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Abstract: Individuals with Parkinson's disease and the aged have a high prevalence of falls. Since an increase in the number of falls is associated with physical and psychological harm, it is prudent that biomechanical metrics be established that will accurately assess an individual's walking balance. In this investigation, we initially used a simple bipedal walking computer model to theoretically establish the relationship between attractor divergence and walking balance. The Lyapunov exponent was used to quantify the amount of divergence present in the walking attractor. Simulations from our model indicated that attractors that have a greater amount of divergence are more susceptible to falls from external perturbations. Based on the results of our simulations, we conducted an initial experiment to explore. If the young, aged and individuals with Parkinson's disease have different degrees of attractor divergence. Our results indicate that individuals with Parkinson's disease and aged have walking patterns with a greater amount of attractor divergence. Based on the results of our simulations, we infer that these participants may have a higher probability of losing their balance.

Introduction

The aged and individuals with Parkinson's disease walk slower, have shorter step length, widened base of support, and an increased amount of step-to-step variations (Morris, Huxham, McGinley, Dodd, & Iansek, 2001; Prince, 1997). These changes have been speculated to be mechanisms for preserving walking balance. However, there is still a high prevalence of falls in the aged and individuals suffering from Parkinson's disease (Grimbergen, Munneke, & Bloem, 2004; Prince, 1997). Falls may cause bodily harm and increase an individual's fear of future falls. Since these factors are associated with an increased health care cost and a reduction in an individual's well-being, it is prudent that clinical metrics be established that accurately predict an individual's walking balance.

Attempts to quantify changes walking balance have ranged from observing differences in the walking kinematics (Parker, Osterning, van Donkelaar & Chou, 2006), variations of the stepping pattern (Bauby & Kuo, 2000; Gabell & Nayak, 1984; Owings & Gradiner, 2004a, 2004b), and responses to external perturbations that induce a slip or trip during gait (Cordero, Koopman, & van der Helm, 2004; Pavol, Ownings Foley, & Gradiner, 1999a, 1999b; Schillings, Van Wezel, & Duysens, 1996). Resilience of the locomotive system to external perturbations is often considered as the gold standard for quantifying walking balance. A stable walking pattern is capable of resisting larger perturbations that may cause a fall, and will return back to the steady state walking pattern at a faster rate after a perturbation is encountered. However, perturbation analysis does have its limitations since it may be difficult to employ in the clinical setting with frail patients. In addition, it may not provide an objective metric of the patient's balance since the patient often switches to a guarded gait after the first perturbation trial, which may limit the range of perturbation that can be used when assessing balance (Pavol et al., 1999a). Alternative metrics that provide similar information as the perturbation analysis are necessary to determine the walking balance of the aged and individuals with Parkinson's disease.

Several investigations have focused their efforts on exploring how statistical measures of the variance of the stepping pattern may be related to walking balance (e.g, coefficient of variation of the variation, standard deviation). These efforts are grounded on the notion that a stable walking pattern has less variability or greater certainty in the stepping pattern. Outcomes from these investigations have determined that the aged and individuals with Parkinson's disease have more variance in their walking pattern, which may be associated with a higher susceptibility of falls (Frenkel–Toledo et al., 2005 Gabell & Nayak, 1984; Kurz & Stergiou, 2003; Owings & Grabiner, 2004b). However, the variance in the walking kinematics does not appear to correlate well with the perturbation analysis results (Li, Haddad, & Hamill, 2005). This brings to question how well the statistical measures of the variance in the stepping pattern assess walking balance. The lack of correlation may be related to the fact that variability measures (i.e., standard deviation and coefficient of variation) do not provided insight on how the motor system responds to disturbances that are present in the walking pattern overtime. The ability to encapsulate the time dependent changes in the walking pattern may provide a better metric for assessing balance.

