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Noise reduction in time series data from dynamical systems.

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NOISE REDUCTION IN TIME SERIES DATA FROM DYNAMICAL SYSTEMS

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NOISE REDUCTION IN TIME SERIES DATA FROM DYNAMICAL SYSTEMS

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The objective of this thesis is to determine if the amount of noise in the observed time series data affected the outcome of the chaotic descriptors. To thoroughly analyze this problem I explain how the data was obtained, find the chaotic descriptors, and then discuss and apply different noise reduction techniques. The data studied in this paper were collected from 10 young and old females while walking on a treadmill. The data contain x and y coordinates for six different parts of the body.

The first step in this project was to do an initial chaotic analysis of the data, which created a baseline to determine progress. I placed the majority of my focus on the maximal Lyapunov exponent, which is an indicator of the amount of divergence in the system. Once the maximal Lyapunov exponent was found for each data set, I compared the old and young populations and determined if there was a significant difference between the two.

I then focus on the reduction of noise in each system. Noise is defined, and I address the two distinct types, *additive measurement noise* and *dynamical noise*. I utilize several techniques to eliminate both types of noise from the data. This enhances the accuracy of the comparisons made between the two age categories. After each noise reduction I conduct a chaotic analysis and compare

both populations with the original analysis.

Finally, the successfully noise reduced data is compiled. I find statistically significant differences between the old and young data in the metatarsal (x-axis), heel (x-axis), and iliac crest (y-axis). I also recommend using more subjects and taking measurements from the same side of the body in future research.

CONTENTS

CHAPTER ONE – INTRODUCTION

The objective of this thesis is to determine if the amount of noise in the observed time series data has affected the outcome of the chaotic descriptors. To thoroughly analyze this problem I will first introduce how the data was obtained, next how to find the chaotic descriptors, and then discuss and apply noise reduction techniques.

The data studied in this paper were collected from 10 young females (mean age: 25.10, SD: 5.30 yr; mean height: 1.70, SD: 0.049 m; mean mass: 63.93, SD 6.53 kg) and 10 old females (mean age: 74.6, SD: 2.55 yr; mean height: 1.59, SD: 0.053 m; mean mass: 64.07, SD 9.69 kg) while walking on a treadmill. Both the old and young subjects were healthy with no restrictions in daily living activities. The data contains x and y coordinates for six different parts of the body (iliac crest, hip, knee, ankle, heel and metatarsal). The elderly and young subjects were measured from different sides of the body, so graphs of the different age groups will have the appearance of being transposed. The elderly data were obtained from the right side of the body and the young were from the left. The data was captured at a rate of 60 data points per second, or at a rate of 60 hertz, and was measured in centimeters. The amount of data that were collected ranges from 2145 to 2913 data points or in other words about 36 to 49 seconds worth of data. This is a sufficient amount of data to estimate the chaotic descriptors of the underlying system (Rosenstein, Collins, & De Luca, 2003).

The first step in this undertaking is to do an initial chaotic analysis of the data.

This will create a baseline that we can look back on and determine the progress that has been made. I will put the majority of my focus on the maximal Lyapunov exponent, which is an indicator of the amount of divergence in the system. This has been done in the past (Buzzi, Stergiou, Kurz, Hageman, & Heidel, 2003) for this data, however, results can vary some-what from one algorithm or tool to another. Therefore, I will conduct another analysis to have a consistent method of measurement both before and after the noise reduction. Once the maximal Lyapunov exponent has been found for each data set I will statistically compare the old and young populations and determine if there is a significant difference between the two. All this will be accomplished in chapter two.

Chapters three and four will focus on reducing the noise in each system. To begin, the term noise should be defined. I will define it as:

> *Any alteration, however small, in the perception or representation of the true state of an object or system.*

Noise can be classified into two distinct types, the first of which is additive measurement noise, which I shall refer to in this paper as ADM noise. This type of noise is not part of the dynamical system. It is common to all collected data and is caused in part by the limits of instrument measurement accuracy. Since chaotic systems are very sensitive to initial conditions, the slightest truncation in precision would misrepresent the system. ADM noise is also introduced into the data by movements of the dynamical system and the measuring devise in reference to each other while the data is being collected. Some authors distinguish between different types of ADM noise. However, for convenience, I will not make any differentiation between the sub-categories of ADM noise.

The other main noise categorization is Dynamical Noise. In the case of this particular data, there are many types of dynamical noise that could possibly skew the theoretical system, but they are in reality an integral part of the actual system. They are not introduced as a part of the measurement, and would still be observed even with a flawless measuring system. An example of this type of noise occurs when learning a task. In the beginning we are very noisy (variable) and it is difficult to successfully accomplish the specific task. Later, after more practice, we can decrease this noise and we are more consistent with the performed task. Due to aging, the nervous system deteriorates and the ability to properly organize the available degrees of freedom may decrease. Therefore, the execution of a specific task (in this case walking) may actually become more difficult and less precise movements can enter the system. Regarding the data that are used in this thesis, it is possible that the elderly data are more contaminated with this dynamical noise. Another example of dynamic noise could occur if the test subject varied her pace. In a perfect test environment this would not happen; the subject would keep a precise pace to display the theoretical system. However, no test subject could possibly keep a perfect cadence. Therefore, the varying of pace is part of the actual system, although it complicates the underlying attractor.

I will utilize several techniques to eliminate both types of noise from the data. This will enhance any comparisons made between the two age categories. After each noise reduction I will conduct a chaotic analysis and compare both populations with the original analysis.

In the final chapter I will make some conclusions about the success of the noise reductions that have been made. I will also comment on any interesting differences between the two age groups and make a few recommendations for future research.

CHAPTER TWO – CHAOTIC ANALYSIS OF THE DATA

There are a variety of things that need to be done in order to determine the chaotic descriptors of the data. The main descriptor I will focus on is the largest or maximal Lyapunov exponent, which I will refer to hereafter as the LyE. To find the LyE in the data I will use algorithms and subroutines developed primarily from Hegger, Kantz, and Schreiber's *Practical implementation of nonlinear time series methods: The TISEAN package* (Hegger, Kantz, & Schreiber, 1999). I will also use Rosenstein, Collins, and De Luca's *A practical method for calculating largest Lyapunov exponents from small data sets* (Rosenstein et al., 1993).

The Lyapunov exponent is a measure of divergence in a system. If the system is diverging exponentially fast then the system is chaotic. There is a Lyapunov exponent for every dimension in phase space of a system. The only Lyapunov exponent that we take interest in for this paper is the maximal Lyapunov exponent. If the LyE is greater than zero then the system is diverging exponentially fast, at least in that dimension, and the entire system is considered chaotic or dynamical (Alligood, Sauer, & Yorke, 1996).

An important aspect one should consider before attempting to find the LyE in any data set is to ensure that the data being studied are nonlinear and can not be explained by a linear model (Kantz & Schreiber, 2004, p. 109). It is conceivable to come up with a linear data set that has a LyE > 0, which is a major characteristic of nonlinear dynamics, but is not chaotic. A common test for nonlinearity is the method of surrogate data. A detailed description of how this can be done is found in A. Galka's *Topics in Nonlinear Time Series Analysis* and in Schreiber's *Interdisciplinary application of nonlinear time series methods* (1999, chap 5). I will not attempt to show that this data is nonlinear since it has already been accomplished by Buzzi, Stergiou, Kurz, Hageman, & Heidel (2003). With the knowledge that the data is nonlinear we can begin to study it.

Because of the amount of data and the number of times I will be calculating the LyE for each of the data sets (after each type of noise reduction) I found it necessary to automate the process of calculating the LyE. The primary routine I have taken and adapted for this calculation is found in Rosenstein et al. (1993). This routine is designed for small noisy data sets and will be optimal for the type of data in this study. The noteworthy inputs needed for this algorithm are:

J = time delay in samples (or time lag)

W = Theiler window (window size skipping temporally close nearest neighbors)

m = embedding dimension

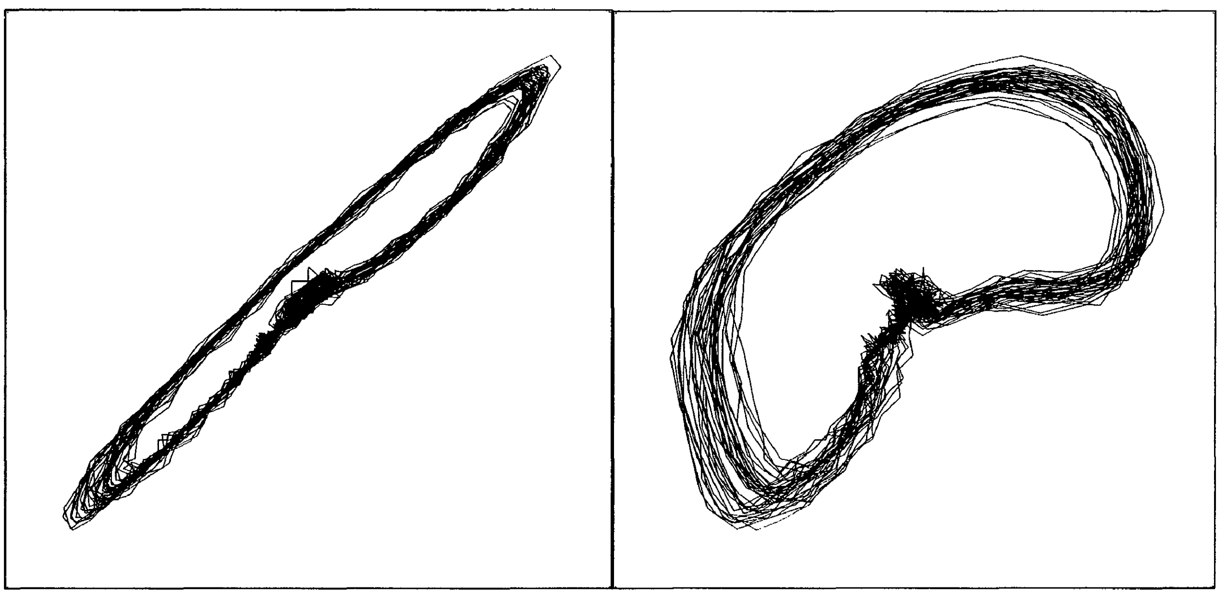
DT = total divergence time in samples

I will discuss finding each of these parameters individually in the order they are listed above.

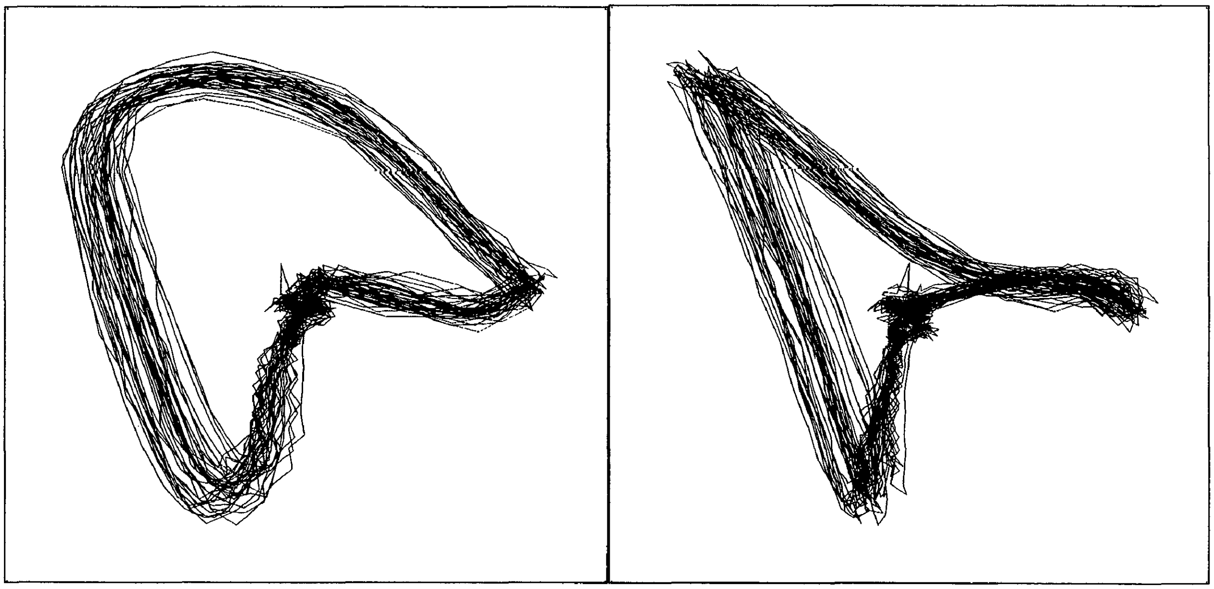
<u>Finding the delay</u>

The time delay or time lag is an integer with units in samples. It is the number of time steps between vectors in the delay reconstruction. There are a few methods to find J; I have utilized two of these methods in conjunction with each other for increased accuracy throughout a variety of data. The first method is to produce the autocorrelation function of the data set and find where the function decays to 0. Choose J as the time step when the function is 0 or less. A description of more detail can be found in Kantz and Schreiber (2004, p. 38). The other method uses the time delay mutual information function. When this function reaches its first minimum, this time step is chosen as the delay (p. 151). For the calculations of both of these functions, I have utilized the TISEAN package routines. Once I calculate both delays from the above methods, I take the smaller of the two as the delay.

An example of choosing the time delay is below:



The above pictures are young subject #5's y-axis knee data. The one on the left is with delay of 1 (i.e. the x-axis is the y(t) and the y-axis is $y(t+J)$), the one on the right is with a delay of 5. The other two below are from the same data, with the one on the left with J = 9 and the one on the right with J = 13.



One way to choose the best delay is to see which delay would best unfold the graph. The optimal delay above looks to be somewhere from J=5 to J=9. This is because it is unfolded to the best extent somewhere between these two delays. The method described above chose the delay for this data to be 9, so it worked fairly well.

Finding the Theiler Window

The next parameter that needs to be estimated is the Theiler window (also known as the Theiler correction). With sampled data from a system, there exists a strong possibility that close data points could skew or bias the analysis too heavily on a single trajectory. This window sets a threshold to eliminate this "trajectory bias." According to Galka, W is not a very critical parameter as long as it is not too small (2000, p.107). Hegger, Kantz, and Schreiber also declare that "typically not much statistics is lost, as long as it remains smaller than about 10% of the data set size" (2000). Using these recommendations I have chosen W to be fixed at 10% of the total number of data points.

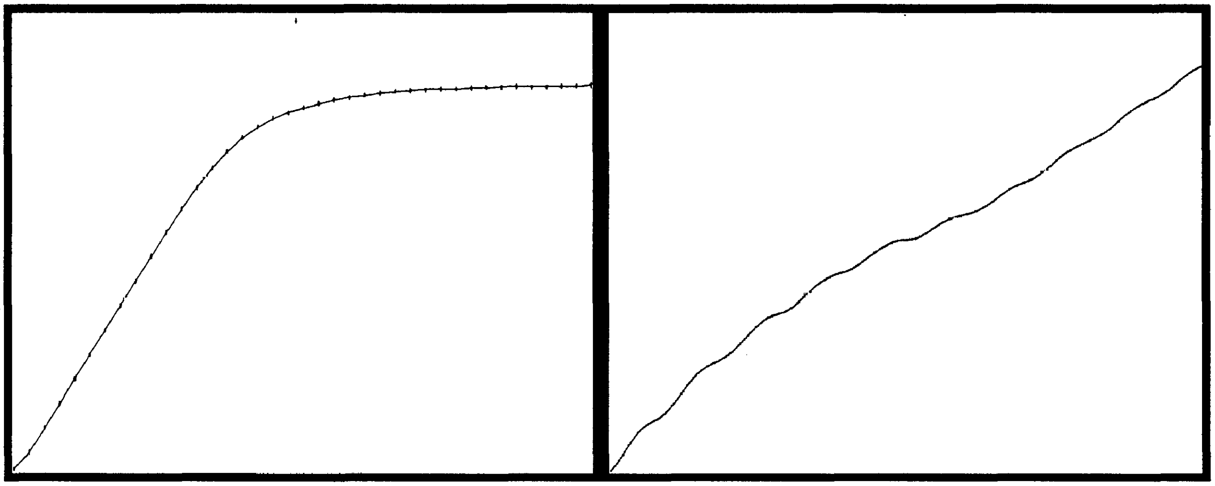
Finding the Embedding Dimension

After choosing a delay and Theiler window the embedding dimension can be calculated. The embedding dimension (*m*) describes the smallest dimension where the attractor of the dynamical system can be unfolded so no artificial crossings of the orbit exist. This can be done by calculating the percentage of false nearest neighbors (FNN) in the system for each dimension (Abarbanel, 1996, chap. 4).

"False Nearest Neighbors" can be defined as two points that are closest to each other in the data set, but on two quickly diverging trajectories. Therefore, starting out in a low dimension the number of FNNs should be high, but as the dimension increases the number of FNNs should fall. Once the percentage of FNNs for a particular dimension falls below a set threshold, it is reasonable to assume the system could be unfolded in that dimension. The threshold where the percentage of FNNs falls to an acceptable level can vary somewhat, but I chose 0.000005 to ensure a high confidence in the embedding dimension. To estimate *m* I used the routine false_nearest from the TISEAN package.

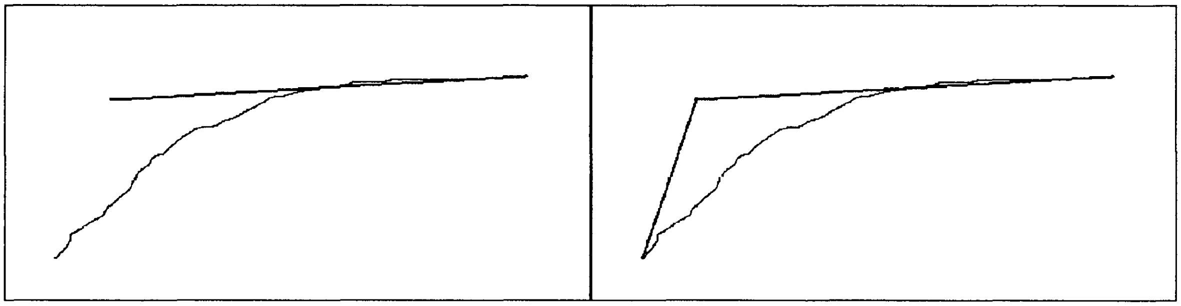
Finding the Divergence Time

The final parameter necessary for the calculation of the LyE is the divergence time (*DT*). This is a measure of how many time steps it takes for two nearest neighbors on average to reach maximum divergence (Rosenstein, 2001). This is done by observing the divergence data until it reaches a plateau (M. T. Rosenstein, personal communication, January 23, 2002). For some data this is very simple, but for others quite complicated.

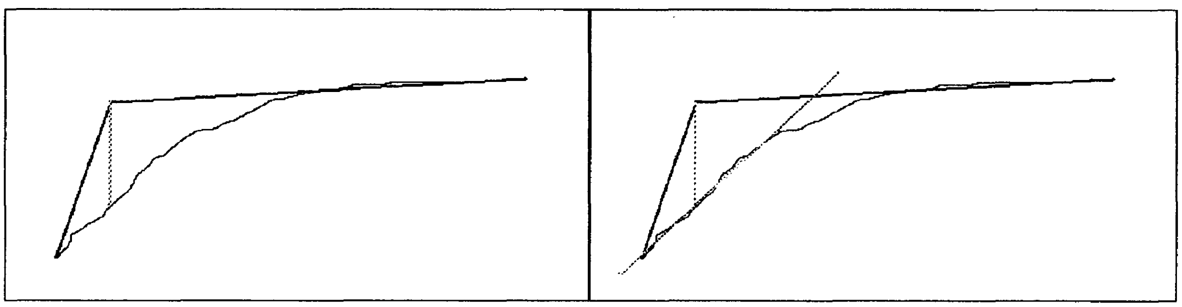


The above plots (not to scale) show an example of two different divergence plots at 40 and 400 time steps respectively. The left is the Henon map data (a=1.4, b=0.3) and the right is the Lorenz data (r=28, b=8/3, sigma=10). It is very clear where the Henon divergence plot plateaus and easy to pick the value for *DT*. However, for the Lorenz divergence plot it is very unclear where it plateaus, and unclear which value to choose for *DT*. It is even more difficult automating the process and getting accurate LyEs.

The process I chose to pick the *DT* is as follows. First calculate the divergence data for each time step. Once that is calculated I find the slope of the best fit line from 1 to n for each n. I then find the first maximum in the best fit slopes and plot that line. Then I estimate the slope of the plateau of that divergence data. I do this by taking the best fit line from the second quartile of the data (i.e. from n/4 to n/2). Then I find the x coordinate of the intersection of the 2 lines. This x coordinate is the time step I will choose for *DT*. See the figures below for an illustration.



The figure on the left shows the best fit line of the second quarter of the divergence data (only half is shown). The picture on the right adds in the first maximum slope line. The picture below on the left adds in the x-intercept chosen as *DT*. Then the picture below on the right shows the best fit line from 0 to *DT*. The slope of the best fit line from 0 to *DT* is the LyE.



Now that the above parameters have been selected we can proceed with running the routine. In the routine the divergence plot is calculated and a cut-off time is already chosen. With that in hand the slope of the best-fit line is calculated from time 0 to *DT*, which is an estimate of the LyE. To ensure the accuracy of the entire process I ran it for both the Henon map and the Lorenz equations. The Lorenz data was calculated in 1/100 of second time steps which cause the LyE to be reduced by a factor of 0.01. The following are the actual results compared to the expected results.

Data	J	m	DT	W	LyE	Exact LyE	Difference
Henon – x-axis	1	2	16	550	0.381	0.422	9.72%
Henon – y-axis	1	2	16	550	0.381	0.422	9.72%
Lorenz – x-axis	16	3	109	300	0.00932	0.00906	-2.87%
Lorenz – y-axis	16	3	90	300	0.01027	0.00906	-13.36%
Lorenz – z-axis	16	3	136	300	0.00806	0.00906	11.04%

Looking at the following results, the percentages off from the true LyE range from -13% to 11%. These LyEs are not perfect by any stretch of the imagination but should produce satisfactory results for the purpose of this paper.

Now that the method to estimate the LyE has been developed, I can begin to apply it to the walking data. It is important to note that since the subjects were on a treadmill and had freedom to move back and forth in the x-axis, the data in that axis will be very noisy before filtering. As mentioned earlier the data was taken at 60 hertz. Therefore to get the true LyE it would need to be multiplied by 60 to be scaled properly in seconds. Since the actual numbers are not the real concern of this paper I will leave the LyE scaled at 1/60 seconds for simplicity. Also, because of the amount of data I will be figuring averages and standard deviation (SD) of the different types of data and put the raw numbers in Appendix A to keep the paper from being cluttered with profuse amounts of data.

