-selected walking speed. The trials were spread over three blocks of three trials each. The subjects then completed the same experimental procedure one week later, totaling two sessions overall.

Data Analysis. Segment pitch angles from the left and right thigh, shank, and foot were extracted from the first 2-minutes of each trial to calculate LyEs. Simulated data consisted of values from a reconstructed Lorenz attractor sampled at 50 Hz. All experimental data was also downsampled to 50 Hz due to computational requirements. The time delay, τ, and embedding dimension, *m*, were found using Average Mutual Information (AMI) and False Nearest Neighbor (FNN) functions, respectively. Evolution time was calculated by multiplying a fixed constant by the sampling rate and applying the ceiling function (ceil()) in MATLAB to round up to the nearest whole integer. For simulated data, evolution time was systematically increased by increasing the fixed constant from 0.05 to 5 by .01, totaling 496 evolution time values. For experimental data, the same procedure occurred but the fixed constant increased from 0.05 to 1.85 by .2 due to computational constraints. Wolf's LyE algorithm was then run on the data with each evolution time (Figure 2).

RESULTS

Multi-level and linear models were used to assess whether the inclusion of fixed effects of evolution time improved prediction of LyE over and above an intercept only model for experimental and simulated data, respectively. *Exp. Data:* A fixed effect of evolution time improved the model fit when predicting LyE (p= <0.0001), with LyE values decreasing 0.43 SDs for every 1 second increase in evolution time, t = -14.7, p = <.0001. *Sim. Data:* LyE values decreased 2.0 SDs for every 1 second increase in the evolution time, t = -24.6, p = <.0001.

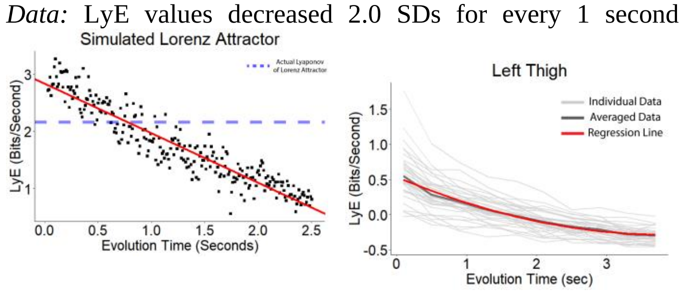

Figure 2. LyE values as a function of evolution time during (left) simulated data and (right) experimental data.

DISCUSSION

Increasing evolution time in Wolf's algorithm substantially negatively biases LyE values for simulated and experimental data. Overall, careful consideration should be taken when choosing the evolution time. An evolution time of 1.5 seconds produced the closest values to the expected simulated value. Therefore, future research should consider using a similar time for experimental data.

REFERENCES

1. Raffalt et al, *Ann Biomed Eng*. 2019;47(4):913-923.
2. Wolf et al, *Phys Nonlinear Phenom*. 1985;285-317

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