Mathematically a system has a greater amount of stability if there is less divergence in the attractor dynamics as time evolves (Abarbanel, 1996; Strogatz, 2004.) During steady state walking, divergence in the attractor dynamics may be created by small disturbances that arise from the natural couplings of the lower extremities and neural noise (Faisal, Selen, & Wolpert, 2008; Hasan, 2005; Kurz, Judkins, Arellano, & Scott–Pandorf, 2008). These local disturbances must be dissipated within or across several strides for the maintenance of walking and balance. If the motor system fails to dissipate these disturbances, they will grow uncontrollably and may result in a loss of balance (Kurz & Stergiou, 2005; Su & Dingwell, 2007). The Lyapunov exponent is a mathematical measure of the amount of divergence present in the attractor dynamics, and has been recently used as a metric of walking balance (Buzzi, Stergiou, Kurz, Hageman, & Heidel, 2003; Dingwell & Cusumano, 2000; England & Granata, 2007; Kurz & Stergiou, 2005, 2007). The larger the value of the Lyapunov exponent, the greater the amount of divergence or instability present in the attractor dynamics. For example, the attractor for a harmonic oscillator has no divergence in its behavior, and the largest Lyapunov exponent is zero (Baker & Gollub, I996). Alternatively, the attractor for the x component of the Lorenz equation ($\sigma = 16$, $R=45.92$, $b=4$) has a high amount of divergence and the Lyapunov exponent is 1.5 (Rosenstein, Collins, & DeLuca, I993). Previous investigations that have used the Lyapunov exponent have revealed that the aged have a greater amount of divergence in their walking attractor dynamics (Buzzi et al., 2003; England & Granata, 2007). Although it is appealing to suggest that the increased amount of divergence is related to a less stable walking pattern, the relationship between the amount of divergence in the attractor and the loss of balance has not been clearly established. Furthermore, it has not been established if Parkinson's disease is related to an additional amount of divergence that is above what has been previously noted in the aged. This information is necessary to determine how much of the divergence present in the attractor dynamics is due to natural aging, and how much may be related to dysfunction of the basal ganglia. Further exploration of these relationships may assist in the management of Parkinson's disease and the development of a new clinically relevant metrics that can be used for the assessment of walking balance.

In the present investigation, we initially used simulations from a simple bipedal walking computer model to further investigate the theoretical relationship between attractor divergence and walking balance. Utilizing these computer simulations, we explored if attractors that have a greater amount of divergence arc more susceptible to falls from external perturbations. To begin to build upon the theoretical outcomes of our simulations, we conducted an initial experiment to further explore if the young, aged and individuals with Parkinson's disease have different degrees of attractor divergence. Since it is well known that the aged and individuals with Parkinson's disease have a higher susceptibility of falls, we hypothesized that their locomotive attractor dynamics would have a larger amount of divergence compared to young controls.

Attractor Divergence Measures

Nonlinear time series analysis techniques were used to quantify the amount of divergence in the locomotive attractor dynamics (Abarbanel, 1996; Kantz & Schreiber, 2003; Stergiou, 2004). All calculations were performed using subroutines from TISEAN (Hegger, Kantz, & Schreiber, 1999). The attractor dynamics were reconstructed based on Taken's embedding theorem, which involved using time-lagged copies of the original joint angle time series (Abarbancl, 1996; Kantz & Schreiber, 2003). These time lag copies were used to create a state vector that described the time evolving dynamics of the locomotive attractor (Eq. 1).

$$y(t) = [x(t), x(t+T), x(t+2T), \dots x(t+(d_E-1)\,T)] \qquad (1)$$

where $y(t)$ is the reconstructed state vector, $x(t)$ is the original time series data, $x(t+iT)$ is time delay copies of $x(t)$, and d_E IS the dimension of the attractor. An average mutual information algorithm was used to determine the appropriate time lag for creating the state vector, and a global false nearest neighbors algorithm was used to determine the attractor dimension (Abarbanel, 1996; Kantz & Schreiber, 2003). The selected time lag was the first local minimum of the average mutual information curve (Fig. 1A), and the embedding dimension was identified when the percent nearest neighbors was zero (Fig. 1B).