Calculating the LyE along with the other parameters, averaging the 10 subjects in each of the categories (BP = Body Part, a = ankle, h = heel, i = iliac crest, k = knee, m

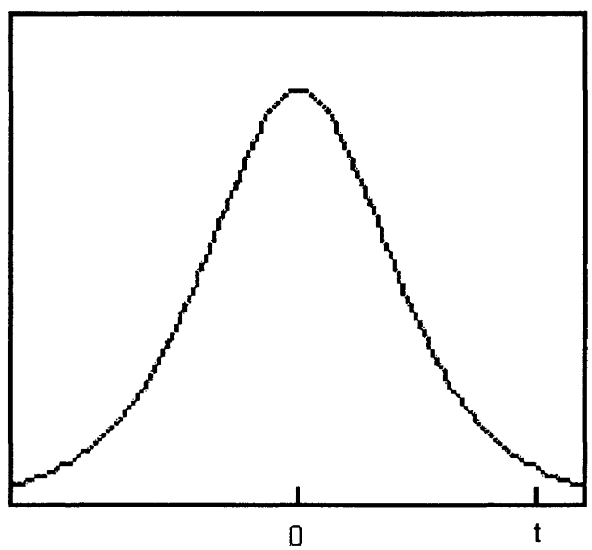
= metatarsal, p = hip, AX = the x or y axis, and AG = the age group old or young) and comparing the age groups I get the following results:

BP	AX	AG	LyE-Ave	LyE-SD	m-Ave	m-SD	J-Ave	J-SD	DT Ave	DT-SD	t-val	p-val
a	x	y	0.06531	0.03653	3.5	0.70711	8.4	6.7198	29.9	9.6891		
a	x	o	0.06572	0.03072	4.1	0.8756	7.6	5.6999	29.4	11.296	-0.025	0.981
a	y	y	0.02107	0.00837	4.7	0.48305	13.5	1.5092	23.8	7.9275		
a	y	o	0.02429	0.00671	4.8	0.42164	13.8	1.3166	20.7	6.237	-0.901	0.391
h	x	y	0.07972	0.02333	3.9	0.73786	4.3	3.8887	27.6	7.5748		
h	x	o	0.0832	0.02565	4.8	0.42164	3.6	1.3499	27.4	6.867	-0.301	0.770
h	y	y	0.02389	0.01051	4.8	0.42164	13.6	0.6992	23.4	7.975		
h	y	o	0.03036	0.02812	4.9	0.31623	13.4	1.075	25.4	7.975	-0.647	0.534
i	x	y	0.04933	0.03226	4.1	0.31623	14.6	1.8974	16.8	7.9415		
i	x	o	0.04165	0.02658	4.4	0.5164	17.1	1.3703	15.3	6.4127	0.551	0.595
i	y	y	0.02297	0.0051	4.6	0.5164	9.8	1.2293	25.9	8.0616		
i	y	o	0.03114	0.00978	4.6	0.5164	10.2	1.7512	18.1	7.1095	-2.222	0.053
k	x	y	0.04747	0.02475	3.1	0.31623	16.7	1.2517	17.7	5.9076		
k	x	o	0.03815	0.02279	3.6	0.69921	17.5	0.8498	19.7	6.3254	0.831	0.427
k	y	y	0.024	0.0085	4.9	0.31623	9.8	1.3984	26.1	11.855		
k	y	o	0.03743	0.03135	4.9	0.31623	10	2.2111	26	15.818	-1.24	0.246
m	x	y	0.08927	0.01964	3.8	0.63246	3.2	1.4757	25.5	5.4416		
m	x	o	0.0878	0.0242	4.8	0.42164	3	1.2472	27.7	7.8464	0.142	0.890
m	y	y	0.03102	0.01422	5	0	7.9	1.5951	24.6	11.587		
m	y	o	0.03759	0.03423	5	0	11.1	1.5239	23.4	11.73	-0.531	0.608
p	x	y	0.0623	0.04505	4	0.4714	14.2	2.2998	14	6.549		
p	x	o	0.04078	0.02725	4.4	0.5164	16.9	1.2867	15.9	5.087	1.226	0.251
p	y	y	0.02959	0.01224	4.3	0.48305	9.9	1.7288	24	10.328		
p	y	o	0.03025	0.00677	4.4	0.5164	9.8	0.7888	23	12.074	-0.141	0.891

Statistically, when comparing two populations (in this case the LyEs of older and younger females) we initially assume the averages are the same. Once two samples are selected from each of the populations and compared together we can calculate a probability that these samples are from populations with the same averages. If the probability is low enough we reject our initial assumption that the averages of the populations are the same. In this paper the average (also known as the mean) of the population and the standard deviation are unknown. Additionally, in general it can not be assumed that the two population standard deviations are equal. With this information the proper statistical formula can be chosen. The formula or test statistic is given below:

$$t = \frac{x1 - x2}{\sqrt{s1^2 + s2^2}\;(n-1)}$$

In the formula x1 and x2 are the sample averages, s1 and s2 are the sample standard deviations and n is the sample size. This formula relates to a probability distribution function (PDF) that looks like the graph below:



This particular PDF is called the "Student's" t-distribution. The area under any PDF equals 1 (i.e. the sum of the probability of anything happening is 100%). The output of the formula is a value on the x-axis of the t distribution, called a t-value. The farther from zero the t-value gets the less probable it is to randomly choose those samples (if they are from populations with the same average). Therefore, a threshold must be chosen where it is improbable enough to reject our initial assumption that the two populations have the same mean. A reasonable threshold is usually chosen around 5% (i.e. on average we will be right 95% of the time). If the population averages (the LyE of old and young females walking) are unequal, the average of the old females could be larger or smaller. Therefore, we divide up the threshold between the two tails on the PDF. The area or probability under each of the tails is 0.025. This equates to t-values of -2.262 or less and 2.262 or greater. Therefore, if a t-value is found in this range the assertion is made that the two average of the populations are unequal. The statistical comparisons will now be labeled as a 2-tailed t-test with 9 degrees of freedom (n-1).

A less-formal way of looking at this is the p-value method. The p-value is the probability that the populations of the two samples have the same mean. If this probability or p-value is around 0.05 or less then we assume the average of the two populations are not the same.

In comparing the old and young data, initially the only body part and axis that shows a significant statistical difference is the iliac crest y-axis. It has a p-value of about 0.05 and shows the older age group with greater variability (i.e. the LyE is larger). So at this point in the analysis there may be a possibility that on average the old subjects have greater variability in the iliac crest / y-axis.

An analysis of this raw data has been done previously (Buzzi et al., 2003). However, the average LyEs differ significantly between the Warr analysis and Buzzi et al's (see the table below).

			Buzzi et al	Warr	Difference
BP	**AX**	**AG**	**LyE-Ave**	**LyE-Ave**	
a	y	y	0.078	0.021	271.4%
a	y	o	0.098	0.024	308.3%
k	y	y	0.127	0.024	429.2%
k	y	o	0.145	0.037	291.9%
p	y	y	0.179	0.030	496.7%
p	y	o	0.219	0.030	630,0%

I will discuss some of the possible reasons for these noteworthy differences.

There are a few explanations for the disparity in the average LyEs between this study and the one done by Buzzi et al. First, different software programs, which utilize different algorithms, were used for discovering the LyE of the data sets. Another difference is that the former study fixed the embedding dimension (*m*) at 5 and the current study lets *m* vary between 2-5 depending on the outcome of the False Nearest Neighbor technique. The last fairly obvious reason is related to the first, but worth mentioning as well. The software used by Buzzi et al does not require an input for the time delay. I will attempt to compare the two studies by making them as similar as possible. To do this I will fix *m* at 5 and *J* (the delay) at 1. The results are in the table below.

			Buzzi et al	Warr Fixed m & J	Difference
BP	**AX**	**AG**	**LyE-Ave**	**LyE-Ave**	
a	y	y	0.078	0.152	-48.7%
a	y	o	0.098	0.149	-34.2%
k	y	y	0.127	0.185	-31.4%
k	y	o	0.145	0.183	-20.8%
p	y	y	0.179	0.220	-18.6%
p	y	o	0.219	0.209	4.8%

As can be seen, the difference in the average LyEs between the two studies is greatly reduced when the *m* and *J* are fixed in the technique currently being used. With this noticeable difference explained, I will continue using the method introduced in this chapter. Nevertheless, both have error tolerances and the analysis of chaotic data is not yet an exact science. In this paper the exact LyEs should not be the focus, but the comparisons between the different age groups and the data before and after the noise reduction techniques.

Another disparity in the analysis of the raw data with that found in Buzzi et al is the statistically significant differences between the age groups. Up to this point I have found no significant statistical differences between the age groups in the hip, knee and ankle in the y-axis. However, Buzzi et al found differences in all three. This again is directly attributable to the varying methods of finding LyEs.

Now that I have completed an initial analysis of the raw data I will begin to implement some noise reduction techniques. The next two chapters will explore some of the possible methods and their outcomes.

CHAPTER THREE – ADDITIVE MEASUREMENT NOISE

As I wrote in the introduction, noise is any alteration, however small, in the perception or representation of the true state of an object or system. Noise in observed data can be broken up into two distinct types, the first of which is external to the system being measured, also referred to as additive measurement (ADM) noise or quantization noise (Galka 2000, p. 30). The other noise that exists in the system itself, or internal to the system is known as Dynamical Noise. In this chapter the noise external to the system will be examined and an attempt will be made to remove it from the data. One caution upfront is that using linear filtering techniques can have detrimental effects on the data, i.e. removing the signal instead of the noise (Galka 2000, p. 46). Therefore, care needs to be given to ensure that the signal is left undisturbed in the data.

Noise external to the system or ADM noise is fairly recognizable but extracting it from data is problematic. The amount of signal and ADM noise in data can be described by the signal-to-noise ratio (SNR)(Hu, Gao, & White 2004). Therefore with a high SNR there is a small amount of measurement noise. The following are some examples of how ADM noise may be introduced into data:

- If the measuring device only has a finite amount of accuracy
- If the measuring devise moved in reference to the system during measurement
- If the markers placed on the body were slightly off, even by a small amount

It is critical to be aware of how the data were collected to recognize the numerous ways the data may have been contaminated by ADM noise.

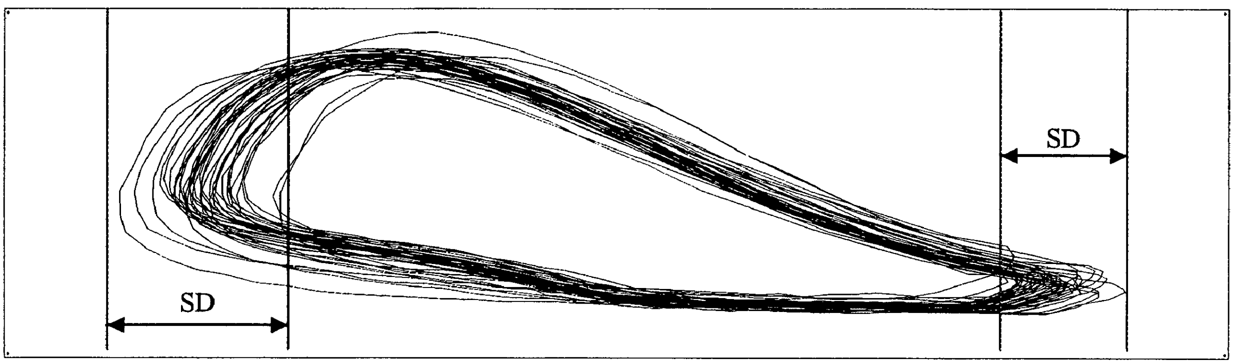
The main component of ADM noise in this particular data belongs to the x-axis. It has been introduced into the data because the subject has the freedom to go back and forth horizontally while walking on the treadmill even though the measurement device is fixed. Since the x-axis data comprises half the data collected, it is significant to include it in the analysis.

The primary technique I will use for the reduction of ADM noise in the x-axis data is to change the frame of reference of the measurement. In these particular data sets I have the luxury of multiple body parts being measured simultaneously. Since this is the case I can change the frame of reference I am using. I do this by comparing one body part to another while holding the other body part fixed on the x-axis. Thus I have the true motion of the body part in the x-axis in reference to the other body part. This eliminates the ADM noise that was introduced from the horizontal movement of the individual on the treadmill in reference to the static measurement device. The body part I chose to be the fixed reference point was the iliac crest, since this body part is fairly stable in the x-axis while walking, in reference to the body as a whole. This technique could be applied to both axes if desired, to use the data collectively in the future for analysis.

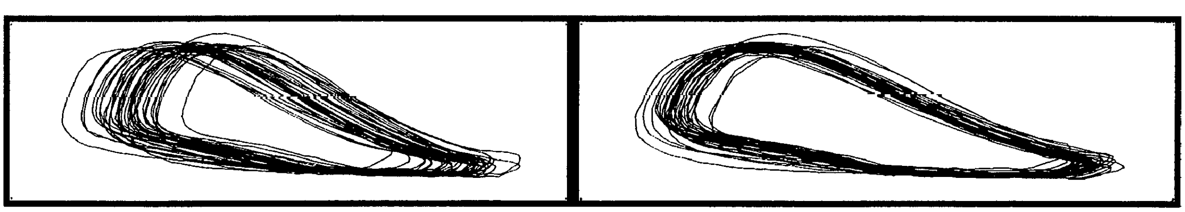
The iliac crest was used for the other five body parts and I will use the hip as the fixed reference body part when applying this technique to the iliac crest. I picked this as the secondary fix reference body part because it is also fairly stable while walking in reference to the other body parts being measured. By using this technique we can be more confident in the x-axis data and include it in our analysis.

It is important to note that since I used this change in frame of reference technique I may have increased the amount of ADM noise that was in the system. This could happen because for any given data point there exists α amount of ADM noise. Since I am adding or subtracting it from another data point with β amount of ADM noise, I have given any data point in the filtered data $\alpha + \beta$ ADM noise. This remaining noise would be on average equal to the original α or β if the ADM noise was white noise. However, Galka suggests it is more likely that it is not random white noise but correlated red noise (2000, p. 30). By applying this method, the most noise that would be introduced into the system is $2*\alpha$ (choosing $|\alpha| > |\beta|$). Therefore, using this technique could possibly add more noise into a data set then it removes. For this reason, I recommend checking the signal-to-ratio before and after its application and discarding the filtered data if the SNR is lower than in the original data. I will not change the frame of reference in the y-axis data since it is more likely to increase in ADM noise.

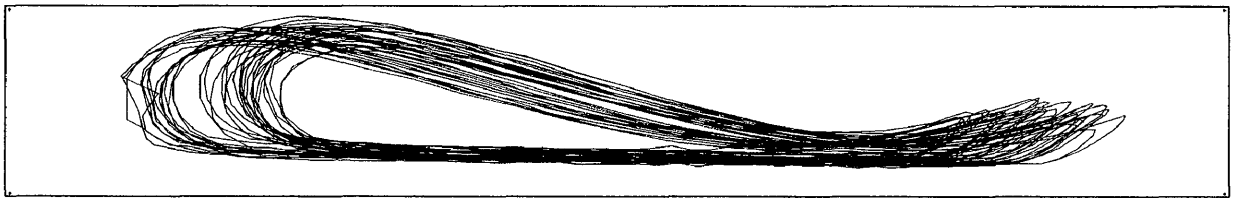
To ensure changing the frame of reference actually does remove more noise in the x-axis then it adds in, I will estimate the SNR for each of the data sets and ensure it increases. If it does, I will keep the changes to the data. Otherwise, they will be discarded. The method I will use to estimate the SNR is to first find the SD of all the local maximums in a data set, then find the SD for all the local minimums in that same data set, and then add the two SDs together. Once this quantity is found for both the raw and changed frame of reference data sets, I will take the quotient of the two respectively. This will test to see if the data is being grouped closer together or spreading it out. If this ratio I have computed is greater than 1 then the technique has been effective in reducing the amount of ADM noise in the data. This estimate is not the actual SNR but without knowing the true signal without the noise, I must approximate it. I will refer to this value as the Maximum and Minimum SD Ratio (MSDR). This calculation gives a general idea if the signal is increasing proportionally to the ADM noise. I will walk through calculating this for subject 10 in the older category, taking the ankle and x-axis. The figure below shows the bands of data in which the local maximums and minimums are located.



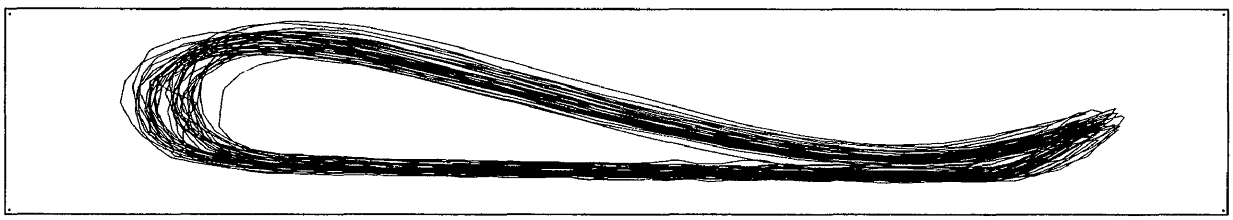
The SDs for the raw data are 14.19 for the local maximums and 11.95 for the local minimums. The SDs for the changed frame of reference data are 5.86 for the local maximums and 6.67 for the local minimums. Therefore, adding the maximums and minimums together, then dividing the values equals 2.09. Since MSDR is greater than 1.0 I have succeeded in reducing the amount of ADM noise. This is fairly obvious from looking at both graphs below, but provides a way to quantify it (the graph of the

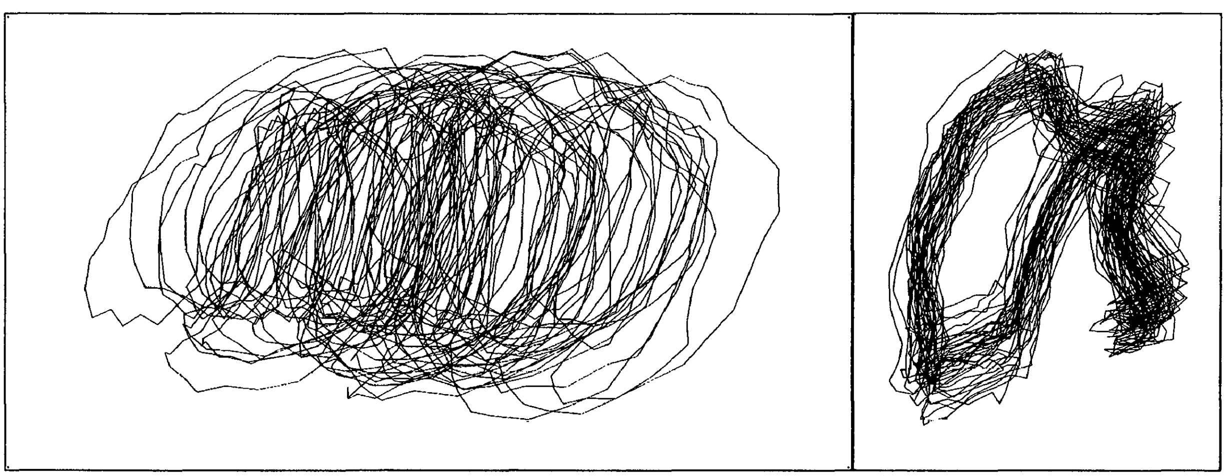


raw data is on the left and the changed frame of reference on the right.) After applying the change in frame of reference technique to the x-axis data I found that it was effective in reducing the ADM noise for all of the data. The details of the MSDR results can be found in Appendix B. The following graphs below are examples of the changes made when applying this technique:



Above are old subject 3's ankle movements before any noise reduction and below is after the frame of reference change.





The pictures above are of young subject 5's iliac crest movements. The one on the left is before any noise reduction and the one on the right is after the frame of reference change.

From the pictures above it is clear that the change in frame of reference technique has greatly reduced the ADM noise in the x-axis. Therefore the x-axis data is now useable for analysis and should provide results that are close to the accuracy of the y-axis data. The x-axis data will be very useful in the remainder of the paper for comparisons to the y-axis data.

Another technique that leverages the same principles as changing the frame of reference is comparing the angles of the body parts in reference to each other. An example of this is measuring the angle between the metatarsal, the ankle, and the knee or the angle between the ankle, the knee and the hip and then analyzing the angle as opposed to the actual displacement of the body part. This has been done in a previous analysis of this data (Buzzi et al., 2003). Because of the similarity of this technique and the one I have used previously, I will not I will not pursue it in this paper.

Now that I have the ability to use the frame of reference noise reduction technique, I have eliminated the majority of the ADM noise in the x-coordinate data without damaging the nonlinear properties of the system. Another apparent source of ADM noise in the data is the finite precision of the measurement device. This is difficult if not impossible to remove from the data without altering the properties of the dynamic system as well. One of the major problems in analyzing chaotic data is its sensitivity to initial conditions. Since there is only a finite precision in the data at hand this could cause problems. However, since I am not predicting future behavior, this problem is minimized. If the data is contaminated with an amount of noise it will not be propagated into the following data because it has been measured again many more times independent of the previous measurements. Thus, it is better to leave this noise in the data with the minor effects it may have.

Another type of ADM noise in the data is that of machine malfunction. In two of the data sets there are problems where the data is zero for a few iterations and then recovers and starts measuring properly again. The only way to fix this noise is to keep the data either before or after the problem, and discard the rest. In subject 2 of the older age category the fix has minimal impact because the error was very early in the data. However, for subject 5 in the older age category it happens at about the 350th time step into the data. This data set is 2913 time steps long. To fix this data about 400 time steps will need to be thrown away. According to Rosenstein et al., this should not have a dramatic effect on the outcome since his routine was designed for small data sets (1993, p.10). To test the assertion that a smaller amount of data shouldn't effect the LyE, I ran a check on subject 7 in the older age category and found only an average change in the LyE to be -2.7% with a SD of 4.5% (this was the data set with the closest number of data points to subject 5). It is also important to note that since subject 5 in the older age category had by far the most data points of any subject, taking 400 away will leave it with about the same number as most other subjects. Therefore the process of discarding data should not significantly affect the estimates of the true LyE.

Now that I have used a few methods to remove the ADM noise from the system we should expect more accurate results in estimating the LyE. After running the same tests in chapter two on the refined data I get the following results:

BP	AX	AG	LyE-Ave	LyE-SD	m-Ave	m-SD	J-Ave	J-SD	DT-Ave	DT-SD	t-val	p-val
a	x	y	0.06979	0.03231	3.7	0.67495	6.8	7.0993	25.5	7.5314		
a	x	o	0.04699	0.02929	3.8	0.91894	13	5.9067	23.7	7.5137	1.569	0.151
a	y	y	0.02107	0.00837	4.7	0.48305	13.5	1.5092	23.8	7.9275		
a	y	o	0.02499	0.0075	4.7	0.48305	13.8	1.3166	20.7	6.237	-1.046	0.323
h	x	y	0.08893	0.03241	3.5	0.70711	4.3	4.99	21.5	5.4006		
h	x	o	0.03665	0.02411	4.3	0.67495	13	5.9067	24.2	4.7796	3.883	0.004
h	y	y	0.02389	0.01051	4.8	0.42164	13.6	0.6992	23.4	7.975		
h	y	o	0.03074	0.02811	4.9	0.31623	13.4	1.075	24.3	8.3273	-0.685	0.511
i	x	y	0.04453	0.04619	4.9	0.31623	11.9	4.6056	15.9	12.188		
i	x	o	0.04978	0.05318	4.9	0.31623	12.7	3.3015	14.8	10.82	-0.224	0.828
i	y	y	0.02297	0.0051	4.6	0.5164	9.8	1.2293	25.9	8.0616		
i	y	o	0.03317	0.0121	4.5	0.52705	10.5	1.7795	17.7	7.5285	-2.33	0.045
k	x	y	0.05009	0.03471	3.4	0.5164	16.4	1.075	17.6	7.6041		
k	x	o	0.04778	0.03562	3.9	0.56765	15.8	1.2293	18.5	7.3068	0.14	0.892
k	y	y	0.024	0.0085	4.9	0.31623	9.8	1.3984	26.1	11.855		
k	y	o	0.038	0.03133	4.8	0.42164	10.4	1.9551	23.6	15.269	-1.293	0.228
m	x	y	0.07757	0.03099	3.7	0.48305	5.7	6.0928	23.8	5.7889		
m	x	o	0.03873	0.0161	3.6	0.5164	14.2	4.4171	21.9	3.8137	3.336	0.009
m	y	y	0.03102	0.01422	5	0	7.9	1.5951	24.6	11.587		
m	y	o	0.03736	0.03437	5	0	11.1	1.5239	23.6	11.568	-0.512	0.621
p	x	y	0.04453	0.04619	4.9	0.31623	11.9	4.6056	15.9	12.188		
p	x	o	0.04978	0.05318	4.9	0.31623	12.7	3.3015	14.8	10.82	-0.224	0.828
p	y	y	0.02959	0.01224	4.3	0.48305	9.9	1.7288	24	10.328		
p	y	o	0.02964	0.00697	4.4	0.5164	9.9	0.8756	22.9	12.279	-0.012	0.991

The complete results from the above table are again contained in Appendix A. There are a couple of significant changes from the noise reduction conducted thus far. First, difference is in the iliac crest / y-axis. On average the older age group shows greater variability then that of the young, with a lower or stronger p-value of 0.045 from the previous 0.053.