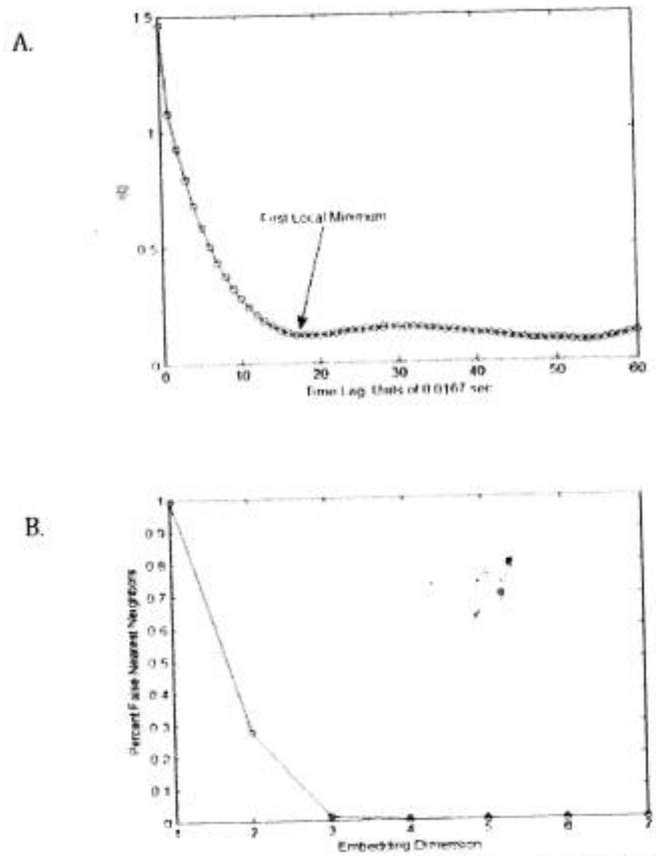

Fig. 1. (A) Time lag is calculated from the first local minimum of the average mutual information algorithm, (B) The embedding dimension is determine to be when the false nearest neighbor algorithm is zero.

An algorithm by Rosentein et al. (1993) was used to calculate the largest Lyapunov exponent of the respective locomotive attractors (Eq. 2).

$$\lambda(i) = \frac{1}{i}\left(\frac{1}{M}\sum_{j=1}^{M-i}\ln\frac{d_j(i)}{d_j(0)}\right) \tag{2}$$

where A is the Lyapunov exponent, *i* is the data sample, *M* was the number of points in the attractor that are considered, d*j*(0) was the initial Euclidean distance between the **j** neighbors, and $d_j(i)$ was the Euclidean distance between the *j* neighbors *i* times later. The magnitude of the largest Lyapunov exponent was estimated from the divergence curve, which consisted of average rate of divergence of neighboring points in the attractor as a function of time. The abscissa of the divergence curve was re-scaled by multiplying it by the average stride frequency. The largest Lyapunov exponent was estimated by using a least squares algorithm to calculate the slope of the initial portion of the divergence curve. A larger slope signified a greater amount of divergence in the attractor dynamics (Rosenstein et al., 1993).

Computer Walking Model

Methods

A simplified walking model was initially used to further establish the relationship between the amount of divergence in the attractor dynamics and walking balance (Fig. 2). Complete details of the derivations of the equations of motion for the walking model are detailed in Garcia et al. (1998). In brevity, the model is a simplified double pendulum system that receives energy from a slightly sloped walking surface (Eqs. 3 and 4).

$$\ddot{\theta}(t) - \sin(\theta(t) - \gamma) = 0 \quad (3)$$

$$\ddot{\theta}(t) - \ddot{\phi}(t) + \dot{\theta}(t)^2 \sin(\phi(t)) - \cos(\theta(t) - \gamma)\sin(\phi(t)) = 0 \quad (4)$$

where Θ was the angle of the stance leg, ϕ was the angle of the swing leg and $\dot{\Theta}$, $\ddot{\Theta}$ and $\ddot{\varphi}$ were the respective time derivatives, γ is the angle of the walking surface, and t is time. Equation 3 represents the stance leg and Eq. 4 represents the swing leg. The governing equations were integrated until the swing leg angle was twice as large as the stance leg angle (e.g., *φ=2Θ).* When this condition was met, a transition equation was applied that switched the roles of the stance and swing leg (Eq. 5).

$$\begin{pmatrix} \theta \\ \dot{\theta} \\ \phi \\ \dot{\phi} \end{pmatrix}^{+} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & 0 & 0 \\ -2 & 0 & 0 & 0 \\ 0 & \cos 2\theta(1-\cos 2\theta) & 0 & 0 \end{pmatrix} \begin{pmatrix} \theta \\ \dot{\theta} \\ \phi \\ \dot{\phi} \end{pmatrix}^{-} + \begin{pmatrix} 0 \\ \sin 2\theta \\ 0 \\ (1-\cos 2\theta)\sin 2\theta \end{pmatrix} J \quad (5)$$

where "+" indicated the behavior of the model just after heel-contact, "-" indicated the behavior of the model just before heel-contact, and J was a toe-off impulse that was directed toward the center of mass and provided additional power for the walking pattern. Further details on the derivation of the transition equation are found in Garcia et al. (Garcia et al., 1998) and Kuo (Kuo, 2002). All stimulations were conducted for 900 continuous footfalls with a ramp angle (γ) of 0.005, and a toe-off impulse (*J*) of 0.01. the values of J and γ were selected because they produced simulations that had similar leg angles as human walking.

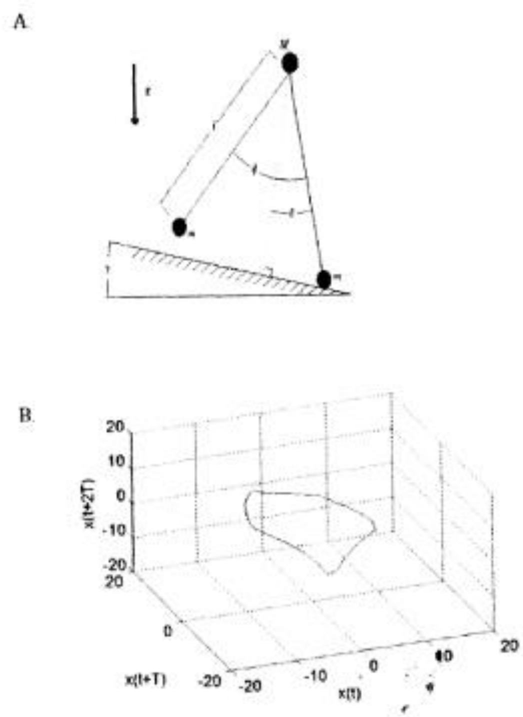

Fig. 2. (A) A simplified walking model that consists of a double pendulum system that captures the stance and swing phase dynamics, (B) projection of the locomotive attractor of the walking model's gait where the time lag (*T*) was 28 samples.

Divergence in the model's attractor was created by adding random noise to the toe-off impulse. We selected to use the toe-off impulse to create divergence in the attractor because several investigations have noted that the aged and Parkinsonian gait is limited by altered ankle joint performance with has been speculated to impact the rhythmicity and timing of the gait pattern (Frenkel – Toledo et al., 2005; Morris et al., 2001; Prince, 1997). The random noise had the form of $\varepsilon \cdot U$ [-1,1] where ε was the amplitude of the random noise distribution, *U*. The amount of divergence in the locomotive attractor was controlled by increasing ε in the range of 0 to 0.002. For each simulation, we calculated the Lyapunov exponent from the linear region of the divergence curve that existed from zero to one stride.

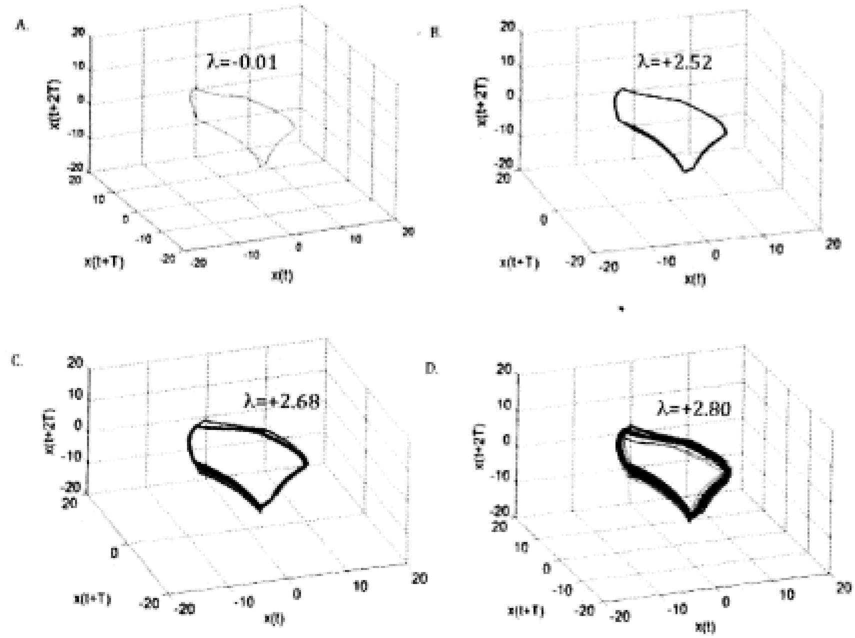

Fig. 3. Projection of the reconstructed attractors when ε was (A) 0, (B) 0.0005, (C) 0.001, (D) 0.002. Qualitatively, the amount of divergence increased as the amount of noise in the toe-off impulse was increased. A time lag (T) of 28 was used to reconstruct the respective attractors.