The other items of interest that have appeared after this noise reduction are the changes in the x-axis of the metatarsal, heel and possibly the ankle. The p-values indicate statistical differences of the mean LyEs in the heel and metatarsal. One might conclude that this would also translate over into the ankle as well since these three body parts are closely connected and would have similar LyEs. However, at this point it has not. These are the changes brought about by changing the frame of reference in the x-axis data.

The effects of the deleting the machine malfunction data are fairly significant as well. Again this only affected the older subjects 2 and 5. I will only list the changes to the y-axis because the x-axis had the frame of reference applied as well. The LyEs for the different body part are listed in the table below.

	Old Subject #2 Y-Axis				**Old Subject #5 Y-Axis**		
Body Part	**Raw LyEs**	**Cleaned LyEs**	**% Change**		**Raw LyEs**	**Cleaned LyEs**	**% Change**
Iliac Crest	0.03103	0.05356	-72.6%		0.03348	0.03121	6.8%
Hip	0.03018	0.0274	9.2%		0.02888	0.02563	11.3%
Knee	0.11156	0.11207	-0.5%		0.02683	0.03202	-19.3%
Ankle	0.03638	0.03647	-0.2%		0.02838	0.03525	-24.2%
Heel	0.11007	0.11036	-0.3%		0.02204	0.02563	-16.3%
Metatarsal	0.01951	0.01952	-0.1%		0.01988	0.01765	11.2%

As shown, the changes in the LyEs were fairly significant in some of the body parts. It was fairly obvious to delete the data before and up to the machine malfunction but we can see it had an impact on the LyE for the data sets. Therefore, it was an important step to see the clearer picture of the dynamics of the data in these two subjects.

The main processes that were used to remove ADM noise were changing the frame of reference and discarding data from 2 subjects that had bad data. Now that the ADM noise has been reduced in the data, the same can begin for the dynamical noise.

CHAPTER FOUR – DYNAMICAL NOISE

The other type of noise found in data is Dynamical Noise. This is noise that is contained in the system and is an integral part of it but would cloud some of the true properties, manifolds, or strange attractors we may be looking for in the system. This type of noise is much harder to recognize and eliminate from the data. Many of the noise reduction methods when applied could reduce either ADM noise or Dynamical noise or both and are unable to distinguish between the two. The only real measure of success is to determine if the total noise in the system has been reduced. Therefore, I will not make a distinction between the two types of noise for the duration of this chapter.

There are a couple of methods developed that I will utilize to reduce the noise in the data. The first is a process entitled "simple nonlinear noise reduction" (Kantz & Schreiber, 2004, chap. 4).

The simple nonlinear noise reduction algorithm replaces the central coordinate of each embedding vector by the local average of this coordinate. This amounts to a locally constant approximation of the dynamics and is based on the assumption that the dynamics is continuous. In a chaotic system it is essential not to replace the first and last coordinates of the embedding vectors by local averages. Due to the instability, initial errors in these coordinates are magnified instead of being averaged out. This noise reduction scheme is implemented quite easily. First an embedding has to be chosen. Except for extremely over-sampled data, it is advantageous to choose a short time delay.

The program always sets the delay to 1. The embedding dimension m should be chosen somewhat higher than that required by the embedding theorems. The radius of the neighborhoods (epsilon) should be taken large enough in order to cover the noise extent, but still smaller than a typical curvature radius. If the noise level is substantially smaller than the typical radius of curvature, neighborhoods of radius about 2-3 times the noise level give the best results with artificial data. After one complete sweep through the time series, all data points are replaced by the corrected values; except for the first and last points no corrections are made. The average correction can be taken as a new neighborhood radius for the next iteration. Note that the neighborhood of each point at least contains the point itself. If that is the only member, the average is simply the uncorrected measurement and no change is made. Thus one can safely perform multiple iterations with decreasing values of epsilon until no further change is made. (Hegger, Kantz, & Schreiber, 1999)

The two parameters that must be chosen for this noise reduction routine are epsilon and the embedding dimension. To pick an appropriate epsilon I assumed the noise level in each data set to be 1%. In other words, I figure the amplitude of the data and assume that 1% of that amplitude can be accounted for noise in the system. Then with the recommendation in the above paragraph, I set epsilon to be about 2 and a half times the noise level, or in this case, 2.5% of the amplitude. The other parameter is the embedding dimension. For this parameter I will take the normal embedding dimension times 2 and add 1 (i.e. 2*m+1) and use it for this routine as well. Additionally, it is safe to run the algorithm more than once. Therefore, I will run it until it produces no additional effect on the data or until I have applied it two times. Now that the parameters and the number of iterations are decided upon I can begin running the simple nonlinear noise reduction routine.

The following table lists the average results of the data after the simple nonlinear noise reduction routine has been applied.

BP	AX	AG	LyE-Ave	LyE-SD	m-Ave	m-SD	J-Ave	J-SD	DT-Ave	DT-SD	t-val	p-val
a	x	y	0.07539	0.028	4.7	0.675	5.5	5.911	30.7	7.587		
a	x	o	0.06325	0.037	4	0.817	10.1	7.047	26.4	6.753	0.781	0.455
a	y	y	0.02612	0.003	4.9	0.316	14.3	0.823	30.3	9.719		
a	y	o	0.02975	0.004	4.9	0.316	13.8	1.317	26.5	10.824	2.025	0.074
h	x	y	0.07723	0.029	4.7	0.675	5.8	6.460	30.7	7.454		
h	x	o	0.06863	0.043	4.2	0.919	10.1	7.047	27.7	12.102	0.503	0.627
h	y	y	0.02995	0.006	5	0.000	13.6	0.699	43.6	20.354		
h	y	o	0.0348	0.017	5	0.000	13.4	1.075	32.8	12.327	0.793	0.448
i	x	y	0.05276	0.051	4.9	0.316	13.1	4.771	14.1	11.367		
i	x	o	0.04604	0.055	4.9	0.316	12.8	3.458	16.3	9.226	0.269	0.794
i	y	y	0.02592	0.009	4.5	0.707	9.8	1.229	26.7	8.577		
i	y	o	0.03294	0.013	4.5	0.527	10.5	1.780	17.8	6.989	1.351	0.210
k	x	y	0.03374	0.011	3.8	0.789	16.4	1.075	28.2	9.426		
k	x	o	0.04193	0.023	3.8	0.422	15.8	1.229	22.2	8.561	0.958	0.363
k	y	y	0.02737	0.010	4.8	0.422	10.2	1.229	26.2	11.868		
k	y	o	0.03311	0.016	4.9	0.316	10.2	1.751	24.1	12.618	0.913	0.385
m	x	y	0.09193	0.037	4.8	0.422	4.4	4.648	30.3	8.577		
m	x	o	0.08229	0.042	4.4	0.843	7.5	7.169	26.2	10.152	0.517	0.618
m	y	y	0.03128	0.007	5	0.000	8.6	1.430	28.4	13.689		
m	y	o	0.05231	0.052	5	0.000	11.2	1.398	22.8	11.970	1.196	0.262
p	x	y	0.05275	0.051	4.9	0.316	13.1	4.771	14.1	11.367		
p	x	o	0.04603	0.055	4.9	0.316	12.8	3.458	16.3	9.226	0.269	0.794
p	y	y	0.0384	0.035	4.6	0.516	9.9	1.595	25.6	13.648		
p	y	o	0.02994	0.007	4.4	0.516	9.8	0.789	23.1	11.318	0.707	0.497

Before commenting on the changes this noise reduction routine has produced in the data we must first determine if the changes are desirable. The noise removed should have an autocorrelation function that decays quickly. It should also have no significant cross-correlations with the new estimated data (Kantz & Schreiber, 1997, p.167). I will define the "autocorrelation function decaying quickly" if it has a zero within 5 steps of the autocorrelation function (i.e. less than 0.25% of each data set). To determine if the cross-correlation is significant, I will take the absolute value of the output and chose the maximum. If this maximum is greater than 0.05 then it has a significant cross-correlation. This means that if there is a cross-correlation less than or equal to 5% then we accept that the data are uncorrelated. After applying these criteria to the output of this noise reduction technique it shows that about 78% of the data sets fail to produce noise that is uncorrelated to the cleaned data. Therefore this method has not been very successful on this particular data. I will scrap the results of this one with the possibility of trying it later.

Another method that could validate the successfulness of a noise reduction technique is to determine if the LyEs of the x and y axis data are converging closer together. From the reconstruction theorem one should be able to reconstruct the multi-dimensional attractor from one dimensional time series data (Diks, 1999, p.15). In theory the x and y data should reconstruct to the same attractor and this would imply that the LyEs of the two reconstructions should be the same. However, in practice this is usually not the case. This can be explained by the amount of data collected and noise in the data sets. If the noise is truly being reduced in the data, it should eliminate some of this problem, and we would see some convergence in the LyEs for the x and y data.

This method does not prove that a noise reduction technique is successful, only that it might be successful, or an additional measure of success. If the LyEs for the x and y axis are diverging after noise reduction has been applied then it could be causing adverse affects on the data. Therefore changes to the data might possibly be rejected. Since this is not a definitive measure but a rough gauge, it should only be used in conjunction or as a secondary check on the success of a noise reduction method. I will use this if needed to choose between two methods.

The next noise reduction method is also developed in Schreiber and Kantz's *Nonlinear Time Series Analysis* (2004, chap 10) entitled "Locally projective nonlinear noise reduction."

Locally projective nonlinear noise reduction is a more sophisticated method that makes use of the hypotheses that the measured data is composed of the output of a low-dimensional dynamical system and of random or high-dimensional noise. This means that in an arbitrarily high-dimensional embedding space the deterministic part of the data would lie on a low-dimensional manifold, while the effect of the noise is to spread the data off this manifold. If we suppose that the amplitude of the noise is sufficiently small, we can expect to find the data distributed closely around this manifold. The idea of the projective nonlinear noise reduction scheme is to identify the manifold and to project the data onto it. The projections are done in a local coordinate system which is defined by the condition that the average of the vectors in the neighborhood is zero. Or, in other words, the origin of the coordinate systems is the centre of mass of the neighborhood. This centre of mass has a bias towards the centre of the curvature. Hence, a projection would not lie on the tangent at the manifold, but on a secant. Now we can compute the centre of mass of these points in the neighborhood. Recall that the most important parameters have to be set individually for each time series. The embedding parameters are usually chosen quite differently from other applications since considerable over-embedding may lead to better noise averaging. Thus, the delay time is preferably set to unity and the embedding dimension is chosen to provide embedding windows of reasonable lengths. Only for highly over-sampled data are larger delays necessary so that a substantial fraction of a cycle can be covered without the need to work in prohibitively high dimensional spaces (Hegger et al, 1999).

Next, one has to decide how many dimensions q to leave for the manifold supposedly containing the attractor. Rather brisk projections can be optimal in the sense of lowest residual deviation from the true signal. Low root-mean-square error can, however, coexist with systematic distortions of the attractor structure. Thus, for a subsequent dimension calculation, a more conservative choice would be in order. Remember, however, that points are only moved towards the local linear subspace and too low a value of q does not do as much harm as may be thought. The noise amplitude to be removed can be selected to some degree by the choice of the neighborhood size. In fact, nonlinear projective filtering can be seen independently of the dynamical systems background as filtering by amplitude rather than by frequency or shape. To allow for a clear separation of noise and signal directions locally, neighborhoods should be at least as large as the supposed noise level, preferably rather larger. This of course competes with curvature effects. For small initial noise levels, it is recommended to also specify a minimal number of neighbors in order to permit stable linearizations (Hegger et al, 1999).

Finally, we should remark that in successful cases most of the filtering is done within the first one to three iterations. Going further is potentially dangerous, since further corrections may lead mainly to distortion. One should watch the correction in each iteration and stop as soon as it doesn't decrease substantially any more. The main assumption for this algorithm to work is that the data is well approximated by a low-dimensional manifold. If this is not the case it is unpredictable what results are created by the algorithm. This means that if one wants to apply this algorithm, one has to carefully study the results. If the assumptions underlying the algorithms are not fulfilled in principle anything can happen. For data which is indeed well approximated by a lower dimensional manifold, the average corrections applied should rapidly decrease with each successful iteration (Hegger et al, 1999).

With this routine four parameters must be chosen. The first is the embedding window, which is given by the formula $(m-1)*J+1$. This can be quite large for some delays, therefore I will cap this parameter at 15. The next parameter is the dimension of the manifold. This should be less than the embedding dimension and doesn't affect the outcome severely if chosen too small. The manifolds for this type of data would typically be 3 dimensional, therefore because of these factors I will fix it at 3. The third parameter is the minimal number of neighbors. I will pick this parameter to be 1% of the data set. This will be typically around 22-24 data points depending on the size of the data set. The last parameter is epsilon or the radius of neighborhoods. As with the previous noise reduction method this parameter is set at the estimated noise level. I will therefore again choose it to be at 1% of the amplitude of the data. This routine can be run multiple times but most of the filtering will be accomplished in the first 3 iterations. Therefore I will run it twice for each of the data sets.

The following table lists the average results of the data after the locally projective nonlinear noise reduction routine has been applied.

BP	AX	AG	LyE-Ave	LyE-SD	m-Ave	m-SD	J-Ave	J-SD	DT-Ave	DT-SD	t-val	p-val
a	x	y	0.08374	0.0366	3.9	0.8756	5.1	6.0636	24	8.1786		
a	x	o	0.06077	0.04155	3.5	0.52705	13	5.9067	23.3	11.422	1.245	0.245
a	y	y	0.02072	0.00841	4.7	0.48305	13.5	1.5092	24.1	7.9784		
a	y	o	0.02321	0.00666	5	0	13.8	1.3166	20.7	6.3779	0.697	0.503
h	x	y	0.08958	0.03747	3.4	0.5164	4.6	4.8808	22.5	5.8926		
h	x	o	0.04763	0.03057	4.1	0.73786	11.6	6.7198	23.1	4.5326	2.602	0.029
h	y	y	0.02296	0.00786	4.8	0.42164	13.6	0.6992	23.7	7.9589		
h	y	o	0.03031	0.02842	5	0	13.4	1.075	24.9	8.5434	0.748	0.474
i	x	y	0.04681	0.05213	5	0	12.9	4.5326	16.3	11.48		
i	x	o	0.03851	0.02344	4.6	0.5164	12.3	3.8887	14.4	11.345	0.436	0.673
i	y	y	0.02436	0.00495	4.5	0.52705	10.1	1.3703	24	6.4636		
i	y	o	0.03368	0.01328	4.4	0.5164	10.4	1.7127	17.8	6.941	1.973	0.080
k	x	y	0.03972	0.02463	3.5	0.52705	16.4	1.075	20.1	7.5196		
k	x	o	0.04873	0.03538	3.9	0.73786	15.8	1.2293	18.4	7.0111	0.627	0.546
k	y	y	0.0228	0.0058	4.9	0.31623	9.8	1.3984	26.8	11.39		
k	y	o	0.03827	0.03331	4.8	0.42164	10.4	1.9551	23.9	14.708	1.372	0.203
m	x	y	0.08505	0.03752	3	0	6.4	6.2752	23.4	6.3631		
m	x	o	0.03425	0.01717	3.9	0.56765	14.2	4.4171	22.4	3.34	3.693	0.005
m	y	y	0.02775	0.0089	5	0	8.2	1.5492	28.9	15.481		
m	y	o	0.03748	0.0343	5	0	11.1	1.5239	23.8	11.858	0.823	0.432
p	x	y	0.0451	0.04328	4.8	0.42164	13.1	4.5326	15.1	9.255		
p	x	o	0.03565	0.02459	4.8	0.42164	12.3	3.6225	16	10.317	0.57	0.583
p	y	y	0.04112	0.04164	4.6	0.5164	9.9	1.7288	23.2	12.848		
p	y	o	0.02991	0.00723	4.5	0.52705	9.7	0.8233	23.7	11.935	0.796	0.447

After testing this method with the decay of the autocorrelation function and the cross-correlation of the noise and the noise reduced data, 93.8% of the data sets pass. Seeing that this technique has been for the most part successful, I will attempt to apply it again to improve the outcome. The table below contains the results.

BP	AX	AG	LyE-Ave	LyE-SD	m-Ave	m-SD	J-Ave	J-SD	DT-Ave	DT-SD	t-val	p-val
a	x	y	0.09357	0.041	3.5	0.850	5.1	6.064	22.6	8.644		
a	x	o	0.06439	0.041	3.4	0.516	13	5.907	21.3	10.350	1.501	0.168
a	y	y	0.02061	0.008	4.8	0.422	13.5	1.509	24.3	8.084		
a	y	o	0.02411	0.008	4.9	0.316	13.8	1.317	20.6	6.381	0.921	0.381

h	x	y	0.08985	0.038	3.5	0.527	4.6	4.881	22.8	6.563		
h	x	o	0.05452	0.032	3.8	0.789	11.6	6.720	22.1	3.957	2.121	0.063
h	y	y	0.02273	0.008	4.9	0.316	13.6	0.699	24.3	8.564		
h	y	o	0.03049	0.029	5	0.000	13.4	1.075	24.7	8.473	0.782	0.454
i	x	y	0.04106	0.038	5	0.000	12.7	4.270	15.8	9.953		
i	x	o	0.03291	0.024	4.8	0.422	13.1	3.695	15	6.667	0.542	0.601
i	y	y	0.02442	0.005	4.5	0.527	10.1	1.370	24.4	6.518		
i	y	o	0.03428	0.013	4.4	0.516	10.3	1.494	18.2	7.021	2.113	0.064
k	x	y	0.04311	0.024	3.4	0.516	16.4	1.075	17.4	5.211		
k	x	o	0.04222	0.034	4.1	0.568	15.7	1.059	20.3	7.499	0.063	0.951
k	y	y	0.02253	0.006	4.9	0.316	9.8	1.229	27.4	10.265		
k	y	o	0.03947	0.033	4.7	0.483	10.4	1.955	23.8	14.520	1.503	0.167
m	x	y	0.08825	0.040	2.9	0.316	6.4	6.275	23.6	6.059		
m	x	o	0.03727	0.017	3.7	0.675	14.2	4.417	22.1	3.510	3.494	0.007
m	y	y	0.03229	0.014	5	0.000	7.9	1.853	24.1	12.023		
m	y	o	0.02865	0.016	5	0.000	10.9	1.663	24.3	11.146	0.509	0.623
p	x	y	0.04444	0.038	4.7	0.483	13.1	4.725	15.4	9.131		
p	x	o	0.03603	0.023	4.6	0.516	12.4	3.658	16.3	10.467	0.575	0.579
p	y	y	0.04123	0.041	4.6	0.516	9.9	1.729	23.2	12.848		
p	y	o	0.031	0.011	4.5	0.527	9.7	0.823	25.6	16.304	0.717	0.492

Once again, testing the noise removed from these results, we find that 94.6% of the data sets pass. This is a little strange because a few of the data sets that failed the first round of this technique passed the test after the method had been applied twice. However, since this noise reduction method has brought the noise within tolerance, we will accept these changes.

I will now retry the attempt of the simple nonlinear noise reduction routine after the locally projective nonlinear noise reduction routine has been applied to the data twice. The following table contains the results.

BP	AX	AG	LyE-Ave	LyE-SD	m-Ave	m-SD	J-Ave	J-SD	DT-Ave	DT-SD	t-val	p-val
a	x	y	0.07708	0.029	4.7	0.675	5.5	5.911	30.4	7.214		
a	x	o	0.06425	0.036	3.7	0.823	11.6	6.720	24.9	7.724	0.831	0.427
a	y	y	0.02603	0.003	5	0.000	14	1.333	33.3	9.522		
a	y	o	0.02939	0.005	4.9	0.316	14	1.333	26.7	11.861	-1.794	0.106
h	x	y	0.08008	0.029	4.6	0.699	5	5.142	30.6	9.359		

h	x	o	0.0723	0.037	4	1.054	10.1	7.047	25.8	11.053	0.497	0.631
h	y	y	0.03088	0.005	5	0.000	13.6	0.699	42.5	15.087		
h	y	o	0.03496	0.017	5	0.000	13.4	1.075	32.9	12.503	-0.675	0.517
i	x	y	0.03984	0.036	4.9	0.316	13.3	4.692	16.5	10.341		
i	x	o	0.03067	0.019	4.9	0.316	12.6	3.406	16	2.667	0.667	0.522
i	y	y	0.0249	0.007	4.6	0.516	10.1	1.370	26.4	8.566		
i	y	o	0.03602	0.011	4.3	0.483	10.4	1.713	18.4	6.835	-2.559	0.031
k	x	y	0.03395	0.011	3.9	0.738	16.4	1.075	27.8	11.516		
k	x	o	0.0408	0.023	3.8	0.422	15.8	1.229	23.3	8.247	-0.822	0.432
k	y	y	0.03121	0.008	4.6	0.516	10.1	1.287	23.6	11.664		
k	y	o	0.03907	0.027	4.9	0.316	10.4	1.955	23.8	12.035	-0.835	0.425
m	x	y	0.09095	0.039	4.4	0.966	6.7	7.119	28.4	7.074		
m	x	o	0.07616	0.040	4.2	0.919	8.5	6.868	25.6	10.596	0.789	0.450
m	y	y	0.03161	0.009	5	0.000	8.7	1.418	29.1	13.486		
m	y	o	0.0496	0.045	5	0.000	11	1.247	24.4	12.112	-1.182	0.267
p	x	y	0.04135	0.036	4.9	0.316	12.8	4.517	16.6	10.341		
p	x	o	0.02854	0.020	4.9	0.316	12.8	3.048	18.6	8.996	0.938	0.373
p	y	y	0.02368	0.008	4.6	0.516	10	1.414	26.7	10.199		
p	y	o	0.02902	0.006	4.6	0.516	10	0.817	23.4	11.654	-1.661	0.131

The simple nonlinear noise reduction routine again has not been as effective as the other but may contribute to the overall noise reduction on some data sets. The 22.5% pass rate for this application of the technique was only slightly higher than the last. However, this is the last noise reduction attempt so I will let this last step stand because it will improve the noise reduction in about 22% of the data sets.