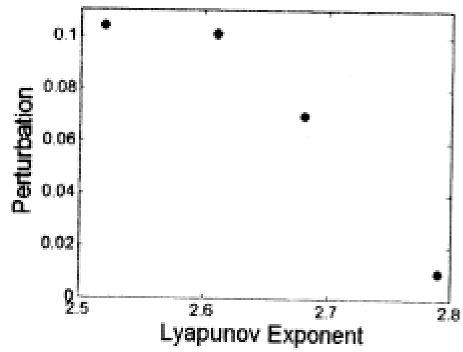

Fig. 4. A walking pattern that had a larger amount of divergence was not able to withstand as large of a perturbation and would fall.

The balance of the walking model with different amounts of attractor divergence was tested by applying an instantaneous horizontal force perturbation to the stance leg at mid-stance. The perturbations were unit less and ranged from 0.016 to 0.107. The perturbation was applied at the 25th-step of the gait pattern, and was systematically increased until the model would fall down. If the model did not fall for 50 simulations, we considered the model to be able to maintain its balance under the respective perturbation level.

Computer Simulations Results

Using the average mutual information and global false nearest neighbors algorithms we determined that an embedding dimension of four and a time lag of 28 were sufficient to reconstruct the model's attractor dynamics. The projection of the model's reconstructed attractor had a limit cycle shape, and the amount of divergence in the attractor was dependent on the noise level in the toe-off impulse dynamics (Fig. 3). As the value of ε*:* was systematically increased, there was a greater amount of divergence present in the attractor dynamics.

The perturbation analysis indicated that an attractor that had a greater amount of divergence could not attenuate as large of a force, and the model was more likely to lose its balance (Fig. 4). Essentially, falling occurred when the force perturbed the attractor's trajectory beyond the basin of attraction. When there was a higher amount of divergence in the attractor dynamics, there was a higher probability that the attractor's trajectory would come closer to the boundaries of the basin of attraction. When this occurred, a lower force value could easily perturb the trajectory to lie outside the basin of attraction and the model would lose its balance. Hence, our simulations predict that an attractor divergence is an indicator of walking balance.

Human Experiments

Methods

Five patients suffering from idiopathic Parkinson's disease (Age=66±7.6 yrs., Height=1.73± 0.1 m, Mass=:26.17±4.5), 15 healthy aged controls (Age=73.4±3.2, Height= 1.59 ± 0.1; Mass = 70.8 ± 9.5), and ten healthy young subjects (Age= 25.89 ± 5 yrs., Height = 1.68 ± 0.1 m; Weight= 67.94 ± 8.1 N) participated in the complementary human experiments. Data collection for the participants with Parkinson's disease took place during the "off' cycle of the dopamine treatment, and they had a Unified Parkinson's Disease Rating Scale (UPDRS) score of 26.17 ± 4.49, which was equivalent to the 53rd percentile of motor function (Fahn, 1987). All participants signed an informed consent that had been previously approved by the Institutional Review Board.

The participants walked on the treadmill for two minutes at a self-selected pace. The average walking speed for the individuals with Parkinson's disease, aged and young were 0.83 ± 0.2, 0.92 ± 0.2, and 1.1 ± 0.1 m/s respectively. A motion capture system (Vicon, Centennial, CO) was used to capture the three-dimensional positions of reflective markers placed on the lower extremity at 60 Hz. The triangulations of markers that were placed on the thigh, and foot segments were used to calculate the sagittal plane hip knee and ankle joint angles. The joint angle time series were analyzed unfiltered in order to get a more accurate representation of the attractor dynamics. The average mutual information and the global false nearest neighbors algorithm were used to determine the respective time lags and embedding dimensions. These parameters were used to reconstruct the lower extremity joints' attractors. The Lyapunov exponents for the respective attractors were calculated from the linear region that existed from 0 to 2 strides of the divergence curve.