There are many other noise reduction and smoothing methods suggested in the literature but will not be pursued in this paper, some of them are listed below:

Gaussian sum filter (Kitagawa & Gersch, 1996, p 201)

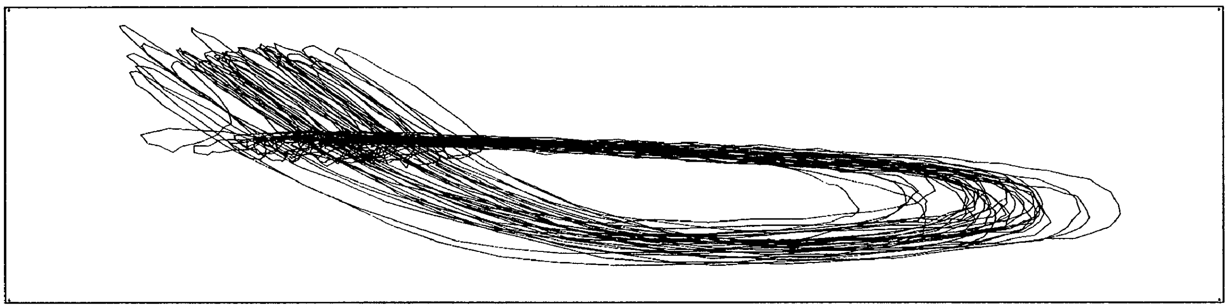
Monte Carlo filter method (Fan & Yao, 2003, p. 181)

Kalman Filter (Tong, 1990, p. 353)

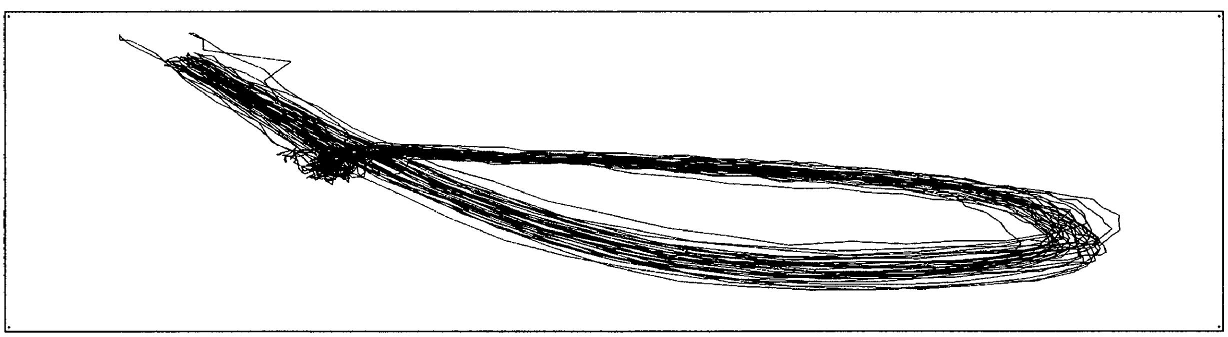
Exponential Smoothing (Bowerman, O'Connell, Koehler, 2005)

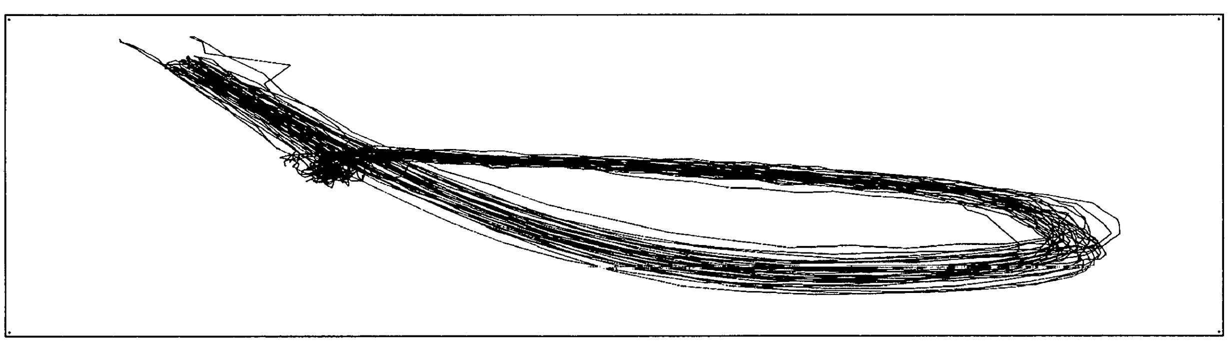
These focus more on smoothing than on noise reduction. In this paper I will not apply these smoothing techniques because of the added complexity. However, these could also reduce the dynamical noise in the data.

Now that the noise reduction methods have been applied I would like to step through some graphs of the data to show visually how the data improved. I will choose two random subjects and highlight one body part. The first is younger subject #4's knee:

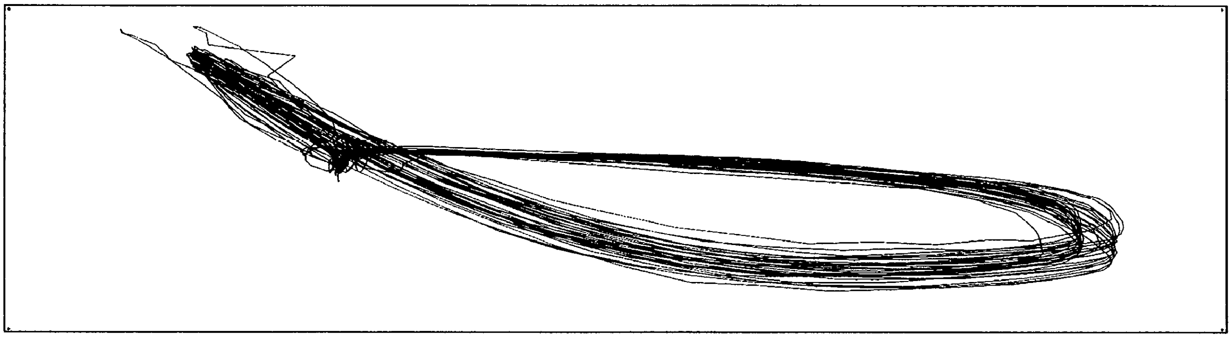


The graph above shows the raw data and the one below shows the data after changing the frame of reference.

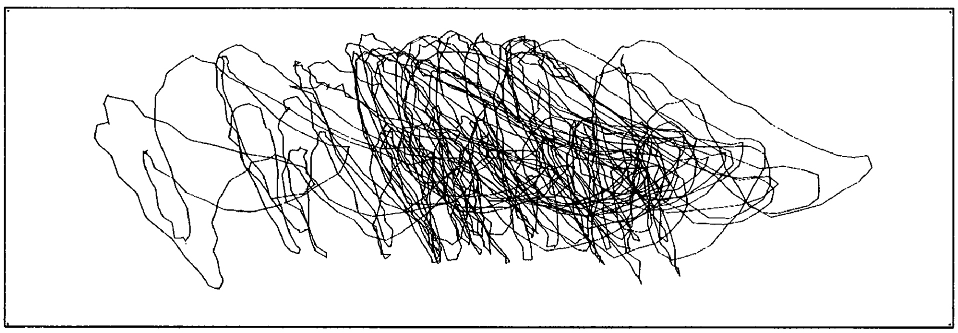




The graph above shows the data after applying the locally projective nonlinear routine twice and the one below shows it after the simple nonlinear noise reduction routine (this last one is rejected because the noise is too correlated with the data).



The other example I will show is old subject 0's hip.



The graph above shows the raw data.

Of the three graphs above the one on the left shows the data after changing the frame of reference (CFR), the one in the middle is the data after applying the locally projective nonlinear routine twice (2*L), and the one on the right is the data after the simple nonlinear noise reduction method (2*L+S). Visually, the differences are not distinct, however, if we look at the LyE, the changes are more apparent. The x-coordinate changes from the left graph to the center graph by 45%. It then decreases from the center graph to the right graph by 57%. The y-coordinate increases from left to center 3%, and then increases again from center to right 7%. See the table below:

Old Subject 0's Hip Data			
	Axis	LyE	% Change
CFR	x	0.03237	
2*L	x	0.04686	45%
2*L+S	x	0.02019	-57%
CFR	y	0.02277	
2*L	y	0.02347	3%
2*L+S	y	0.02507	7%

From these two examples it is clear the noise reductions have been quite productive. I have now completed the noise reduction phase of the paper. In the next chapter I will decide which data sets I will keep to best represent the true noise free data.

CHAPTER FIVE – CONCLUSION

Now that a few noise reduction methods have been applied to the data it is easier to draw conclusions about the walking data that were originally sought. When this project was originally proposed it was postulated that the young females would have more variability walking. However, after the data was analyzed this did not appear to be the case. Now that the noise has been reduced in the system we can see if it has had any effect on the outcome.

It is clear from the noise reduction attempts that one must be very cautious when applying and accepting each phase in the noise reduction process. In chapter four I rejected some of the attempts because they removed more then just the noise in the data. Others clearly took out data that didn't correlate with the improved data. Although I have just skimmed the surface on different techniques and procedures it is clear we have been able to improve the data's usefulness and accuracy.

To merge the effects of the noise reduction techniques I have selectively chosen the data sets farthest along in the noise reduction process that are still effective by the criteria mentioned before. For some data sets this would be directly after changing the frame of reference, and others after all three other noise reduction attempts. Once merged, I again averaged the LyE of each body part, age, and axis and compared this average of both age groups. I compared the populations with a two-tailed t-test with 9 degrees of freedom. I also calculated the p-values for the comparisons of the age groups.

The results for each data set where I stopped the noise reduction process can be found at the end of Appendix A. The following table includes the final results from the noise reduction methods completed in this paper.

BP	AX	AG	LyE-Ave	LyE-SD	m-Ave	m-SD	J-Ave	J-SD	DT-Ave	DT-SD	t-val	p-val
a	x	y	0.09211	0.0405	3.5	0.84984	5.1	6.06355	22.6	8.64356		
a	x	o	0.06439	0.04122	3.4	0.5164	13	5.90668	21.3	10.34999	1.439	0.184
a	y	y	0.02061	0.00819	4.8	0.42164	13.5	1.50923	24.3	8.08359		
a	y	o	0.02411	0.00792	4.9	0.31623	13.8	1.31656	20.6	6.38053	-0.921	0.381
h	x	y	0.08986	0.03653	3.5	0.52705	4.5	4.92725	22.5	6.81909		
h	x	o	0.05452	0.03215	3.8	0.78881	11.6	6.71979	22.1	3.95671	2.179	0.057
h	y	y	0.02273	0.0081	4.9	0.31623	13.6	0.69921	24.3	8.56414		
h	y	o	0.03049	0.02864	5	0	13.4	1.07497	24.7	8.47283	-0.782	0.454
i	x	y	0.03944	0.03624	4.9	0.31623	13.2	4.63801	16.3	9.95601		
i	x	o	0.04723	0.05368	4.9	0.31623	12.9	3.41402	16.6	9.24001	-0.361	0.726
i	y	y	0.02613	0.00633	4.4	0.5164	10.1	1.37032	24.6	6.46701		
i	y	o	0.03638	0.01198	4.3	0.48305	10.3	1.49443	18.3	6.961	-2.27	0.049
k	x	y	0.04311	0.02402	3.4	0.5164	16.4	1.07497	17.4	5.2111		
k	x	o	0.0427	0.0342	4.1	0.56765	15.7	1.05935	20.9	8.37257	0.029	0.977
k	y	y	0.02253	0.00606	4.9	0.31623	9.8	1.22927	27.4	10.26537		
k	y	o	0.03947	0.03327	4.7	0.48305	10.4	1.95505	23.8	14.52048	-1.503	0.167
m	x	y	0.08836	0.04029	2.9	0.31623	6.4	6.27517	23.5	6.15088		
m	x	o	0.03727	0.01746	3.7	0.67495	14.2	4.41714	22.1	3.5103	3.491	0.007
m	y	y	0.03229	0.01406	5	0	7.9	1.85293	24.1	12.02266		
m	y	o	0.02865	0.01616	5	0	10.9	1.66333	24.3	11.146	0.509	0.623
p	x	y	0.04114	0.03545	4.9	0.31623	12.8	4.51664	16.6	10.34086		
p	x	o	0.04396	0.05497	4.9	0.31623	13.2	2.74064	17.3	9.10494	-0.129	0.900
p	y	y	0.03816	0.04173	4.5	0.52705	10	1.41421	23.7	12.15685		
p	y	o	0.03257	0.0096	4.3	0.48305	9.8	0.78881	23.3	11.69093	0.392	0.704

The question can now again be asked if one age group has more variability than the other. There are some indications of statistical differences in the variability of some body parts between the old and young subjects while walking. Again, I will classify anything as significant that has a p-value of around 0.05 or less. The most significant difference is in the x-axis of the metatarsal. This difference is very pronounced with the young female's data being more variable or chaotic. The heel x-axis also has a statistical difference with the younger age group being more variable. The last difference is in the iliac crest y-axis but this time with the older females' possessing the most variability. This last difference was suggested by the p-value in the original data before noise reduction. However, the first two differences only appeared after noise reduction.

If future research is done in this area I would recommend a couple of changes in the way the data are collected. As previously mentioned, the data was taken from different sides of the body. The old subjects were measured on the right side and the young on the left. Because individuals possess a dominate leg just as a dominate hand, it could introduce some problems in the comparisons of the two age groups. If dominance in legs were equally distributed between the right and left in the population it would not be a skewing factor. However, I would recommend taking this factor into consideration if additional subjects are measured. Another change that may enhance the power of statistics is a greater sample size. I understand that additional subjects add time and money into any project, but it would also enhance the accuracy of the comparisons between the old and young age categories. For the comparisons that were done earlier in the paper between the two age groups, the underlying assumption that had to be made is that of normality. In other words, I assumed that the distributions of the Lyapunov exponents of all old and young body parts are normal. This assumption is fairly common, but if not accurate, all comparisons would be invalidated. To avoid this possibility the optimal sample size would be 30 or more for both age groups.

Noise reduction plays a factor in all data analysis because all data is corrupted at least with a small amount of measurement noise. For data from dynamical systems the noise can drastically change the properties of the system being studied. Therefore it is even more critical to reduce the noise from this type of data.

After the noise reduction in this paper, it appears that two parameters of young females' data are more chaotic, and one with the older age category. Some of the other parameters may in fact be different as well, but from this sample I am unable to distinguish any further differences. The fact that there appears to be differences in the average variability of the two age groups and some of the body parts could have important implications in the field of human performance. I will leave further conclusions to those versed in that discipline.

APPENDIX A – LyE Calculations for All of the Data

LyE Calculations for the Raw Data (Subject = SB, AG = Age Group, Body Part = BP, Axis = AX)																		
SB	**AG**	**BP**	**AX**	**LyE**	**m**	**J**	**W**	**DT**		**SB**	**AG**	**BP**	**AX**	**LyE**	**m**	**J**	**W**	**DT**
1	o	i	x	0.02756	4	18	220	25		1	y	i	x	0.03126	4	14	238	17
1	o	i	y	0.02115	5	11	220	12		1	y	i	y	0.01821	5	9	238	37
1	o	p	x	0.02503	5	18	220	18		1	y	p	x	0.08045	4	15	238	9
1	o	p	y	0.03636	4	11	220	13		1	y	p	y	0.02484	4	10	238	23
1	o	k	x	0.08073	3	18	220	13		1	y	k	x	0.05372	3	18	238	13
1	o	k	y	0.0145	5	12	220	30		1	y	k	y	0.02425	5	11	238	18
1	o	a	x	0.10661	4	4	220	20		1	y	a	x	0.09907	4	2	238	24
1	o	a	y	0.02142	5	13	220	17		1	y	a	y	0.02367	4	15	238	24
1	o	h	x	0.06958	5	4	220	33		1	y	h	x	0.09682	4	2	238	24
1	o	h	y	0.02299	5	13	220	27		1	y	h	y	0.02069	5	15	238	21
1	o	m	x	0.08188	5	3	220	34		1	y	m	x	0.10216	4	2	238	24
1	o	m	y	0.02403	5	11	220	28		1	y	m	y	0.06589	5	5	238	14
2	o	i	x	0.05241	4	18	238	14		2	y	i	x	0.02836	4	13	239	21
2	o	i	y	0.03103	5	9	238	17		2	y	i	y	0.02266	4	9	239	30
2	o	p	x	0.0525	4	18	238	14		2	y	p	x	0.02519	4	14	239	17
2	o	p	y	0.03018	5	10	238	15		2	y	p	y	0.02336	4	9	239	28
2	o	k	x	0.04314	3	18	238	17		2	y	k	x	0.03327	3	16	239	17
2	o	k	y	0.11156	5	13	238	7		2	y	k	y	0.01616	5	9	239	34
2	o	a	x	0.08975	5	4	238	20		2	y	a	x	0.03102	3	16	239	21
2	o	a	y	0.03638	5	14	238	13		2	y	a	y	0.01861	5	12	239	16
2	o	h	x	0.08959	5	4	238	23		2	y	h	x	0.07768	5	3	239	20
2	o	h	y	0.11007	5	14	238	9		2	y	h	y	0.04997	5	13	239	10
2	o	m	x	0.10727	4	3	238	22		2	y	m	x	0.10348	4	2	239	18
2	o	m	y	0.01951	5	13	238	16		2	y	m	y	0.02473	5	8	239	23
3	o	i	x	0.01272	5	16	255	19		3	y	i	x	0.10345	4	16	239	5
3	o	i	y	0.02006	5	10	255	26		3	y	i	y	0.01865	5	10	239	40
3	o	p	x	0.01269	5	16	255	19		3	y	p	x	0.10284	4	15	239	5
3	o	p	y	0.03786	4	9	255	19		3	y	p	y	0.01819	5	11	239	27
3	o	k	x	0.03051	3	18	255	22		3	y	k	x	0.0245	4	18	239	19
3	o	k	y	0.02159	5	11	255	13		3	y	k	y	0.0209	5	10	239	36
3	o	a	x	0.09043	3	5	255	24		3	y	a	x	0.09734	3	4	239	23
3	o	a	y	0.02068	5	14	255	19		3	y	a	y	0.04147	5	15	239	14
3	o	h	x	0.07145	5	4	255	28		3	y	h	x	0.05914	5	4	239	44
3	o	h	y	0.021	5	13	255	31		3	y	h	y	0.03299	5	14	239	22
3	o	m	x	0.07448	5	4	255	28		3	y	m	x	0.07839	3	6	239	28
3	o	m	y	0.02114	5	10	255	34		3	y	m	y	0.02534	5	10	239	28
4	o	i	x	0.01305	5	18	214	22		4	y	i	x	0.0187	5	15	239	19
4	o	i	y	0.04978	4	9	214	13		4	y	i	y	0.02257	5	11	239	18
4	o	p	x	0.01946	5	17	214	19		4	y	p	x	0.05372	3	14	239	15

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
4	o	p	y	0.03423	5	9	214	14		4	y	p	y	0.03163	4	10	239	25
4	o	k	x	0.06688	4	18	214	12		4	y	k	x	0.08174	3	18	239	11
4	o	k	y	0.02961	5	8	214	19		4	y	k	y	0.03568	4	11	239	18
4	o	a	x	0.04589	3	15	214	21		4	y	a	x	0.09053	5	2	239	26
4	o	a	y	0.02899	5	12	214	19		4	y	a	y	0.02417	4	13	239	26
4	o	h	x	0.08895	4	5	214	23		4	y	h	x	0.09641	4	2	239	25
4	o	h	y	0.0232	5	12	214	29		4	y	h	y	0.02104	5	14	239	37
4	o	m	x	0.1151	5	2	214	20		4	y	m	x	0.06562	5	4	239	33
4	o	m	y	0.02837	5	11	214	20		4	y	m	y	0.0197	5	9	239	45
5	o	i	x	0.07075	5	18	291	5		5	y	i	x	0.0305	4	13	238	18
5	o	i	y	0.03348	5	9	291	19		5	y	i	y	0.02907	4	9	238	22
5	o	p	x	0.08855	4	18	291	8		5	y	p	x	0.02762	4	13	238	18
5	o	p	y	0.02888	5	10	291	19		5	y	p	y	0.02883	4	9	238	23
5	o	k	x	0.01396	5	18	291	21		5	y	k	x	0.04194	3	15	238	15
5	o	k	y	0.02683	5	7	291	40		5	y	k	y	0.02062	5	9	238	26
5	o	a	x	0.09731	5	3	291	21		5	y	a	x	0.0356	4	7	238	44
5	o	a	y	0.02838	5	13	291	20		5	y	a	y	0.01411	5	12	238	39
5	o	h	x	0.05869	5	6	291	24		5	y	h	x	0.06934	3	5	238	32
5	o	h	y	0.02204	5	13	291	28		5	y	h	y	0.01475	5	13	238	30
5	o	m	x	0.08978	5	3	291	23		5	y	m	x	0.07287	4	3	238	31
5	o	m	y	0.01988	5	13	291	14		5	y	m	y	0.02412	5	8	238	20
6	o	i	x	0.09255	5	14	262	8		6	y	i	x	0.03536	4	12	239	24
6	o	i	y	0.01925	5	14	262	14		6	y	i	y	0.02549	4	10	239	21
6	o	p	x	0.08259	5	15	262	8		6	y	p	x	0.03583	4	10	239	22
6	o	p	y	0.01868	4	9	262	50		6	y	p	y	0.01971	4	9	239	47
6	o	k	x	0.02169	4	16	262	33		6	y	k	x	0.03456	3	16	239	24
6	o	k	y	0.01604	5	10	262	59		6	y	k	y	0.01696	5	8	239	50
6	o	a	x	0.04957	5	3	262	46		6	y	a	x	0.02817	3	17	239	43
6	o	a	y	0.01827	5	14	262	36		6	y	a	y	0.02552	4	14	239	22
6	o	h	x	0.06403	5	2	262	38		6	y	h	x	0.0732	4	3	239	28
6	o	h	y	0.02155	5	13	262	37		6	y	h	y	0.02154	4	13	239	31
6	o	m	x	0.0648	5	2	262	37		6	y	m	x	0.08758	4	2	239	25
6	o	m	y	0.02161	5	9	262	38		6	y	m	y	0.01876	5	8	239	40
7	o	i	x	0.04616	4	18	269	15		7	y	i	x	0.04659	4	13	239	13
7	o	i	y	0.03822	4	10	269	20		7	y	i	y	0.03373	4	8	239	22
7	o	p	x	0.05286	4	18	269	14		7	y	p	x	0.03271	4	12	239	15
7	o	p	y	0.02398	5	9	269	32		7	y	p	y	0.02913	5	7	239	17
7	o	k	x	0.0532	3	17	269	15		7	y	k	x	0.03158	3	15	239	26
7	o	k	y	0.02744	5	8	269	29		7	y	k	y	0.02386	5	8	239	20
7	o	a	x	0.0159	5	16	269	47		7	y	a	x	0.03202	3	15	239	33
7	o	a	y	0.02333	5	14	269	25		7	y	a	y	0.01796	5	11	239	32
7	o	h	x	0.11319	5	2	269	23		7	y	h	x	0.03395	3	15	239	32
7	o	h	y	0.02104	5	13	269	27		7	y	h	y	0.02507	4	13	239	18
7	o	m	x	0.10269	5	2	269	24		7	y	m	x	0.06542	4	5	239	29
7	o	m	y	0.08976	5	13	269	10		7	y	m	y	0.02789	5	6	239	33
8	o	i	x	0.05602	4	17	231	9		8	y	i	x	0.02114	4	16	239	32
8	o	i	y	0.0402	4	12	231	12		8	y	i	y	0.01986	5	11	239	21

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
8	o	p	x	0.01793	4	18	231	23		8	y	p	x	0.01608	5	17	239	24
8	o	p	y	0.03864	4	11	231	13		8	y	p	y	0.03788	5	12	239	11
8	o	k	x	0.03662	3	18	231	20		8	y	k	x	0.03318	3	17	239	25
8	o	k	y	0.07624	5	13	231	9		8	y	k	y	0.02189	5	11	239	16
8	o	a	x	0.08002	4	3	231	28		8	y	a	x	0.11304	3	3	239	21
8	o	a	y	0.01294	5	17	231	19		8	y	a	y	0.01647	5	15	239	20
8	o	h	x	0.07097	5	3	231	30		8	y	h	x	0.10773	4	2	239	24
8	o	h	y	0.01643	5	16	231	20		8	y	h	y	0.01923	5	14	239	19
8	o	m	x	0.07159	5	3	231	29		8	y	m	x	0.10824	4	2	239	23
8	o	m	y	0.11318	5	11	231	7		8	y	m	y	0.02608	5	10	239	12
9	o	i	x	0.0221	4	18	269	19		9	y	i	x	0.07809	4	18	238	10
9	o	i	y	0.02786	5	10	269	14		9	y	i	y	0.02091	5	12	238	17
9	o	p	x	0.02387	4	15	269	21		9	y	p	x	0.15777	4	18	238	6
9	o	p	y	0.03089	4	10	269	21		9	y	p	y	0.06011	4	13	238	11
9	o	k	x	0.01592	4	18	269	26		9	y	k	x	0.10085	3	18	238	9
9	o	k	y	0.02072	5	10	269	21		9	y	k	y	0.04227	5	12	238	11
9	o	a	x	0.04611	4	7	269	43		9	y	a	x	0.09764	4	2	238	22
9	o	a	y	0.02981	4	13	269	22		9	y	a	y	0.01327	5	15	238	16
9	o	h	x	0.0658	5	4	269	36		9	y	h	x	0.07428	4	4	238	30
9	o	h	y	0.01938	5	14	269	17		9	y	h	y	0.01639	5	14	238	18
9	o	m	x	0.0471	5	6	269	42		9	y	m	x	0.08496	3	4	238	28
9	o	m	y	0.01892	5	11	269	27		9	y	m	y	0.0438	5	8	238	13
0	o	i	x	0.02316	4	16	232	17		0	y	i	x	0.09986	4	16	239	9
0	o	i	y	0.03036	4	8	232	34		0	y	i	y	0.01856	5	9	239	31
0	o	p	x	0.03237	4	16	232	15		0	y	p	x	0.09083	4	14	239	9
0	o	p	y	0.02277	4	10	232	34		0	y	p	y	0.02222	4	9	239	28
0	o	k	x	0.01884	4	16	232	18		0	y	k	x	0.03936	3	16	239	18
0	o	k	y	0.02976	4	8	232	33		0	y	k	y	0.01745	5	9	239	32
0	o	a	x	0.03557	3	16	232	24		0	y	a	x	0.0287	3	16	239	42
0	o	a	y	0.02271	4	14	232	17		0	y	a	y	0.01544	5	13	239	29
0	o	h	x	0.1398	4	2	232	16		0	y	h	x	0.10868	3	3	239	17
0	o	h	y	0.02587	4	13	232	29		0	y	h	y	0.01721	5	13	239	28
0	o	m	x	0.12329	4	2	232	18		0	y	m	x	0.12401	3	2	239	16
0	o	m	y	0.01946	5	9	232	40		0	y	m	y	0.03391	5	7	239	18

LyE Calculations for the ADM Noise Reduced Data
(Subject = SB, AG = Age Group, Body Part = BP, Axis = AX)

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
1	o	i	x	0.0161	5	15	220	15		1	y	i	x	0.02081	5	17	238	16
1	o	i	y	0.02115	5	11	220	12		1	y	i	y	0.01821	5	9	238	37
1	o	p	x	0.0161	5	15	220	15		1	y	p	x	0.02081	5	17	238	16
1	o	p	y	0.03636	4	11	220	13		1	y	p	y	0.02484	4	10	238	23
1	o	k	x	0.02516	4	15	220	22		1	y	k	x	0.02245	4	17	238	31
1	o	k	y	0.0145	5	12	220	30		1	y	k	y	0.02425	5	11	238	18

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
1	o	a	x	0.0438	3	15	220	22		1	y	a	x	0.10887	3	2	238	20
1	o	a	y	0.02142	5	13	220	17		1	y	a	y	0.02367	4	15	238	24
1	o	h	x	0.02808	4	15	220	23		1	y	h	x	0.09339	4	2	238	22
1	o	h	y	0.02299	5	13	220	27		1	y	h	y	0.02069	5	15	238	21
1	o	m	x	0.02893	4	15	220	22		1	y	m	x	0.06262	3	6	238	31
1	o	m	y	0.02403	5	11	220	28		1	y	m	y	0.06589	5	5	238	14
2	o	i	x	0.07362	5	15	238	10		2	y	i	x	0.16419	5	16	239	4
2	o	i	y	0.05356	4	12	238	11		2	y	i	y	0.02266	4	9	239	30
2	o	p	x	0.07362	5	15	238	10		2	y	p	x	0.16419	5	16	239	4
2	o	p	y	0.0274	5	11	238	12		2	y	p	y	0.02336	4	9	239	28
2	o	k	x	0.08266	4	16	238	9		2	y	k	x	0.12717	3	16	239	7
2	o	k	y	0.11207	5	13	238	7		2	y	k	y	0.01616	5	9	239	34
2	o	a	x	0.0158	5	16	238	21		2	y	a	x	0.08537	4	3	239	19
2	o	a	y	0.03647	5	14	238	13		2	y	a	y	0.01861	5	12	239	16
2	o	h	x	0.0228	4	16	238	18		2	y	h	x	0.11514	3	2	239	16
2	o	h	y	0.11036	5	14	238	9		2	y	h	y	0.04997	5	13	239	10
2	o	m	x	0.06303	3	16	238	15		2	y	m	x	0.1079	4	2	239	16
2	o	m	y	0.01952	5	13	238	16		2	y	m	y	0.02473	5	8	239	23
3	o	i	x	0.18934	5	13	255	5		3	y	i	x	0.02498	5	8	239	21
3	o	i	y	0.02006	5	10	255	26		3	y	i	y	0.01865	5	10	239	40
3	o	p	x	0.18934	5	13	255	5		3	y	p	x	0.02498	5	8	239	21
3	o	p	y	0.03786	4	9	255	19		3	y	p	y	0.01819	5	11	239	27
3	o	k	x	0.01767	4	17	255	20		3	y	k	x	0.02766	4	18	239	18
3	o	k	y	0.02159	5	11	255	13		3	y	k	y	0.0209	5	10	239	36
3	o	a	x	0.03345	3	17	255	21		3	y	a	x	0.02236	4	18	239	43
3	o	a	y	0.02068	5	14	255	19		3	y	a	y	0.04147	5	15	239	14
3	o	h	x	0.09398	5	2	255	19		3	y	h	x	0.04328	3	18	239	19
3	o	h	y	0.021	5	13	255	31		3	y	h	y	0.03299	5	14	239	22
3	o	m	x	0.03827	3	16	255	19		3	y	m	x	0.03836	3	17	239	29
3	o	m	y	0.02114	5	10	255	34		3	y	m	y	0.02534	5	10	239	28
4	o	i	x	0.03684	5	14	214	12		4	y	i	x	0.05383	5	15	239	11
4	o	i	y	0.04978	4	9	214	13		4	y	i	y	0.02257	5	11	239	18
4	o	p	x	0.03684	5	14	214	12		4	y	p	x	0.05383	5	15	239	11
4	o	p	y	0.03423	5	9	214	14		4	y	p	y	0.03163	4	10	239	25
4	o	k	x	0.11289	3	14	214	10		4	y	k	x	0.04466	3	17	239	16
4	o	k	y	0.02961	5	8	214	19		4	y	k	y	0.03568	4	11	239	18
4	o	a	x	0.02901	4	14	214	25		4	y	a	x	0.07741	5	2	239	25
4	o	a	y	0.02899	5	12	214	19		4	y	a	y	0.02417	4	13	239	26
4	o	h	x	0.03235	4	14	214	22		4	y	h	x	0.07486	5	2	239	25
4	o	h	y	0.0232	5	12	214	29		4	y	h	y	0.02104	5	14	239	37
4	o	m	x	0.0444	3	14	214	19		4	y	m	x	0.0616	4	4	239	30
4	o	m	y	0.02837	5	11	214	20		4	y	m	y	0.0197	5	9	239	45
5	o	i	x	0.01312	5	15	251	17		5	y	i	x	0.01012	5	16	238	23
5	o	i	y	0.03121	5	9	251	21		5	y	i	y	0.02907	4	9	238	22
5	o	p	x	0.01312	5	15	251	17		5	y	p	x	0.01012	5	16	238	23
5	o	p	y	0.02563	5	10	251	21		5	y	p	y	0.02883	4	9	238	23
5	o	k	x	0.06656	3	16	251	14		5	y	k	x	0.03632	3	15	238	17

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
5	o	k	y	0.03202	4	11	251	16		5	y	k	y	0.02062	5	9	238	26
5	o	a	x	0.09484	5	2	251	21		5	y	a	x	0.06108	4	3	238	28
5	o	a	y	0.03525	4	13	251	20		5	y	a	y	0.01411	5	12	238	39
5	o	h	x	0.01923	5	16	251	27		5	y	h	x	0.08095	3	3	238	24
5	o	h	y	0.02563	5	13	251	17		5	y	h	y	0.01475	5	13	238	30
5	o	m	x	0.02951	4	15	251	25		5	y	m	x	0.08499	4	2	238	22
5	o	m	y	0.01765	5	13	251	16		5	y	m	y	0.02412	5	8	238	20
6	o	i	x	0.0131	5	9	262	40		6	y	i	x	0.0133	5	9	239	38
6	o	i	y	0.01925	5	14	262	14		6	y	i	y	0.02549	4	10	239	21
6	o	p	x	0.0131	5	9	262	40		6	y	p	x	0.0133	5	9	239	38
6	o	p	y	0.01868	4	9	262	50		6	y	p	y	0.01971	4	9	239	47
6	o	k	x	0.01546	5	15	262	30		6	y	k	x	0.01847	4	16	239	30
6	o	k	y	0.01604	5	10	262	59		6	y	k	y	0.01696	5	8	239	50
6	o	a	x	0.02658	3	16	262	37		6	y	a	x	0.08838	3	2	239	22
6	o	a	y	0.01827	5	14	262	36		6	y	a	y	0.02552	4	14	239	22
6	o	h	x	0.06286	5	2	262	29		6	y	h	x	0.07811	4	2	239	23
6	o	h	y	0.02155	5	13	262	37		6	y	h	y	0.02154	4	13	239	31
6	o	m	x	0.06873	4	2	262	27		6	y	m	x	0.08204	4	2	239	23
6	o	m	y	0.02161	5	9	262	38		6	y	m	y	0.01876	5	8	239	40
7	o	i	x	0.01734	5	11	269	22		7	y	i	x	0.02277	5	7	239	23
7	o	i	y	0.03822	4	10	269	20		7	y	i	y	0.03373	4	8	239	22
7	o	p	x	0.01734	5	11	269	22		7	y	p	x	0.02277	5	7	239	23
7	o	p	y	0.02398	5	9	269	32		7	y	p	y	0.02913	5	7	239	17
7	o	k	x	0.02417	4	15	269	21		7	y	k	x	0.04277	4	15	239	13
7	o	k	y	0.02744	5	8	269	29		7	y	k	y	0.02386	5	8	239	20
7	o	a	x	0.02392	4	15	269	36		7	y	a	x	0.0343	3	15	239	28
7	o	a	y	0.02333	5	14	269	25		7	y	a	y	0.01796	5	11	239	32
7	o	h	x	0.02444	4	15	269	32		7	y	h	x	0.0674	4	4	239	27
7	o	h	y	0.02104	5	13	269	27		7	y	h	y	0.02507	4	13	239	18
7	o	m	x	0.02649	4	15	269	26		7	y	m	x	0.07712	4	3	239	25
7	o	m	y	0.08976	5	13	269	10		7	y	m	y	0.02789	5	6	239	33
8	o	i	x	0.05763	4	12	231	0		8	y	i	x	0.03759	5	7	239	0
8	o	i	y	0.0402	4	12	231	12		8	y	i	y	0.01986	5	11	239	21
8	o	p	x	0.05763	4	12	231	0		8	y	p	x	0.03759	5	7	239	0
8	o	p	y	0.03864	4	11	231	13		8	y	p	y	0.03788	5	12	239	11
8	o	k	x	0.08563	4	18	231	10		8	y	k	x	0.05408	3	16	239	15
8	o	k	y	0.07624	5	13	231	9		8	y	k	y	0.02189	5	11	239	16
8	o	a	x	0.06901	3	18	231	12		8	y	a	x	0.09929	4	2	239	21
8	o	a	y	0.01294	5	17	231	19		8	y	a	y	0.01647	5	15	239	20
8	o	h	x	0.01615	5	18	231	28		8	y	h	x	0.15731	3	2	239	13
8	o	h	y	0.01643	5	16	231	20		8	y	h	y	0.01923	5	14	239	19
8	o	m	x	0.01999	4	18	231	23		8	y	m	x	0.11666	4	2	239	19
8	o	m	y	0.11318	5	11	231	7		8	y	m	y	0.02608	5	10	239	12
9	o	i	x	0.04823	5	17	269	11		9	y	i	x	0.07267	4	17	238	0
9	o	i	y	0.02786	5	10	269	14		9	y	i	y	0.02091	5	12	238	17
9	o	p	x	0.04823	5	17	269	11		9	y	p	x	0.07267	4	17	238	0
9	o	p	y	0.03089	4	10	269	21		9	y	p	y	0.06011	4	13	238	11

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
9	o	k	x	0.02523	4	17	269	25		9	y	k	x	0.0951	3	18	238	11
9	o	k	y	0.02072	5	10	269	21		9	y	k	y	0.04227	5	12	238	11
9	o	a	x	0.09614	5	2	269	20		9	y	a	x	0.02444	4	18	238	31
9	o	a	y	0.02981	4	13	269	22		9	y	a	y	0.01327	5	15	238	16
9	o	h	x	0.02696	4	17	269	25		9	y	h	x	0.06549	3	6	238	30
9	o	h	y	0.01938	5	14	269	17		9	y	h	y	0.01639	5	14	238	18
9	o	m	x	0.02757	4	16	269	24		9	y	m	x	0.02722	4	17	238	28
9	o	m	y	0.01892	5	11	269	27		9	y	m	y	0.0438	5	8	238	13
0	o	i	x	0.03252	5	6	232	16		0	y	i	x	0.02501	5	7	239	23
0	o	i	y	0.03036	4	8	232	34		0	y	i	y	0.01856	5	9	239	31
0	o	p	x	0.03237	4	16	232	15		0	y	p	x	0.02501	5	7	239	23
0	o	p	y	0.02277	4	10	232	34		0	y	p	y	0.02222	4	9	239	28
0	o	k	x	0.02233	4	15	232	24		0	y	k	x	0.03225	3	16	239	18
0	o	k	y	0.02976	4	8	232	33		0	y	k	y	0.01745	5	9	239	32
0	o	a	x	0.03731	3	15	232	22		0	y	a	x	0.09639	3	3	239	18
0	o	a	y	0.02271	4	14	232	17		0	y	a	y	0.01544	5	13	239	29
0	o	h	x	0.03961	3	15	232	19		0	y	h	x	0.11333	3	2	239	16
0	o	h	y	0.02587	4	13	232	29		0	y	h	y	0.01721	5	13	239	28
0	o	m	x	0.04042	3	15	232	19		0	y	m	x	0.11719	3	2	239	15
0	o	m	y	0.01946	5	9	232	40		0	y	m	y	0.03391	5	7	239	18

LyE Calculations for the Simple Nonlinear Noise Reduced Data
(Subject = SB, AG = Age Group, Body Part = BP, Axis = AX)

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
1	o	i	x	0.01606	5	15	220	15		1	y	i	x	0.0125	5	17	238	17
1	o	i	y	0.0211	5	11	220	12		1	y	i	y	0.0184	5	9	238	37
1	o	p	x	0.01606	5	15	220	15		1	y	p	x	0.0125	5	17	238	17
1	o	p	y	0.02052	5	11	220	15		1	y	p	y	0.01755	5	9	238	34
1	o	k	x	0.02906	4	15	220	26		1	y	k	x	0.04282	3	17	238	22
1	o	k	y	0.01582	5	12	220	31		1	y	k	y	0.02448	5	11	238	19
1	o	a	x	0.04711	3	15	220	24		1	y	a	x	0.10898	5	2	238	24
1	o	a	y	0.02978	5	13	220	19		1	y	a	y	0.03219	4	15	238	36
1	o	h	x	0.0296	4	15	220	28		1	y	h	x	0.10703	5	2	238	26
1	o	h	y	0.02716	5	13	220	29		1	y	h	y	0.03296	5	15	238	36
1	o	m	x	0.13895	5	2	220	19		1	y	m	x	0.06053	4	6	238	41
1	o	m	y	0.02769	5	11	220	28		1	y	m	y	0.01828	5	10	238	59
2	o	i	x	0.08967	5	17	238	9		2	y	i	x	0.16321	5	16	239	4
2	o	i	y	0.05436	4	12	238	11		2	y	i	y	0.01749	5	9	239	39
2	o	p	x	0.08967	5	17	238	9		2	y	p	x	0.16321	5	16	239	4
2	o	p	y	0.02999	5	10	238	16		2	y	p	y	0.01759	5	9	239	38
2	o	k	x	0.05645	4	16	238	12		2	y	k	x	0.0189	5	16	239	30
2	o	k	y	0.05404	5	11	238	10		2	y	k	y	0.01942	5	10	239	37
2	o	a	x	0.02663	4	16	238	25		2	y	a	x	0.08367	5	3	239	25
2	o	a	y	0.02242	5	14	238	15		2	y	a	y	0.02475	5	13	239	28

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
2	o	h	x	0.11167	3	16	238	9		2	y	h	x	0.09232	5	3	239	22
2	o	h	y	0.08272	5	14	238	11		2	y	h	y	0.02633	5	13	239	29
2	o	m	x	0.05262	3	16	238	18		2	y	m	x	0.14798	5	2	239	18
2	o	m	y	0.17758	5	13	238	6		2	y	m	y	0.02826	5	8	239	23
3	o	i	x	0.18878	5	13	255	5		3	y	i	x	0.11475	5	18	239	7
3	o	i	y	0.02	5	10	255	26		3	y	i	y	0.01916	5	10	239	39
3	o	p	x	0.18868	5	13	255	5		3	y	p	x	0.11469	5	18	239	7
3	o	p	y	0.03662	4	10	255	19		3	y	p	y	0.13147	4	11	239	7
3	o	k	x	0.02268	4	17	255	22		3	y	k	x	0.03972	3	18	239	21
3	o	k	y	0.02773	5	11	255	15		3	y	k	y	0.05203	4	11	239	11
3	o	a	x	0.12266	4	2	255	20		3	y	a	x	0.04192	3	18	239	26
3	o	a	y	0.0285	5	14	255	27		3	y	a	y	0.02667	5	15	239	17
3	o	h	x	0.11706	4	2	255	21		3	y	h	x	0.02437	4	18	239	45
3	o	h	y	0.02842	5	13	255	35		3	y	h	y	0.04485	5	14	239	24
3	o	m	x	0.10661	5	2	255	22		3	y	m	x	0.02566	4	17	239	45
3	o	m	y	0.02313	5	10	255	34		3	y	m	y	0.02757	5	10	239	29
4	o	i	x	0.01878	5	12	214	16		4	y	i	x	0.05376	5	15	239	11
4	o	i	y	0.05039	4	9	214	14		4	y	i	y	0.02264	5	11	239	19
4	o	p	x	0.01878	5	12	214	16		4	y	p	x	0.05376	5	15	239	11
4	o	p	y	0.03403	5	9	214	15		4	y	p	y	0.02387	5	10	239	28
4	o	k	x	0.09552	3	14	214	11		4	y	k	x	0.01781	5	17	239	40
4	o	k	y	0.02976	5	8	214	20		4	y	k	y	0.03819	4	11	239	23
4	o	a	x	0.04967	3	14	214	24		4	y	a	x	0.09894	5	2	239	28
4	o	a	y	0.03295	5	12	214	22		4	y	a	y	0.02359	5	15	239	38
4	o	h	x	0.04903	3	14	214	23		4	y	h	x	0.0994	5	2	239	26
4	o	h	y	0.02656	5	12	214	29		4	y	h	y	0.0288	5	14	239	92
4	o	m	x	0.03408	4	14	214	24		4	y	m	x	0.09755	5	2	239	28
4	o	m	y	0.0265	5	11	214	21		4	y	m	y	0.03194	5	9	239	37
5	o	i	x	0.01315	5	15	251	17		5	y	i	x	0.0103	5	16	238	23
5	o	i	y	0.03099	5	9	251	22		5	y	i	y	0.03132	4	9	238	22
5	o	p	x	0.01315	5	15	251	17		5	y	p	x	0.01027	5	16	238	23
5	o	p	y	0.04043	4	9	251	20		5	y	p	y	0.02991	4	9	238	23
5	o	k	x	0.03011	4	16	251	20		5	y	k	x	0.02986	4	15	238	29
5	o	k	y	0.02791	5	11	251	19		5	y	k	y	0.01906	5	10	238	33
5	o	a	x	0.10308	5	2	251	25		5	y	a	x	0.04623	5	5	238	44
5	o	a	y	0.03588	5	13	251	31		5	y	a	y	0.024	5	14	238	44
5	o	h	x	0.10536	5	2	251	24		5	y	h	x	0.07742	5	3	238	30
5	o	h	y	0.04075	5	13	251	26		5	y	h	y	0.02612	5	13	238	48
5	o	m	x	0.1166	5	2	251	22		5	y	m	x	0.09112	5	3	238	27
5	o	m	y	0.02229	5	13	251	17		5	y	m	y	0.02974	5	8	238	22
6	o	i	x	0.0131	5	9	262	40		6	y	i	x	0.01239	5	10	239	33
6	o	i	y	0.01409	5	14	262	15		6	y	i	y	0.04599	3	10	239	20
6	o	p	x	0.0131	5	9	262	40		6	y	p	x	0.01239	5	10	239	33
6	o	p	y	0.02057	4	9	262	50		6	y	p	y	0.02149	4	9	239	49
6	o	k	x	0.02607	4	15	262	34		6	y	k	x	0.02316	4	16	239	46
6	o	k	y	0.02256	5	10	262	52		6	y	k	y	0.02155	5	9	239	49
6	o	a	x	0.08345	5	2	262	29		6	y	a	x	0.07112	5	3	239	30