A one-way analysis of variances was performed for each joint to determine if there was statistical significance between the Lyapunov exponent means of the respective groups. A Tukey HSD post-hoc was used to determine any statistical differences that existed between the groups at a 0.05 alpha level.

Results from Human Experiments

The average time lag calculated for the hip, knee and ankle were 15, 13 and 13 respectively. The average embedding dimension for the hip, knee and ankle were 4, 5 and 5 respectively. Qualitatively, the attractor dynamics of the aged and participants with Parkinson's disease had an increased amount of divergence compared to the young (Fig. 5). The statistical analysis confirmed this qualitative observation. There was a significant difference in the amount of divergence in the hip joint attractors ($F(2,30)=56.76$; $p<0.0001$; Fig. 6A). The divergence 111 the elderly ($p<$ 0.0001), and Parkinsonian hip joint attractors ($p<0.0001$) were significantly greater than the young. Additionally, there was a greater amount of divergence in the hip joint attractor of the Parkinsonian than the aged participants ($p<0.0001$).

There was a significant difference in the amount of divergence in the knee joint attractor dynamics ($F(2,30)=26.22$; $p<0.0001$; Fig. 6B). The amount of divergence present in the elderly ($p=0.009$) and Parkinsonian attractors (p<0.0001) were significantly greater than the young. Additionally, the Parkmsoman attractor has a significantly greater amount of divergence than the aged (p<0.0001).

There was a significant difference in the amount of divergence in the ankle joint attractor dynamics ($F(2,30)=23.6$; $p<0.0001$; Fig. 6C; Table 1). Where the amount of divergence present in the Parkinsonian attractor was significantly greater than the young ($p<0.0001$) and the aged ($p<0.0001$). However, the amount of divergence was not significantly different between the aged and young ankle joint attractors ($p=$ 0.961).

Table 1. Largest Lyapunov exponents for the respective joints (Mean + SEM).

	Ankle	Knee	Hip
Parkinson's Disease	1.34 ± 0.1	1.19 ± 0.2	0.87 ± 0.3
Aged	1.01 ± 0.1	0.98 ± 0.1	0.53 ± 0.1
Young	0.41 ± 0.1	0.58 ± 0.1	0.38 ± 0.1

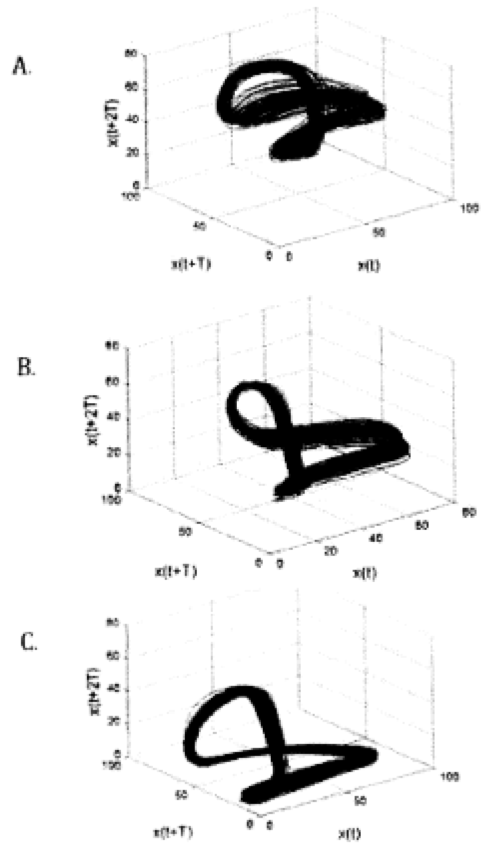

Fig. 5. Exemplary knee joint attractors for an individual with Parkinson's disease (A), aged (B) and young (C). Similar amount of attractor divergence were found at the other lower extremity joints.