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
6	o	a	y	0.02763	5	14	262	51		6	y	a	y	0.02651	5	14	239	44
6	o	h	x	0.07787	5	2	262	32		6	y	h	x	0.06115	5	3	239	40
6	o	h	y	0.03387	5	13	262	59		6	y	h	y	0.02807	5	13	239	52
6	o	m	x	0.08529	5	2	262	29		6	y	m	x	0.11885	5	2	239	28
6	o	m	y	0.02254	5	10	262	38		6	y	m	y	0.02529	5	8	239	37
7	o	i	x	0.02869	4	12	269	15		7	y	i	x	0.02369	5	7	239	23
7	o	i	y	0.03989	4	10	269	20		7	y	i	y	0.02684	5	8	239	26
7	o	p	x	0.02869	4	12	269	15		7	y	p	x	0.02369	5	7	239	23
7	o	p	y	0.03516	4	9	269	27		7	y	p	y	0.02539	5	8	239	20
7	o	k	x	0.04541	3	15	269	22		7	y	k	x	0.03024	4	15	239	31
7	o	k	y	0.02881	5	8	269	30		7	y	k	y	0.02923	5	8	239	24
7	o	a	x	0.02613	4	15	269	41		7	y	a	x	0.02934	4	15	239	43
7	o	a	y	0.02361	5	14	269	31		7	y	a	y	0.02741	5	14	239	24
7	o	h	x	0.01881	5	15	269	41		7	y	h	x	0.06695	5	4	239	34
7	o	h	y	0.02871	5	13	269	32		7	y	h	y	0.02658	5	13	239	28
7	o	m	x	0.11742	5	2	269	23		7	y	m	x	0.08886	5	3	239	29
7	o	m	y	0.07424	5	13	269	11		7	y	m	y	0.03444	5	6	239	31
8	o	i	x	0.01834	5	12	231	17		8	y	i	x	0.03762	5	7	239	0
8	o	i	y	0.03651	4	12	231	13		8	y	i	y	0.02001	5	11	239	21
8	o	p	x	0.01837	5	12	231	17		8	y	p	x	0.03762	5	7	239	0
8	o	p	y	0.02613	5	11	231	14		8	y	p	y	0.03791	5	12	239	11
8	o	k	x	0.05944	4	18	231	13		8	y	k	x	0.04966	3	16	239	20
8	o	k	y	0.06727	5	13	231	10		8	y	k	y	0.02189	5	11	239	19
8	o	a	x	0.04342	3	18	231	18		8	y	a	x	0.09826	5	2	239	25
8	o	a	y	0.0327	5	17	231	15		8	y	a	y	0.02255	5	15	239	29
8	o	h	x	0.01556	5	18	231	53		8	y	h	x	0.11009	5	2	239	22
8	o	h	y	0.02391	5	16	231	37		8	y	h	y	0.02605	5	14	239	31
8	o	m	x	0.03663	3	18	231	30		8	y	m	x	0.10812	5	2	239	23
8	o	m	y	0.10423	5	11	231	8		8	y	m	y	0.03739	5	10	239	15
9	o	i	x	0.04195	5	17	269	12		9	y	i	x	0.07349	4	18	238	0
9	o	i	y	0.02796	5	10	269	13		9	y	i	y	0.02902	4	12	238	17
9	o	p	x	0.04195	5	17	269	12		9	y	p	x	0.07349	4	18	238	0
9	o	p	y	0.03156	4	10	269	21		9	y	p	y	0.06088	4	13	238	11
9	o	k	x	0.02889	4	17	269	35		9	y	k	x	0.04222	4	18	238	15
9	o	k	y	0.02193	5	10	269	23		9	y	k	y	0.02623	5	12	238	13
9	o	a	x	0.10716	5	2	269	24		9	y	a	x	0.09964	5	2	238	26
9	o	a	y	0.02998	5	13	269	33		9	y	a	y	0.02337	5	15	238	23
9	o	h	x	0.12105	5	2	269	21		9	y	h	x	0.04183	3	18	238	32
9	o	h	y	0.02867	5	14	269	28		9	y	h	y	0.02867	5	14	238	38
9	o	m	x	0.11272	5	2	269	22		9	y	m	x	0.05442	5	5	238	39
9	o	m	y	0.02335	5	11	269	28		9	y	m	y	0.03433	5	10	238	13
0	o	i	x	0.03185	5	6	232	17		0	y	i	x	0.02585	5	7	239	23
0	o	i	y	0.03406	4	8	232	32		0	y	i	y	0.02832	4	9	239	27
0	o	p	x	0.03185	5	6	232	17		0	y	p	x	0.02585	5	7	239	23
0	o	p	y	0.02436	4	10	232	34		0	y	p	y	0.01797	5	9	239	35
0	o	k	x	0.02567	4	15	232	27		0	y	k	x	0.04306	3	16	239	28
0	o	k	y	0.03525	4	8	232	31		0	y	k	y	0.02165	5	9	239	34

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
0	o	a	x	0.02319	4	15	232	34		0	y	a	x	0.0758	5	3	239	36
0	o	a	y	0.03409	4	14	232	21		0	y	a	y	0.03014	5	13	239	20
0	o	h	x	0.04027	3	15	232	25		0	y	h	x	0.0917	5	3	239	30
0	o	h	y	0.02727	5	13	232	42		0	y	h	y	0.03103	5	13	239	58
0	o	m	x	0.02197	4	15	232	53		0	y	m	x	0.12624	5	2	239	25
0	o	m	y	0.02155	5	9	232	37		0	y	m	y	0.04554	5	7	239	18

LyE Calculations for the Locally Projective Nonlinear Noise Reduced Data (Subject = SB, AG = Age Group, Body Part = BP, Axis = AX)																		
SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
1	o	i	x	0.02095	4	15	220	16		1	y	i	x	0.01005	5	16	238	18
1	o	i	y	0.01987	5	11	220	13		1	y	i	y	0.01953	5	12	238	21
1	o	p	x	0.01614	5	14	220	17		1	y	p	x	0.01726	4	17	238	17
1	o	p	y	0.03669	4	11	220	13		1	y	p	y	0.01503	5	10	238	32
1	o	k	x	0.02044	5	15	220	24		1	y	k	x	0.02263	4	17	238	31
1	o	k	y	0.01483	5	12	220	30		1	y	k	y	0.02385	5	11	238	19
1	o	a	x	0.04373	3	15	220	22		1	y	a	x	0.1079	4	2	238	21
1	o	a	y	0.02172	5	13	220	17		1	y	a	y	0.02345	4	15	238	24
1	o	h	x	0.04479	3	15	220	21		1	y	h	x	0.06421	4	4	238	30
1	o	h	y	0.02249	5	13	220	28		1	y	h	y	0.02079	5	15	238	21
1	o	m	x	0.02923	4	15	220	22		1	y	m	x	0.06324	3	6	238	31
1	o	m	y	0.02454	5	11	220	28		1	y	m	y	0.01617	5	10	238	59
2	o	i	x	0.09246	4	17	238	0		2	y	i	x	0.16327	5	16	239	4
2	o	i	y	0.05361	4	12	238	11		2	y	i	y	0.0231	4	9	239	30
2	o	p	x	0.09187	5	17	238	9		2	y	p	x	0.1299	5	16	239	5
2	o	p	y	0.03092	5	10	238	15		2	y	p	y	0.02318	4	9	239	29
2	o	k	x	0.084	4	16	238	9		2	y	k	x	0.0161	4	16	239	20
2	o	k	y	0.11864	4	13	238	8		2	y	k	y	0.01631	5	9	239	35
2	o	a	x	0.13118	3	16	238	8		2	y	a	x	0.10028	5	2	239	17
2	o	a	y	0.03659	5	14	238	13		2	y	a	y	0.01887	5	12	239	16
2	o	h	x	0.02279	4	16	238	18		2	y	h	x	0.13042	3	2	239	16
2	o	h	y	0.1109	5	14	238	9		2	y	h	y	0.03987	5	13	239	11
2	o	m	x	0.02736	4	16	238	17		2	y	m	x	0.12546	3	2	239	16
2	o	m	y	0.01996	5	13	238	16		2	y	m	y	0.02482	5	8	239	23
3	o	i	x	0.04238	5	7	255	13		3	y	i	x	0.11772	5	18	239	7
3	o	i	y	0.02001	5	10	255	26		3	y	i	y	0.01936	5	10	239	39
3	o	p	x	0.03436	5	9	255	15		3	y	p	x	0.11784	5	18	239	7
3	o	p	y	0.03746	4	9	255	20		3	y	p	y	0.15573	5	11	239	4
3	o	k	x	0.01327	5	17	255	22		3	y	k	x	0.04848	3	18	239	17
3	o	k	y	0.02224	5	11	255	13		3	y	k	y	0.0211	5	10	239	36
3	o	a	x	0.03358	3	17	255	21		3	y	a	x	0.02238	4	18	239	43
3	o	a	y	0.02066	5	14	255	19		3	y	a	y	0.04101	5	15	239	14
3	o	h	x	0.09777	5	2	255	19		3	y	h	x	0.04275	3	18	239	19
3	o	h	y	0.02143	5	13	255	32		3	y	h	y	0.03305	5	14	239	22

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
3	o	m	x	0.02494	4	16	255	21		3	y	m	x	0.03842	3	17	239	29
3	o	m	y	0.02119	5	10	255	34		3	y	m	y	0.02535	5	10	239	28
4	o	i	x	0.03662	5	14	214	12		4	y	i	x	0.05533	5	15	239	11
4	o	i	y	0.04955	4	9	214	14		4	y	i	y	0.02281	5	11	239	19
4	o	p	x	0.01875	5	13	214	14		4	y	p	x	0.05479	5	15	239	11
4	o	p	y	0.03525	5	9	214	14		4	y	p	y	0.03127	4	10	239	26
4	o	k	x	0.11239	3	14	214	10		4	y	k	x	0.02076	4	17	239	30
4	o	k	y	0.0298	5	8	214	20		4	y	k	y	0.03505	4	11	239	19
4	o	a	x	0.02883	4	14	214	25		4	y	a	x	0.08154	5	2	239	25
4	o	a	y	0.02923	5	12	214	19		4	y	a	y	0.02298	4	13	239	27
4	o	h	x	0.03249	4	14	214	22		4	y	h	x	0.08789	4	2	239	23
4	o	h	y	0.0233	5	12	214	29		4	y	h	y	0.02118	5	14	239	37
4	o	m	x	0.04495	3	14	214	19		4	y	m	x	0.04451	3	11	239	34
4	o	m	y	0.02854	5	11	214	20		4	y	m	y	0.02005	5	9	239	45
5	o	i	x	0.01553	5	14	251	18		5	y	i	x	0.01028	5	16	238	23
5	o	i	y	0.04355	4	9	251	19		5	y	i	y	0.02948	4	9	238	23
5	o	p	x	0.01325	5	15	251	17		5	y	p	x	0.01041	5	16	238	23
5	o	p	y	0.03024	5	9	251	22		5	y	p	y	0.02903	4	9	238	23
5	o	k	x	0.06676	3	16	251	14		5	y	k	x	0.03707	3	15	238	17
5	o	k	y	0.02493	5	11	251	18		5	y	k	y	0.02083	5	9	238	26
5	o	a	x	0.10929	4	2	251	20		5	y	a	x	0.06287	4	3	238	28
5	o	a	y	0.02799	5	13	251	21		5	y	a	y	0.01415	5	12	238	39
5	o	h	x	0.09527	5	2	251	21		5	y	h	x	0.0702	4	3	238	27
5	o	h	y	0.0257	5	13	251	17		5	y	h	y	0.01485	5	13	238	31
5	o	m	x	0.02988	4	15	251	25		5	y	m	x	0.10297	3	2	238	20
5	o	m	y	0.01747	5	13	251	16		5	y	m	y	0.04095	5	6	238	16
6	o	i	x	0.01342	5	9	262	41		6	y	i	x	0.01354	5	9	239	39
6	o	i	y	0.01401	5	14	262	15		6	y	i	y	0.02565	4	10	239	21
6	o	p	x	0.01328	5	9	262	41		6	y	p	x	0.01705	4	10	239	28
6	o	p	y	0.01919	4	9	262	50		6	y	p	y	0.02016	4	9	239	46
6	o	k	x	0.0218	4	15	262	28		6	y	k	x	0.01852	4	16	239	30
6	o	k	y	0.01633	5	10	262	58		6	y	k	y	0.01741	5	8	239	49
6	o	a	x	0.01942	4	16	262	48		6	y	a	x	0.09712	3	2	239	22
6	o	a	y	0.01825	5	14	262	36		6	y	a	y	0.02573	4	14	239	22
6	o	h	x	0.07553	4	2	262	26		6	y	h	x	0.09483	3	2	239	21
6	o	h	y	0.02203	5	13	262	36		6	y	h	y	0.02156	4	13	239	31
6	o	m	x	0.07765	4	2	262	26		6	y	m	x	0.09635	3	2	239	21
6	o	m	y	0.02124	5	9	262	40		6	y	m	y	0.01901	5	8	239	41
7	o	i	x	0.02932	4	12	269	15		7	y	i	x	0.02371	5	7	239	23
7	o	i	y	0.03819	4	10	269	20		7	y	i	y	0.03449	4	8	239	22
7	o	p	x	0.03442	4	11	269	18		7	y	p	x	0.02353	5	7	239	23
7	o	p	y	0.0242	5	9	269	32		7	y	p	y	0.02994	5	7	239	17
7	o	k	x	0.02411	4	15	269	21		7	y	k	x	0.04269	4	15	239	13
7	o	k	y	0.02835	5	8	269	28		7	y	k	y	0.02395	5	8	239	21
7	o	a	x	0.02407	4	15	269	36		7	y	a	x	0.03384	3	15	239	29
7	o	a	y	0.02323	5	14	269	25		7	y	a	y	0.01727	5	11	239	33
7	o	h	x	0.02458	4	15	269	32		7	y	h	x	0.06786	4	4	239	27

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
7	o	h	y	0.02111	5	13	269	27		7	y	h	y	0.02526	4	13	239	18
7	o	m	x	0.02622	4	15	269	26		7	y	m	x	0.08651	3	3	239	23
7	o	m	y	0.08959	5	13	269	10		7	y	m	y	0.02859	5	6	239	33
8	o	i	x	0.05922	4	12	231	0		8	y	i	x	0.02276	5	8	239	0
8	o	i	y	0.03837	4	11	231	13		8	y	i	y	0.02019	5	11	239	21
8	o	p	x	0.05934	4	12	231	0		8	y	p	x	0.02301	5	8	239	0
8	o	p	y	0.0394	4	11	231	13		8	y	p	y	0.03792	5	12	239	11
8	o	k	x	0.08677	4	18	231	10		8	y	k	x	0.06203	3	16	239	14
8	o	k	y	0.07705	5	13	231	9		8	y	k	y	0.02235	5	11	239	17
8	o	a	x	0.06927	3	18	231	12		8	y	a	x	0.14768	3	2	239	14
8	o	a	y	0.0153	5	17	231	18		8	y	a	y	0.01669	5	15	239	20
8	o	h	x	0.01633	5	18	231	28		8	y	h	x	0.17059	3	2	239	13
8	o	h	y	0.01632	5	16	231	20		8	y	h	y	0.01933	5	14	239	19
8	o	m	x	0.0201	4	18	231	23		8	y	m	x	0.13526	3	2	239	18
8	o	m	y	0.11326	5	11	231	7		8	y	m	y	0.02612	5	10	239	12
9	o	i	x	0.04186	5	17	269	12		9	y	i	x	0.02641	5	17	238	15
9	o	i	y	0.02829	5	10	269	14		9	y	i	y	0.02132	5	12	238	17
9	o	p	x	0.04199	5	17	269	12		9	y	p	x	0.03203	5	17	238	14
9	o	p	y	0.02264	5	10	269	24		9	y	p	y	0.05127	5	13	238	10
9	o	k	x	0.02544	4	17	269	25		9	y	k	x	0.09597	3	18	238	11
9	o	k	y	0.02022	5	10	269	22		9	y	k	y	0.02953	5	12	238	12
9	o	a	x	0.11081	4	2	269	19		9	y	a	x	0.08465	5	2	238	23
9	o	a	y	0.02397	5	13	269	23		9	y	a	y	0.01133	5	15	238	17
9	o	h	x	0.02709	4	17	269	25		9	y	h	x	0.06597	3	6	238	30
9	o	h	y	0.02039	5	14	269	18		9	y	h	y	0.01652	5	14	238	18
9	o	m	x	0.02126	5	16	269	26		9	y	m	x	0.0359	3	17	238	26
9	o	m	y	0.01939	5	11	269	27		9	y	m	y	0.04214	5	8	238	14
0	o	i	x	0.0333	5	6	232	17		0	y	i	x	0.02506	5	7	239	23
0	o	i	y	0.03135	4	8	232	33		0	y	i	y	0.02766	4	9	239	27
0	o	p	x	0.03314	5	6	232	17		0	y	p	x	0.02522	5	7	239	23
0	o	p	y	0.02307	4	10	232	34		0	y	p	y	0.01768	5	9	239	34
0	o	k	x	0.03234	3	15	232	21		0	y	k	x	0.03299	3	16	239	18
0	o	k	y	0.03031	4	8	232	33		0	y	k	y	0.01763	5	9	239	34
0	o	a	x	0.03749	3	15	232	22		0	y	a	x	0.09913	3	3	239	18
0	o	a	y	0.01518	5	14	232	16		0	y	a	y	0.01572	5	13	239	29
0	o	h	x	0.03969	3	15	232	19		0	y	h	x	0.10106	3	3	239	19
0	o	h	y	0.01947	5	13	232	33		0	y	h	y	0.01718	5	13	239	29
0	o	m	x	0.04092	3	15	232	19		0	y	m	x	0.12189	3	2	239	16
0	o	m	y	0.01959	5	9	232	40		0	y	m	y	0.03434	5	7	239	18

LyE Calculations for Locally Projective Nonlinear (applied twice) Noise Reduced Data (Subject = SB, AG = Age Group, Body Part = BP, Axis = AX)

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
1	o	i	x	0.01422	5	15	220	16		1	y	i	x	0.01352	5	17	238	17
1	o	i	y	0.0203	5	11	220	13		1	y	i	y	0.01958	5	12	238	21
1	o	p	x	0.0142	5	15	220	16		1	y	p	x	0.01801	4	17	238	17
1	o	p	y	0.0404	4	10	220	15		1	y	p	y	0.0151	5	10	238	32
1	o	k	x	0.02052	5	15	220	24		1	y	k	x	0.04702	3	17	238	16
1	o	k	y	0.01489	5	12	220	30		1	y	k	y	0.0241	5	11	238	19
1	o	a	x	0.04416	3	15	220	22		1	y	a	x	0.12048	3	2	238	20
1	o	a	y	0.02207	5	13	220	17		1	y	a	y	0.02364	4	15	238	24
1	o	h	x	0.04443	3	15	220	21		1	y	h	x	0.0642	4	4	238	30
1	o	h	y	0.02331	5	13	220	27		1	y	h	y	0.02217	5	15	238	22
1	o	m	x	0.04753	3	15	220	19		1	y	m	x	0.06471	3	6	238	30
1	o	m	y	0.0246	5	11	220	28		1	y	m	y	0.06755	5	5	238	14
2	o	i	x	0.09464	4	17	238	0		2	y	i	x	0.12899	5	16	239	5
2	o	i	y	0.05419	4	12	238	11		2	y	i	y	0.02347	4	9	239	30
2	o	p	x	0.08086	5	17	238	10		2	y	p	x	0.1039	4	16	239	7
2	o	p	y	0.03092	5	10	238	16		2	y	p	y	0.02335	4	9	239	29
2	o	k	x	0.07512	4	16	238	10		2	y	k	x	0.01617	4	16	239	20
2	o	k	y	0.11948	4	13	238	8		2	y	k	y	0.01625	5	9	239	36
2	o	a	x	0.13065	3	16	238	8		2	y	a	x	0.1018	5	2	239	17
2	o	a	y	0.03717	5	14	238	13		2	y	a	y	0.01994	5	12	239	17
2	o	h	x	0.05203	3	16	238	16		2	y	h	x	0.13119	3	2	239	16
2	o	h	y	0.11166	5	14	238	9		2	y	h	y	0.03997	5	13	239	11
2	o	m	x	0.02789	4	16	238	17		2	y	m	x	0.13223	3	2	239	16
2	o	m	y	0.01996	5	13	238	16		2	y	m	y	0.02506	5	8	239	23
3	o	i	x	0.03515	5	9	255	15		3	y	i	x	0.08228	5	13	239	9
3	o	i	y	0.02033	5	10	255	26		3	y	i	y	0.01947	5	10	239	39
3	o	p	x	0.03474	5	9	255	15		3	y	p	x	0.11793	5	18	239	7
3	o	p	y	0.036	4	10	255	19		3	y	p	y	0.15499	5	11	239	4
3	o	k	x	0.01315	5	17	255	22		3	y	k	x	0.04831	3	18	239	17
3	o	k	y	0.0223	5	11	255	13		3	y	k	y	0.02135	5	10	239	36
3	o	a	x	0.03424	3	17	255	21		3	y	a	x	0.02235	4	18	239	43
3	o	a	y	0.02058	5	14	255	19		3	y	a	y	0.04108	5	15	239	14
3	o	h	x	0.10056	5	2	255	19		3	y	h	x	0.043	3	18	239	19
3	o	h	y	0.02115	5	13	255	31		3	y	h	y	0.03337	5	14	239	22
3	o	m	x	0.02475	4	16	255	21		3	y	m	x	0.03837	3	17	239	29
3	o	m	y	0.0212	5	10	255	34		3	y	m	y	0.02552	5	10	239	28
4	o	i	x	0.02047	5	12	214	18		4	y	i	x	0.05756	5	16	239	11
4	o	i	y	0.04979	4	9	214	14		4	y	i	y	0.02302	5	11	239	19
4	o	p	x	0.02064	5	12	214	18		4	y	p	x	0.05743	5	16	239	11
4	o	p	y	0.05143	4	8	214	15		4	y	p	y	0.03168	4	10	239	26
4	o	k	x	0.11267	3	14	214	10		4	y	k	x	0.04523	3	17	239	16
4	o	k	y	0.03996	4	8	214	19		4	y	k	y	0.03543	4	11	239	19
4	o	a	x	0.02905	4	14	214	25		4	y	a	x	0.08381	5	2	239	25
4	o	a	y	0.02922	5	12	214	19		4	y	a	y	0.01786	5	13	239	29
4	o	h	x	0.0326	4	14	214	22		4	y	h	x	0.09654	4	2	239	22

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
4	o	h	y	0.02333	5	12	214	29		4	y	h	y	0.02117	5	14	239	37
4	o	m	x	0.04493	3	14	214	19		4	y	m	x	0.04471	3	11	239	34
4	o	m	y	0.02888	5	11	214	20		4	y	m	y	0.02002	5	9	239	45
5	o	i	x	0.01332	5	15	251	17		5	y	i	x	0.01029	5	16	238	23
5	o	i	y	0.03231	5	9	251	22		5	y	i	y	0.01907	5	9	238	27
5	o	p	x	0.01341	5	15	251	18		5	y	p	x	0.01023	5	16	238	23
5	o	p	y	0.02798	5	10	251	20		5	y	p	y	0.0293	4	9	238	23
5	o	k	x	0.02469	4	16	251	29		5	y	k	x	0.02071	4	15	238	19
5	o	k	y	0.02539	5	11	251	18		5	y	k	y	0.02121	5	9	238	26
5	o	a	x	0.11484	4	2	251	20		5	y	a	x	0.08055	3	3	238	24
5	o	a	y	0.03633	4	13	251	20		5	y	a	y	0.01413	5	12	238	39
5	o	h	x	0.11268	4	2	251	19		5	y	h	x	0.07028	4	3	238	27
5	o	h	y	0.02613	5	13	251	17		5	y	h	y	0.01474	5	13	238	31
5	o	m	x	0.02998	4	15	251	25		5	y	m	x	0.10668	3	2	238	21
5	o	m	y	0.01739	5	13	251	16		5	y	m	y	0.04138	5	6	238	16
6	o	i	x	0.01388	5	15	262	15		6	y	i	x	0.01265	5	10	239	33
6	o	i	y	0.01487	5	13	262	16		6	y	i	y	0.02597	4	10	239	21
6	o	p	x	0.01314	5	9	262	42		6	y	p	x	0.01751	4	10	239	28
6	o	p	y	0.01352	5	9	262	67		6	y	p	y	0.02016	4	9	239	46
6	o	k	x	0.02179	4	15	262	28		6	y	k	x	0.01847	4	16	239	30
6	o	k	y	0.0166	5	10	262	57		6	y	k	y	0.01721	5	9	239	46
6	o	a	x	0.01974	4	16	262	47		6	y	a	x	0.1001	3	2	239	22
6	o	a	y	0.01838	5	14	262	36		6	y	a	y	0.02606	4	14	239	22
6	o	h	x	0.079	4	2	262	26		6	y	h	x	0.10111	3	2	239	21
6	o	h	y	0.02193	5	13	262	37		6	y	h	y	0.01802	5	13	239	36
6	o	m	x	0.07998	4	2	262	26		6	y	m	x	0.09965	3	2	239	22
6	o	m	y	0.02137	5	9	262	40		6	y	m	y	0.01918	5	8	239	41
7	o	i	x	0.02618	4	10	269	27		7	y	i	x	0.02411	5	7	239	23
7	o	i	y	0.03849	4	10	269	20		7	y	i	y	0.03507	4	8	239	22
7	o	p	x	0.02882	4	12	269	15		7	y	p	x	0.02379	5	7	239	23
7	o	p	y	0.02404	5	9	269	33		7	y	p	y	0.02997	5	7	239	17
7	o	k	x	0.0239	4	15	269	21		7	y	k	x	0.04295	4	15	239	13
7	o	k	y	0.02814	5	8	269	29		7	y	k	y	0.02402	5	8	239	21
7	o	a	x	0.04755	3	15	269	17		7	y	a	x	0.03415	3	15	239	29
7	o	a	y	0.02337	5	14	269	25		7	y	a	y	0.01715	5	11	239	33
7	o	h	x	0.04065	3	15	269	26		7	y	h	x	0.06814	4	4	239	27
7	o	h	y	0.0212	5	13	269	27		7	y	h	y	0.02522	4	13	239	18
7	o	m	x	0.02631	4	15	269	26		7	y	m	x	0.08991	3	3	239	23
7	o	m	y	0.06808	5	13	269	11		7	y	m	y	0.0294	5	6	239	32
8	o	i	x	0.03438	5	15	231	13		8	y	i	x	0.02304	5	8	239	0
8	o	i	y	0.03926	4	11	231	13		8	y	i	y	0.02078	5	11	239	21
8	o	p	x	0.06053	4	12	231	0		8	y	p	x	0.03759	5	7	239	0
8	o	p	y	0.03934	4	11	231	13		8	y	p	y	0.03779	5	12	239	11
8	o	k	x	0.0824	4	17	231	10		8	y	k	x	0.06252	3	16	239	14
8	o	k	y	0.07668	5	13	231	9		8	y	k	y	0.01801	5	10	239	26
8	o	a	x	0.06904	3	18	231	12		8	y	a	x	0.15226	3	2	239	14
8	o	a	y	0.01495	5	17	231	18		8	y	a	y	0.01646	5	15	239	20

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
8	o	h	x	0.0165	5	18	231	28		8	y	h	x	0.16807	3	2	239	13
8	o	h	y	0.01625	5	16	231	20		8	y	h	y	0.01933	5	14	239	19
8	o	m	x	0.02893	3	18	231	23		8	y	m	x	0.13413	3	2	239	19
8	o	m	y	0.046	5	9	231	11		8	y	m	y	0.02824	5	10	239	13
9	o	i	x	0.04307	5	17	269	12		9	y	i	x	0.03259	5	17	238	14
9	o	i	y	0.0417	4	10	269	14		9	y	i	y	0.02973	4	12	238	17
9	o	p	x	0.04705	4	17	269	13		9	y	p	x	0.03249	5	17	238	14
9	o	p	y	0.02287	5	10	269	24		9	y	p	y	0.05198	5	13	238	10
9	o	k	x	0.02546	4	17	269	25		9	y	k	x	0.09639	3	18	238	11
9	o	k	y	0.02079	5	10	269	22		9	y	k	y	0.02954	5	12	238	12
9	o	a	x	0.11707	4	2	269	19		9	y	a	x	0.13948	3	2	238	14
9	o	a	y	0.02405	5	13	269	23		9	y	a	y	0.01405	5	15	238	16
9	o	h	x	0.0271	4	17	269	25		9	y	h	x	0.05353	4	6	238	34
9	o	h	y	0.0206	5	14	269	18		9	y	h	y	0.01631	5	14	238	18
9	o	m	x	0.02136	5	16	269	26		9	y	m	x	0.03583	3	17	238	26
9	o	m	y	0.01933	5	11	269	27		9	y	m	y	0.03144	5	10	238	11
0	o	i	x	0.03379	5	6	232	17		0	y	i	x	0.0256	5	7	239	23
0	o	i	y	0.03156	4	8	232	33		0	y	i	y	0.02809	4	9	239	27
0	o	p	x	0.04686	4	6	232	16		0	y	p	x	0.02551	5	7	239	24
0	o	p	y	0.02347	4	10	232	34		0	y	p	y	0.01798	5	9	239	34
0	o	k	x	0.02255	4	15	232	24		0	y	k	x	0.03334	3	16	239	18
0	o	k	y	0.0305	4	8	232	33		0	y	k	y	0.01815	5	9	239	33
0	o	a	x	0.0376	3	15	232	22		0	y	a	x	0.10067	3	3	239	18
0	o	a	y	0.01494	5	14	232	16		0	y	a	y	0.0157	5	13	239	29
0	o	h	x	0.0396	3	15	232	19		0	y	h	x	0.10245	3	3	239	19
0	o	h	y	0.01939	5	13	232	32		0	y	h	y	0.01705	5	13	239	29
0	o	m	x	0.04099	3	15	232	19		0	y	m	x	0.1363	2	2	239	16
0	o	m	y	0.01971	5	9	232	40		0	y	m	y	0.03508	5	7	239	18

LyE Calculations for the Locally Projective Nonlinear (applied twice) and then the Simple Nonlinear Noise Reduced Data
(Subject = SB, AG = Age Group, Body Part = BP, Axis = AX)

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
1	o	i	x	0.01425	5	15	220	16		1	y	i	x	0.01303	5	17	238	17
1	o	i	y	0.03842	4	11	220	13		1	y	i	y	0.01993	5	12	238	21
1	o	p	x	0.01426	5	15	220	16		1	y	p	x	0.01207	5	15	238	20
1	o	p	y	0.02694	5	10	220	17		1	y	p	y	0.01498	5	10	238	32
1	o	k	x	0.02844	4	15	220	27		1	y	k	x	0.04176	3	17	238	21
1	o	k	y	0.01593	5	12	220	31		1	y	k	y	0.02544	5	11	238	19
1	o	a	x	0.04738	3	15	220	24		1	y	a	x	0.11129	5	2	238	25
1	o	a	y	0.03058	5	13	220	19		1	y	a	y	0.0258	5	15	238	37
1	o	h	x	0.0486	3	15	220	22		1	y	h	x	0.10662	5	2	238	26
1	o	h	y	0.02721	5	13	220	29		1	y	h	y	0.03399	5	15	238	35
1	o	m	x	0.13993	5	2	220	18		1	y	m	x	0.04242	3	17	238	40

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
1	o	m	y	0.02914	5	11	220	29		1	y	m	y	0.01885	5	10	238	60
2	o	i	x	0.08038	5	17	238	10		2	y	i	x	0.08708	4	16	239	8
2	o	i	y	0.04344	4	12	238	12		2	y	i	y	0.0179	5	9	239	44
2	o	p	x	0.08077	5	17	238	10		2	y	p	x	0.08719	4	16	239	8
2	o	p	y	0.03107	5	10	238	16		2	y	p	y	0.02644	4	9	239	29
2	o	k	x	0.05489	4	16	238	12		2	y	k	x	0.03394	4	16	239	17
2	o	k	y	0.10565	5	13	238	8		2	y	k	y	0.03948	5	9	239	14
2	o	a	x	0.10352	3	16	238	10		2	y	a	x	0.08484	5	3	239	25
2	o	a	y	0.02196	5	14	238	15		2	y	a	y	0.02625	5	13	239	31
2	o	h	x	0.11865	3	16	238	9		2	y	h	x	0.09404	5	3	239	22
2	o	h	y	0.08273	5	14	238	11		2	y	h	y	0.02965	5	13	239	28
2	o	m	x	0.04978	3	16	238	20		2	y	m	x	0.15347	5	2	239	17
2	o	m	y	0.15933	5	13	238	7		2	y	m	y	0.02101	5	9	239	29
3	o	i	x	0.03349	5	9	255	16		3	y	i	x	0.11835	5	18	239	7
3	o	i	y	0.02097	5	10	255	26		3	y	i	y	0.01975	5	10	239	39
3	o	p	x	0.03491	5	9	255	15		3	y	p	x	0.11747	5	18	239	7
3	o	p	y	0.03923	4	10	255	19		3	y	p	y	0.01852	5	11	239	27
3	o	k	x	0.02303	4	17	255	23		3	y	k	x	0.04398	3	18	239	19
3	o	k	y	0.02927	5	11	255	16		3	y	k	y	0.04385	4	11	239	12
3	o	a	x	0.04028	3	17	255	27		3	y	a	x	0.04235	3	18	239	26
3	o	a	y	0.02808	5	14	255	27		3	y	a	y	0.02617	5	15	239	18
3	o	h	x	0.09946	5	2	255	24		3	y	h	x	0.04136	3	18	239	23
3	o	h	y	0.02872	5	13	255	35		3	y	h	y	0.04287	5	14	239	25
3	o	m	x	0.10228	5	2	255	23		3	y	m	x	0.03884	3	17	239	37
3	o	m	y	0.02354	5	10	255	36		3	y	m	y	0.02869	5	10	239	28
4	o	i	x	0.01992	5	12	214	17		4	y	i	x	0.05733	5	16	239	11
4	o	i	y	0.05242	4	9	214	13		4	y	i	y	0.02293	5	11	239	20
4	o	p	x	0.02069	5	12	214	18		4	y	p	x	0.05586	5	15	239	11
4	o	p	y	0.03514	5	9	214	15		4	y	p	y	0.03359	4	11	239	22
4	o	k	x	0.08456	3	14	214	12		4	y	k	x	0.01773	5	17	239	39
4	o	k	y	0.03165	5	8	214	21		4	y	k	y	0.03933	4	11	239	23
4	o	a	x	0.0308	4	14	214	28		4	y	a	x	0.10196	5	2	239	28
4	o	a	y	0.03143	5	12	214	23		4	y	a	y	0.02664	5	15	239	44
4	o	h	x	0.04969	3	14	214	22		4	y	h	x	0.10344	5	2	239	25
4	o	h	y	0.02689	5	12	214	29		4	y	h	y	0.03342	5	14	239	67
4	o	m	x	0.03469	4	14	214	24		4	y	m	x	0.10096	5	2	239	25
4	o	m	y	0.03086	5	11	214	20		4	y	m	y	0.03255	5	9	239	37
5	o	i	x	0.01622	5	14	251	19		5	y	i	x	0.01081	5	16	238	23
5	o	i	y	0.03323	5	9	251	22		5	y	i	y	0.03147	4	9	238	23
5	o	p	x	0.01341	5	15	251	18		5	y	p	x	0.01083	5	16	238	23
5	o	p	y	0.02739	5	10	251	21		5	y	p	y	0.01842	5	9	238	29
5	o	k	x	0.03077	4	16	251	23		5	y	k	x	0.02329	5	15	238	39
5	o	k	y	0.02734	5	11	251	20		5	y	k	y	0.03218	4	10	238	27
5	o	a	x	0.11021	5	2	251	23		5	y	a	x	0.04678	5	5	238	42
5	o	a	y	0.03643	5	13	251	33		5	y	a	y	0.02489	5	14	238	43
5	o	h	x	0.10837	5	2	251	23		5	y	h	x	0.07115	5	3	238	33
5	o	h	y	0.04	5	13	251	26		5	y	h	y	0.02571	5	13	238	51

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
5	o	m	x	0.11489	5	2	251	22		5	y	m	x	0.08704	5	3	238	27
5	o	m	y	0.0254	5	11	251	26		5	y	m	y	0.02915	5	8	238	23
6	o	i	x	0.0203	5	12	262	18		6	y	i	x	0.01315	5	10	239	35
6	o	i	y	0.0149	5	14	262	15		6	y	i	y	0.02874	4	10	239	23
6	o	p	x	0.01334	5	9	262	43		6	y	p	x	0.01371	5	10	239	35
6	o	p	y	0.02054	4	9	262	51		6	y	p	y	0.02273	4	9	239	47
6	o	k	x	0.02656	4	15	262	34		6	y	k	x	0.02284	4	16	239	49
6	o	k	y	0.02374	5	10	262	49		6	y	k	y	0.02211	5	9	239	50
6	o	a	x	0.08061	5	2	262	31		6	y	a	x	0.07174	5	3	239	31
6	o	a	y	0.02801	5	14	262	56		6	y	a	y	0.02673	5	14	239	44
6	o	h	x	0.08184	5	2	262	30		6	y	h	x	0.06204	5	3	239	39
6	o	h	y	0.03483	5	13	262	59		6	y	h	y	0.02903	5	13	239	58
6	o	m	x	0.08757	5	2	262	29		6	y	m	x	0.10528	5	2	239	29
6	o	m	y	0.02281	5	10	262	37		6	y	m	y	0.02607	5	8	239	36
7	o	i	x	0.03049	4	12	269	16		7	y	i	x	0.02471	5	7	239	24
7	o	i	y	0.04056	4	10	269	20		7	y	i	y	0.03985	4	8	239	25
7	o	p	x	0.03051	4	12	269	16		7	y	p	x	0.02415	5	7	239	24
7	o	p	y	0.02265	5	10	269	28		7	y	p	y	0.02677	5	8	239	21
7	o	k	x	0.02829	4	15	269	27		7	y	k	x	0.02997	4	15	239	31
7	o	k	y	0.02935	5	8	269	30		7	y	k	y	0.02981	5	8	239	25
7	o	a	x	0.02604	4	15	269	39		7	y	a	x	0.02913	4	15	239	43
7	o	a	y	0.02348	5	14	269	30		7	y	a	y	0.02693	5	11	239	39
7	o	h	x	0.04402	3	15	269	28		7	y	h	x	0.06543	5	4	239	35
7	o	h	y	0.02884	5	13	269	32		7	y	h	y	0.02766	5	13	239	30
7	o	m	x	0.04775	3	15	269	26		7	y	m	x	0.09131	5	3	239	27
7	o	m	y	0.07546	5	13	269	11		7	y	m	y	0.03669	5	6	239	30
8	o	i	x	0.01912	5	12	231	18		8	y	i	x	0.0228	5	8	239	0
8	o	i	y	0.03866	4	11	231	15		8	y	i	y	0.0184	5	11	239	22
8	o	p	x	0.01925	5	12	231	18		8	y	p	x	0.03763	5	7	239	0
8	o	p	y	0.02881	5	12	231	12		8	y	p	y	0.03891	4	12	239	12
8	o	k	x	0.07551	3	18	231	14		8	y	k	x	0.05107	3	16	239	19
8	o	k	y	0.06785	5	13	231	10		8	y	k	y	0.03121	4	11	239	17
8	o	a	x	0.0437	3	18	231	18		8	y	a	x	0.10188	5	2	239	25
8	o	a	y	0.03182	5	17	231	15		8	y	a	y	0.02157	5	15	239	30
8	o	h	x	0.01574	5	18	231	53		8	y	h	x	0.12599	4	2	239	21
8	o	h	y	0.02334	5	16	231	38		8	y	h	y	0.02679	5	14	239	31
8	o	m	x	0.0498	3	15	231	17		8	y	m	x	0.11242	5	2	239	22
8	o	m	y	0.08399	5	11	231	9		8	y	m	y	0.03869	5	10	239	16
9	o	i	x	0.03856	5	17	269	13		9	y	i	x	0.02395	5	18	238	16
9	o	i	y	0.04299	4	10	269	15		9	y	i	y	0.02237	5	12	238	18
9	o	p	x	0.03806	5	17	269	13		9	y	p	x	0.02737	5	17	238	15
9	o	p	y	0.03338	4	10	269	21		9	y	p	y	0.01789	5	12	238	14
9	o	k	x	0.0296	4	17	269	34		9	y	k	x	0.04233	4	18	238	15
9	o	k	y	0.0233	5	10	269	22		9	y	k	y	0.02586	5	12	238	14
9	o	a	x	0.1216	4	2	269	22		9	y	a	x	0.10269	5	2	238	24
9	o	a	y	0.03657	4	15	269	25		9	y	a	y	0.02357	5	15	238	25
9	o	h	x	0.11639	5	2	269	22		9	y	h	x	0.03908	4	10	238	51

SB	AG	BP	AX	LyE	m	J	W	DT		SB	AG	BP	AX	LyE	m	J	W	DT
9	o	h	y	0.03084	5	14	269	27		9	y	h	y	0.02849	5	14	238	41
9	o	m	x	0.11307	5	2	269	23		9	y	m	x	0.0434	3	17	238	35
9	o	m	y	0.02399	5	11	269	29		9	y	m	y	0.0367	5	10	238	13
0	o	i	x	0.03399	5	6	232	17		0	y	i	x	0.02716	5	7	239	24
0	o	i	y	0.03461	4	8	232	33		0	y	i	y	0.02766	4	9	239	29
0	o	p	x	0.02019	5	10	232	19		0	y	p	x	0.02718	5	7	239	23
0	o	p	y	0.02507	4	10	232	34		0	y	p	y	0.01856	5	9	239	34
0	o	k	x	0.02635	4	15	232	27		0	y	k	x	0.03257	4	16	239	29
0	o	k	y	0.03665	4	8	232	31		0	y	k	y	0.0228	5	9	239	35
0	o	a	x	0.03837	3	15	232	27		0	y	a	x	0.07812	5	3	239	35
0	o	a	y	0.02555	5	14	232	24		0	y	a	y	0.0318	5	13	239	22
0	o	h	x	0.04022	3	15	232	25		0	y	h	x	0.09161	5	3	239	31
0	o	h	y	0.02623	5	13	232	43		0	y	h	y	0.03122	5	13	239	59
0	o	m	x	0.0218	4	15	232	54		0	y	m	x	0.1344	5	2	239	25
0	o	m	y	0.02151	5	9	232	40		0	y	m	y	0.0477	5	7	239	19

LyE Calculations for the Final Data

(Clean of 2*L+S is the Locally Projective twice and the Simple Nonlinear once)

(Clean of 2*L is the Locally Projective twice)

(Clean of CFR is Changed Frame of Reference)

(Clean of L is Locally Projective once)

Clean	Name	LyE	m	J	W	DT		Clean	Name	LyE	m	J	W	DT
2*L+S	1oix	0.01425	5	15	220	16		2*L+S	1yix	0.01303	5	17	238	17
2*L+S	1oiy	0.03842	4	11	220	13		2*L+S	1yiy	0.01993	5	12	238	21
2*L+S	1opx	0.01426	5	15	220	16		2*L+S	1ypx	0.01207	5	15	238	20
2*L	1opy	0.0404	4	10	220	15		2*L+S	1ypy	0.01498	5	10	238	32
2*L	1okx	0.02052	5	15	220	24		2*L	1ykx	0.04702	3	17	238	16
2*L	1oky	0.01489	5	12	220	30		2*L	1yky	0.0241	5	11	238	19
2*L	1oax	0.04416	3	15	220	22		CFR	1yax	0.10887	3	2	238	20
2*L	1oay	0.02207	5	13	220	17		2*L	1yay	0.02364	4	15	238	24
2*L	1ohx	0.04443	3	15	220	21		2*L	1yhx	0.0642	4	4	238	30
2*L	1ohy	0.02331	5	13	220	27		2*L	1yhy	0.02217	5	15	238	22
2*L	1omx	0.04753	3	15	220	19		2*L	1ymx	0.06471	3	6	238	30
2*L	1omy	0.0246	5	11	220	28		2*L	1ymy	0.06755	5	5	238	14
2*L+S	2oix	0.08038	5	17	238	10		2*L+S	2yix	0.08708	4	16	239	8
2*L	2oiy	0.05419	4	12	238	11		2*L	2yiy	0.02347	4	9	239	30
2*L+S	2opx	0.08077	5	17	238	10		2*L+S	2ypx	0.08719	4	16	239	8
2*L+S	2opy	0.03107	5	10	238	16		2*L+S	2ypy	0.02644	4	9	239	29
2*L	2okx	0.07512	4	16	238	10		2*L	2ykx	0.01617	4	16	239	20
2*L	2oky	0.11948	4	13	238	8		2*L	2yky	0.01625	5	9	239	36
2*L	2oax	0.13065	3	16	238	8		2*L	2yax	0.1018	5	2	239	17
2*L	2oay	0.03717	5	14	238	13		2*L	2yay	0.01994	5	12	239	17
2*L	2ohx	0.05203	3	16	238	16		2*L	2yhx	0.13119	3	2	239	16
2*L	2ohy	0.11166	5	14	238	9		2*L	2yhy	0.03997	5	13	239	11

Clean	Name	LyE	m	J	W	DT		Clean	Name	LyE	m	J	W	DT
2*L	2omx	0.02789	4	16	238	17		2*L	2ymx	0.13223	3	2	239	16
2*L	2omy	0.01996	5	13	238	16		2*L	2ymy	0.02506	5	8	239	23
CFR	3oix	0.18934	5	13	255	5		2*L+S	3yix	0.11835	5	18	239	7
2*L+S	3oiy	0.02097	5	10	255	26		2*L	3yiy	0.01947	5	10	239	39
CFR	3opx	0.18934	5	13	255	5		2*L+S	3ypx	0.11747	5	18	239	7
2*L	3opy	0.036	4	10	255	19		2*L	3ypy	0.15499	5	11	239	4
2*L	3okx	0.01315	5	17	255	22		2*L	3ykx	0.04831	3	18	239	17
2*L	3oky	0.0223	5	11	255	13		2*L	3yky	0.02135	5	10	239	36
2*L	3oax	0.03424	3	17	255	21		2*L	3yax	0.02235	4	18	239	43
2*L	3oay	0.02058	5	14	255	19		2*L	3yay	0.04108	5	15	239	14
2*L	3ohx	0.10056	5	2	255	19		2*L	3yhx	0.043	3	18	239	19
2*L	3ohy	0.02115	5	13	255	31		2*L	3yhy	0.03337	5	14	239	22
2*L	3omx	0.02475	4	16	255	21		2*L	3ymx	0.03837	3	17	239	29
2*L	3omy	0.0212	5	10	255	34		2*L	3ymy	0.02552	5	10	239	28
CFR	4oix	0.03684	5	14	214	12		CFR	4yix	0.05383	5	15	239	11
2*L	4oiy	0.04979	4	9	214	14		2*L+S	4yiy	0.02293	5	11	239	20
2*L+S	4opx	0.02069	5	12	214	18		CFR	4ypx	0.05383	5	15	239	11
2*L	4opy	0.05143	4	8	214	15		2*L+S	4ypy	0.03359	4	11	239	22
2*L	4okx	0.11267	3	14	214	10		2*L	4ykx	0.04523	3	17	239	16
2*L	4oky	0.03996	4	8	214	19		2*L	4yky	0.03543	4	11	239	19
2*L	4oax	0.02905	4	14	214	25		2*L	4yax	0.08381	5	2	239	25
2*L	4oay	0.02922	5	12	214	19		2*L	4yay	0.01786	5	13	239	29
2*L	4ohx	0.0326	4	14	214	22		2*L	4yhx	0.09654	4	2	239	22
2*L	4ohy	0.02333	5	12	214	29		2*L	4yhy	0.02117	5	14	239	37
2*L	4omx	0.04493	3	14	214	19		2*L	4ymx	0.04471	3	11	239	34
2*L	4omy	0.02888	5	11	214	20		2*L	4ymy	0.02002	5	9	239	45
2*L+S	5oix	0.01622	5	14	251	19		2*L+S	5yix	0.01081	5	16	238	23
2*L+S	5oiy	0.03323	5	9	251	22		2*L+S	5yiy	0.03147	4	9	238	23
2*L+S	5opx	0.01341	5	15	251	18		2*L+S	5ypx	0.01083	5	16	238	23
2*L+S	5opy	0.02739	5	10	251	21		2*L	5ypy	0.0293	4	9	238	23
2*L	5okx	0.02469	4	16	251	29		2*L	5ykx	0.02071	4	15	238	19
2*L	5oky	0.02539	5	11	251	18		2*L	5yky	0.02121	5	9	238	26
2*L	5oax	0.11484	4	2	251	20		2*L	5yax	0.08055	3	3	238	24
2*L	5oay	0.03633	4	13	251	20		2*L	5yay	0.01413	5	12	238	39
2*L	5ohx	0.11268	4	2	251	19		2*L	5yhx	0.07028	4	3	238	27
2*L	5ohy	0.02613	5	13	251	17		2*L	5yhy	0.01474	5	13	238	31
2*L	5omx	0.02998	4	15	251	25		2*L	5ymx	0.10668	3	2	238	21
2*L	5omy	0.01739	5	13	251	16		2*L	5ymy	0.04138	5	6	238	16
CFR	6oix	0.0131	5	9	262	40		2*L	6yix	0.01265	5	10	239	33
2*L	6oiy	0.01487	5	13	262	16		2*L	6yiy	0.02597	4	10	239	21
CFR	6opx	0.0131	5	9	262	40		2*L+S	6ypx	0.01371	5	10	239	35
2*L+S	6opy	0.02054	4	9	262	51		2*L	6ypy	0.02016	4	9	239	46
2*L+S	6okx	0.02656	4	15	262	34		2*L	6ykx	0.01847	4	16	239	30
2*L	6oky	0.0166	5	10	262	57		2*L	6yky	0.01721	5	9	239	46
2*L	6oax	0.01974	4	16	262	47		L	6yax	0.09712	3	2	239	22
2*L	6oay	0.01838	5	14	262	36		2*L	6yay	0.02606	4	14	239	22
2*L	6ohx	0.079	4	2	262	26		2*L	6yhx	0.10111	3	2	239	21

Clean	Name	LyE	m	J	W	DT		Clean	Name	LyE	m	J	W	DT
2*L	6ohy	0.02193	5	13	262	37		2*L	6yhy	0.01802	5	13	239	36
2*L	6omx	0.07998	4	2	262	26		2*L	6ymx	0.09965	3	2	239	22
2*L	6omy	0.02137	5	9	262	40		2*L	6ymy	0.01918	5	8	239	41
2*L+S	7oix	0.03049	4	12	269	16		2*L+S	7yix	0.02471	5	7	239	24
2*L	7oiy	0.03849	4	10	269	20		2*L+S	7yiy	0.03985	4	8	239	25
2*L+S	7opx	0.03051	4	12	269	16		2*L+S	7ypx	0.02415	5	7	239	24
2*L+S	7opy	0.02265	5	10	269	28		2*L+S	7ypy	0.02677	5	8	239	21
2*L	7okx	0.0239	4	15	269	21		2*L	7ykx	0.04295	4	15	239	13
2*L	7oky	0.02814	5	8	269	29		2*L	7yky	0.02402	5	8	239	21
2*L	7oax	0.04755	3	15	269	17		2*L	7yax	0.03415	3	15	239	29
2*L	7oay	0.02337	5	14	269	25		2*L	7yay	0.01715	5	11	239	33
2*L	7ohx	0.04065	3	15	269	26		2*L	7yhx	0.06814	4	4	239	27
2*L	7ohy	0.0212	5	13	269	27		2*L	7yhy	0.02522	4	13	239	18
2*L	7omx	0.02631	4	15	269	26		2*L	7ymx	0.08991	3	3	239	23
2*L	7omy	0.06808	5	13	269	11		2*L	7ymy	0.0294	5	6	239	32
2*L+S	8oix	0.01912	5	12	231	18		2*L+S	8yix	0.0228	5	8	239	0
2*L	8oiy	0.03926	4	11	231	13		2*L	8yiy	0.02078	5	11	239	21
2*L+S	8opx	0.01925	5	12	231	18		2*L+S	8ypx	0.03763	5	7	239	0
2*L	8opy	0.03934	4	11	231	13		2*L+S	8ypy	0.03891	4	12	239	12
2*L	8okx	0.0824	4	17	231	10		2*L	8ykx	0.06252	3	16	239	14
2*L	8oky	0.07668	5	13	231	9		2*L	8yky	0.01801	5	10	239	26
2*L	8oax	0.06904	3	18	231	12		2*L	8yax	0.15226	3	2	239	14
2*L	8oay	0.01495	5	17	231	18		2*L	8yay	0.01646	5	15	239	20
2*L	8ohx	0.0165	5	18	231	28		CFR	8yhx	0.15731	3	2	239	13
2*L	8ohy	0.01625	5	16	231	20		2*L	8yhy	0.01933	5	14	239	19
2*L	8omx	0.02893	3	18	231	23		L	8ymx	0.13526	3	2	239	18
2*L	8omy	0.046	5	9	231	11		2*L	8ymy	0.02824	5	10	239	13
2*L+S	9oix	0.03856	5	17	269	13		2*L+S	9yix	0.02395	5	18	238	16
2*L+S	9oiy	0.04299	4	10	269	15		2*L	9yiy	0.02973	4	12	238	17
2*L+S	9opx	0.03806	5	17	269	13		2*L+S	9ypx	0.02737	5	17	238	15
2*L+S	9opy	0.03338	4	10	269	21		2*L+S	9ypy	0.01789	5	12	238	14
2*L	9okx	0.02546	4	17	269	25		2*L	9ykx	0.09639	3	18	238	11
2*L	9oky	0.02079	5	10	269	22		2*L	9yky	0.02954	5	12	238	12
2*L	9oax	0.11707	4	2	269	19		2*L	9yax	0.13948	3	2	238	14
2*L	9oay	0.02405	5	13	269	23		2*L	9yay	0.01405	5	15	238	16
2*L	9ohx	0.0271	4	17	269	25		2*L	9yhx	0.05353	4	6	238	34
2*L	9ohy	0.0206	5	14	269	18		2*L	9yhy	0.01631	5	14	238	18
2*L	9omx	0.02136	5	16	269	26		2*L	9ymx	0.03583	3	17	238	26
2*L	9omy	0.01933	5	11	269	27		2*L	9ymy	0.03144	5	10	238	11
2*L+S	0oix	0.03399	5	6	232	17		2*L+S	0yix	0.02716	5	7	239	24
2*L	0oiy	0.03156	4	8	232	33		2*L+S	0yiy	0.02766	4	9	239	29
2*L+S	0opx	0.02019	5	10	232	19		2*L+S	0ypx	0.02718	5	7	239	23
2*L	0opy	0.02347	4	10	232	34		2*L+S	0ypy	0.01856	5	9	239	34
2*L	0okx	0.02255	4	15	232	24		2*L	0ykx	0.03334	3	16	239	18
2*L	0oky	0.0305	4	8	232	33		2*L	0yky	0.01815	5	9	239	33
2*L	0oax	0.0376	3	15	232	22		2*L	0yax	0.10067	3	3	239	18
2*L	0oay	0.01494	5	14	232	16		2*L	0yay	0.0157	5	13	239	29

Clean	Name	LyE	m	J	W	DT		Clean	Name	LyE	m	J	W	DT
2*L	0ohx	0.0396	3	15	232	19		CFR	0yhx	0.11333	3	2	239	16
2*L	0ohy	0.01939	5	13	232	32		2*L	0yhy	0.01705	5	13	239	29
2*L	0omx	0.04099	3	15	232	19		2*L	0ymx	0.1363	2	2	239	16
2*L	0omy	0.01971	5	9	232	40		2*L	0ymy	0.03508	5	7	239	18

APPENDIX B – MSDR Data after Changing the Frame of Reference

Ankle				Knee			
Old		Young		Old		Young	
6oax	1.21669	9yax	1.54933	6okx	1.12644	6ykx	1.17496
4oax	1.63704	6yax	1.65531	8okx	1.83529	4ykx	1.46125
2oax	2.01332	8yax	1.77017	7okx	2.09155	5ykx	1.51437
0oax	2.08622	4yax	1.80525	1okx	2.23801	2ykx	1.52701
7oax	2.3027	7yax	1.93492	4okx	2.26985	3ykx	1.62234
1oax	2.63906	3yax	1.95424	0okx	2.29489	9ykx	1.71747
5oax	2.77251	2yax	2.28367	3okx	2.83723	7ykx	1.741
8oax	2.9296	5yax	2.35631	2okx	2.90761	8ykx	2.63285
3oax	3.02439	1yax	2.61383	5okx	2.95979	1ykx	3.19483
9oax	3.43688	0yax	3.76507	9okx	4.86337	0ykx	6.0486
Heel				**Metatarsal**			
Old		Young		Old		Young	
6ohx	1.14732	9yhx	1.43286	6omx	1.17975	9ymx	1.44344
4ohx	1.48718	6yhx	1.57153	4omx	1.40728	8ymx	1.58693
2ohx	1.81583	8yhx	1.61433	2omx	1.69943	6ymx	1.5902
8ohx	1.94751	4yhx	1.71925	0omx	1.9405	4ymx	1.59859
0ohx	2.01979	3yhx	1.85875	7omx	1.94904	7ymx	1.73778
7ohx	2.14687	7yhx	1.90055	8omx	2.21241	3ymx	1.75175
1ohx	2.40937	5yhx	2.11447	1omx	2.36205	5ymx	2.09228
5ohx	2.62541	2yhx	2.21254	3omx	2.61445	2ymx	2.15409
3ohx	2.80917	1yhx	2.38453	5omx	2.6901	1ymx	2.2078
9ohx	3.07419	0yhx	3.33842	9omx	2.95961	0ymx	3.45681
Iliac Crest				**Hip**			
Old		Young		Old		Young	
6oix	1.71664	6yix	1.38609	6opx	2.74633	7ypx	2.33832
7oix	3.58191	7yix	1.55446	7opx	3.70754	6ypx	2.36491
8oix	3.79583	0yix	2.05622	8opx	4.48261	0ypx	2.41867
3oix	4.74306	4yix	2.71958	3opx	4.73477	4ypx	2.90315
0oix	4.89661	5yix	3.44568	9opx	5.09455	9ypx	3.62853
9oix	5.45528	1yix	3.71541	2opx	5.12383	1ypx	3.66199
4oix	5.48736	9yix	3.75141	5opx	5.31545	8ypx	4.50283
2oix	5.71217	3yix	3.93797	4opx	5.86822	3ypx	4.58326
5oix	6.76482	8yix	3.97346	0opx	6.5912	5ypx	4.94037
1oix	8.98114	2yix	5.0874	1opx	8.1293	2ypx	6.7642

APPENDIX C – Source Code to for Finding the LyE

FINDLE - The routine to find the LyE of a data set.

```
      SUBROUTINE FINDLE(FILENAM)
      INTEGER NUMBER, DT1
      REAL CHAOS (5)
      REAL DATA (6000)
      CHARACTER*30 FILENAM, FILENAM1
      CALL READIT(FILENAM, DATA, NUMBER)
      CALL DELAY(FILENAM, NUMBER, CHAOS(3))
      CHAOS(4) = NUMBER/10
      CALL DIMEN(FILENAM, NUMBER, CHAOS)
      CALL DT(CHAOS(5),NUMBER)
      FILENAM1 = 'temp\0'
      OPEN (10, FILE = 'temp', STATUS = 'UNKNOWN')
      WRITE (10, 100) FILENAM, INT(CHAOS(2)),
     &            INT(CHAOS(3)), INT(CHAOS(4)), INT(CHAOS(5))
  100 FORMAT (A11, 1X, '1' , 1X, '1', 1X,
     &        '0', 1X , I1, 1X, I2, 1X, I4, 1X, I4)
      CLOSE (10)
      DT1 = LYAP(FILENAM1,CHAOS(1))
      CHAOS(5) = DT1 * 1.0
      FILENAM1 = FILENAM(1:INDEX(FILENAM,'.')+3)//'\0'
      OPEN (UNIT = 10, FILE = 'LEData.DAT',
     &   ACCESS = 'APPEND', STATUS = 'UNKNOWN')
      WRITE(10, 200) FILENAM, CHAOS(1), INT(CHAOS(2)),
     &   INT(CHAOS(3)), INT(CHAOS(4)), INT(CHAOS(5))
  200 FORMAT (A11, 1X, F9.5, 1X, I1, 1X, I2, 1X,
     &        I3, 1X, I4)
      CLOSE(10)
      RETURN
      END
```

Comments - The array CHAOS has 5 elements. The first is the LyE, the second is m, third is J, forth is W and fifth is DT. I will list the routines I call in the above program except for those I have downloaded from other sources or can easily explain its functionality.

READIT - Just reads a data file and puts it in the array DATA for easy access.

DELAY - Calls both TISEAN Routines MUTUAL and AUTOCOR and takes the minimum of the two and choses that value for J. I set the maximum delay at 18.

DIMEN - Calls the program FALSE_NEAREST from TISEAN and sets m to the value returned.

DT - Just sets the preliminary value of DT to one half of the data points.

LYAP - is the routine from Rosenstein that calculates the LyE--I have included the function from that program that I made significant changes to.

```
float SaveL1Results(char *fileRoot)
{
  int   i, testN, stop, stop1, j, flag1=0, flag2=0, k=0, DT, start, T,
aveD;
  float slope[gTest[0].divergeT], hertz, slope1[2], intercept[2];
  float Adjstd[gTest[0].divergeT];
  float std[gTest[0].divergeT];
  double sumxy, sumx, sumy, sumx2;
  double max1=0;
  char  str[256];
  FILE  *outFile;
  hertz = 1;
  sumy = 0; sumx = 0; sumxy = 0; sumx2 = 0;
 for(j=1; j<=gTest[0].divergeT-1; j++)
  {sumx=0;
   sumy=0;
   sumxy=0;
   sumx2=0;
  for(i=0; i<=j; i++)
    { sumx = sumx + i/hertz;
      sumx2 = sumx2 + (i/hertz)*(i/hertz);
      sumxy = sumxy + i*(gDivergence[i][0])/hertz;
      sumy = sumy + (gDivergence[i][0]);
    };
  slope[j] = ((j+1)*(sumxy)-(sumx)*(sumy))/((j+1)*(sumx2)-
(sumx)*(sumx));
  };
  aveD = sumy/(gTest[0].divergeT-1);
  sumy = 0; sumx = 0; sumxy = 0; sumx2 = 0;
  T=10;
  for(i=0; i<=T; i++)
    { sumx = sumx + i/hertz;
      sumx2 = sumx2 + (i/hertz)*(i/hertz);
      sumxy = sumxy + i*(gDivergence[i][0])/hertz;
      sumy = sumy + (gDivergence[i][0]);
      slope1[0] = ((i+1)*(sumxy)-(sumx)*(sumy))/((i+1)*(sumx2)-
(sumx)*(sumx));
      intercept[0] = sumy/(i+1)-slope1[0]*sumx/(i+1);
   };
  sumy = 0; sumx = 0; sumxy = 0; sumx2 = 0;
  start=((gMaxDivergeT-2)/2);
  for(i=start; i<=(gMaxDivergeT-2); i++)
    { sumx = sumx + i/hertz;
      sumx2 = sumx2 + (i/hertz)*(i/hertz);
      sumxy = sumxy + i*(gDivergence[i][0])/hertz;
```

```
      sumy = sumy + (gDivergence[i][0]);
      slope1[1] = ((i+1)*(sumxy)-(sumx)*(sumy))/((i+1)*(sumx2)-
(sumx)*(sumx));
      intercept[1] = sumy/(gMaxDivergeT-start)-
slope1[1]*sumx/(gMaxDivergeT-start);
    };
  if (slope1[1]>=slope1[0])
     {dt1=0;
      return(slope1[1]-slope1[0]);
     };
 if (slope1[1]<0) slope1[1]=0;
 stop=(-(intercept[0]-aveD)/(slope1[0]));
 if (stop>=(gMaxDivergeT-2) || stop<=2)
     {dt1=aveD;
      return(slope1[0]-slope1[1]);
     };
 dt1=stop;
 return(slope[stop]);
}
```

APPENDIX D – Source Code for Noise Reduction

CORECT is the routine to change the frame of reference of the data.

```
      SUBROUTINE CORECT(SUBJCT, WALKDATA, MAXREC, BDYPRT,
     &                  AGE, AXIS, NUMBER, SMOOTH,
     &                  I, J, K, A)
      INTEGER MAXREC, BDYPRT, SUBJCT, AGE, AXIS, SMOOTH
      INTEGER NUMBER (AGE, SUBJCT, SMOOTH)
      REAL WALKDATA (AGE, AXIS, BDYPRT, SUBJCT, SMOOTH, MAXREC)
      INTEGER I,J,K,M,X,A
      REAL INCREM, INCRE2, INCREMY, INCREY2
      DO 500 I = 1, AGE
         DO 400 J = 1, SUBJCT
            DO 333 K = 1, BDYPRT
               DO 366 M = 1, AXIS
                  WALKDATA(I,M,K,J,3,1) = WALKDATA(I,M,K,J,2,1)
  366          CONTINUE
  333       CONTINUE
            DO 300 X = 2, NUMBER(I, J, 2)
               INCREM = WALKDATA(I,1,1,J,2,1) - WALKDATA(I,1,1,J,2,X)
               INCRE2 = WALKDATA(I,1,2,J,2,1) - WALKDATA(I,1,2,J,2,X)
               INCREMY = WALKDATA(I,2,1,J,2,1) - WALKDATA(I,2,1,J,2,X)
               INCREY2 = WALKDATA(I,2,2,J,2,1) - WALKDATA(I,2,2,J,2,X)
               WALKDATA(I,1,1,J,3,X) = INCRE2 + WALKDATA(I,1,1,J,2,X)
               WALKDATA(I,2,1,J,3,X) = INCREY2 + WALKDATA(I,2,1,J,2,X)
               DO 200 K = 2, BDYPRT
                  WALKDATA(I,1,K,J,3,X) = INCREM +
WALKDATA(I,1,K,J,2,X)
                  WALKDATA(I,2,K,J,3,X) =INCREMY +
WALKDATA(I,2,K,J,2,X)
  200          CONTINUE
  300       CONTINUE
            NUMBER(I,J,3) = NUMBER(I,J,2)
  400    CONTINUE
  500 CONTINUE
      RETURN
      END
```

NR - is program that calls the TISEAN Routines CLEAN and NRLAZY. The calls to the routines DELAY, DIMEN, and READIT are the same as in Appendix C.

```
      PROGRAM NR
      CHARACTER*30 FILENAM
      REAL DATA (6000), MAXX, MINX, EPS
      REAL TEMP (6000)
      REAL CHAOS (5)
      INTEGER NUMBER, SMOOTH, B, C, D, E, F, G, I, TRIP
      INTEGER NEIGH, EW
```

```
      WRITE (6,*) 'Enter a data file'
      READ *, FILENAM
      WRITE (6,*) 'Enter a number, 5 for nrlazy or 7 for clean'
      READ *, SMOOTH
      CALL READIT(FILENAM,DATA,NUMBER)
      MINX - DATA(1)
      MAXX = DATA(1)
      DO 100 I = 1, NUMBER
         IF (MAXX .LT. DATA(I)) MAXX = DATA(I)
         IF (MINX .GT. DATA(I)) MINX = DATA(I)
100   CONTINUE
      EPS = (MAXX-MINX)
      CALL DELAY(FILENAM,NUMBER,CHAOS(3))
      CALL DIMEN(FILENAM,NUMBER,CHAOS)
      WRITE(6,*) NUMBER, EPS, MAXX, MINX
      IF (SMOOTH .EQ. 7) THEN
         EW = INT((CHAOS(2)-1)*CHAOS(3)+1)
         IF (EW.GT.15) EW = 15
         NEIGH = NUMBER*0.01
         DO 401 I = 1, 2
             CALL CLEAN(NUMBER,DATA,TEMP,EW
     &                 ,NEIGH,3,EPS*0.01)
401      CONTINUE
      ELSEIF (SMOOTH .EQ. 5) THEN
         EPS = EPS*0.035
         DO 600 I = 1, 3
            CALL NRLAZY(NUMBER,DATA,TEMP,
     &                    INT(CHAOS(2)),EPS)
            DO 700 J = 1, NUMBER
               IF (DATA(J) .NE. TEMP(J)) TRIP = 1
               DATA(J)=TEMP(J)
700         CONTINUE
            IF (TRIP .EQ. 0) GOTO 601
            TRIP = 0
            EPS = EPS-EPS*0.0
            IF (EPS .LE. 0) GOTO 601
600      CONTINUE
601   CONTINUE
      WRITE(6,*) I, EPS
      ELSE
      ENDIF
      E = INDEX(FILENAM,'.')
      D = (ICHAR(FILENAM(E-3:E-3))-48)*100
      C = (ICHAR(FILENAM(E-2:E-2))-48)*10
      B = (ICHAR(FILENAM(E-1:E-1))-48)
      F = D+C+B
      G = F*SMOOTH
      FILENAM(E-3:E-3)=CHAR(MOD(G/100,10)+48)
      FILENAM(E-2:E-2)=CHAR(MOD(G/10,10)+48)
      FILENAM(E-1:E-1)=CHAR(MOD(G,10)+48)
      CALL CLEANRITE(FILENAM,TEMP,NUMBER)
      RETURN
      END
```

APPENDIX E – Source Code to Check the Noise Removed

NOISECK - is program that tests the noise removed from the original data.

```
      PROGRAM NOISECK

      CHARACTER*30 FILENAM1, FILENAM2, FILENAM3
      REAL DATA1 (6000), DATA2 (6000), MAXX, MINX, EPS
      REAL TEMP (6000), XCOR (400), MAXCC, ZERO
      REAL CHAOS (5), SUMX, SUMY, SUMX2, SUMY2
      INTEGER NUMBER1,NUMBER2,SMOOTH, B, D, HI, COUNT
      INTEGER NEIGH, EW, E, F, G, I, TRIP, CORNUM
      DOUBLE PRECISION VAR1, VAR2, AV1, AV2, C

      WRITE (6,*) 'Enter an old data file'
      READ *, FILENAM1
      CALL READIT(FILENAM1,DATA1,NUMBER1)
      WRITE (6,*) 'Enter a cleaned data file'
      READ *, FILENAM2
      CALL READIT(FILENAM2,DATA2,NUMBER2)
      DO 30 I = 1, NUMBER1
         TEMP(I) = DATA1(I)-DATA2(I)
   30 CONTINUE
      FILENAM3 = 'temp\0'
      CALL CLEANRITE(FILENAM3,TEMP,NUMBER1)
      CALL AUTOCOR(FILENAM3, ZERO, NUMBER1)
      DO 21 I = 1, NUMBER1
         SUMX = SUMX+DATA2(I)
         SUMY = SUMY+TEMP(I)
         SUMX2 = SUMX2 + DATA2(I)*DATA2(I)
         SUMY2 = SUMY2 + TEMP(I)*TEMP(I)
   21 CONTINUE
      VAR1=SQRT((NUMBER1*SUMX2-SUMX*SUMX)/(NUMBER1*NUMBER1))
      VAR2=SQRT((NUMBER1*SUMY2-SUMY*SUMY)/(NUMBER1*NUMBER1))
      AV1 = SUMX/NUMBER1
      AV2 = SUMY/NUMBER1
      DO 22 I = 1, NUMBER1
         TEMP(I) = TEMP(I)-AV2
         DATA2(I) = DATA2(I)-AV1
   22 CONTINUE
      CORNUM = 201
      DO 50 I = 1, CORNUM
         C = 0.0
         COUNT = 0
         DO 80 J = I, NUMBER1
            HI=J+I-(CORNUM-1)/2-2
            IF ((HI.GE.0).AND.(HI.LT.NUMBER1)) THEN
               COUNT=COUNT+1
               C = C + DATA2(J)*TEMP(HI+1)
            ENDIF
   80    CONTINUE
```

```
        XCOR(I) = (C)/(COUNT*VAR1*VAR2)
 50  CONTINUE
     MAXCC = XCOR(1)
     DO 44 I = 1, CORNUM
        IF (MAXCC.LT.ABS(XCOR(I))) MAXCC=ABS(XCOR(I))
 44  CONTINUE
     OPEN (UNIT = 10, FILE = 'NOISEData.DAT',
    &   ACCESS = 'APPEND', STATUS = 'UNKNOWN')
     WRITE(10, 200) FILENAM1, FILENAM2, INT(ZERO), MAXCC
200  FORMAT (A15, 1X, A15, 1X, I4, 1X, F7.4)
     RETURN
     END
```

Comments - The call to READIT is the same as in Appendix C. It also calculates the cross correlation of the new data file with the noise removed (previously) from the old file.

CLEANRITE - writes a temporary file for autocor to use.

AUTOCOR - calculates the autocorrelation function and returns which time step the first zero occurs in the variable ZERO.

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