Discussion

It has been well established that the diminished capacity of the neuromuscular system results in a high prevalence of falls in the aged and individuals with Parkinson's disease (Grimbergen et al., 2004; Prince, 1997; Morris et al., 200 1). Although this may be true, biomechanical metrics that assess walking balance have not been rigorously established (Brauer, Bums, & Galley, 2000; Owings, Pavol, Foley, & Grabiner, 2000). The results of our simulations indicate that the amount of attractor divergence may provide a metric for assessing walking balance. When our walking model had a higher amount of divergence in the attractor dynamics, it was more likely to fall when an external perturbation was encountered. A higher amount of divergence was associated with an increase in fall susceptibility because the attractor trajectories approached the limits of the basin of attraction. When this occurs, a smaller external perturbation could force the locomotive system to exceed the basin of attraction and a fall would occur. Su and Dingwell (2007) recently presented similar theoretical concepts based on the divergence present in the walking pattern of a simple walking model that was subjected to various levels of irregularities in the walking surface. However, they never applied a perturbation to the walking model to assess the relationship between attractor divergence and walking balance. Our simulation have extended these original concepts by demonstrating that the amount of divergence present in the attractor is related to fall susceptibility of the model. Furthermore, our simulation and human experiments suggest that an increased amount of attractor divergence can arise from within the motor system.

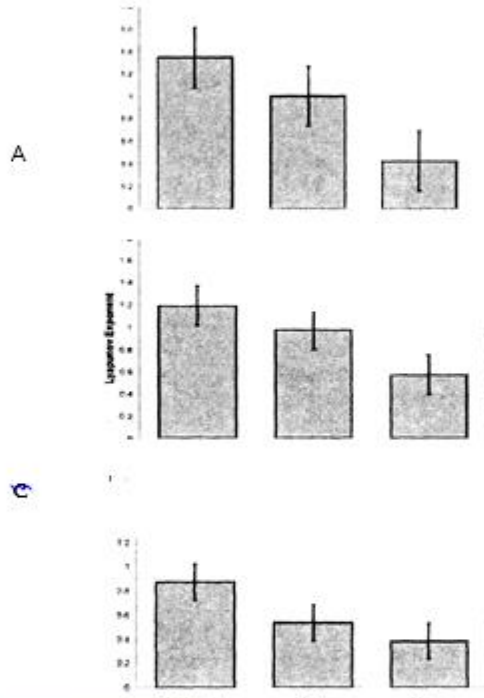

Fig. 6. Lyapunov exponents for the (A) ankle, (B) knee and (C) hip joints' attractors.

We evaluated the amount of attractor divergence in the young, aged and individuals with Parkinson's disease as an initial step towards developing a biomechanical metric that could be used to assess walking balance. Our results illustrated that the aged and Parkinsonian gait dynamics have a greater amount of divergence. Based on the results of our simulations, our results imply that the individuals with Parkinson's disease have poorer walking balance and a greater susceptibility to falls than the aged and the young. This notion appears to align with the current literature that has documented that individuals with Parkinson's disease have a high fall rate (Grimbergen et al., 2004; Prince, 1997). This suggests that quantifying the amount of divergence present in the walking pattern may provide a clinical means for identify individuals with Parkinson's disease who have a higher probability of future falls. Currently, there are no metrics that can provide such information. Hence, our results are promising and warrant further exploration. The limitation of our results is that it is not apparent how large the attractor divergence must be before a loss of walking balance occurs. Our results should be challenged by exploring if the amount of divergence in the attractor can differentiate between fallers and non-fallers. Furthermore, we suggest that our results should be challenged by exploring if the amount of divergence is reduced after the implementation of a balance rehabilitation protocol. This scientific information will further establish if the amount of divergence in the attractor is a good metric for identify patients with walking balance impairments that have a higher propensity for falls.

Our results indicate that individuals with Parkinson's disease have a greater amount of attractor divergence compared to the aged. This suggests that disruption of the basal ganglia function may result in an increased amount of divergence in the attractor dynamics that is beyond what normally occurs with aging. Previously it has been suggested that an increased amount of divergence in the movement pattern of individuals with Parkinson's disease may arise from the gain of low-level deterministic neural processes that are less adaptable (Schmit et al., 2006). As such, it is possible that the individuals with Parkinson's disease may be more susceptible to falls because their walking pattern is not only closer to the stability boundaries, but it is less adaptable to perturbations that may be encountered (Schmit et al., 2006).

We cannot ignore the fact that the amount of divergence present in the attractor dynamics could arise from factors other than the basal ganglia. It is possible that the amount of divergence could have al o arisen from instabilities in the mechanical couplings of the limbs (Kurz et al, 2008). Potentially, these instabilities may have grown sufficiently large because the individuals with Parkinson's disease lack kinesthetic awareness (Maschke, Gomez, Tuite, & Konczak, 2003). Alternatively, it is possible that the amount of divergence in the attractor dynamics may have been related to the external perturbations that were created by the treadmill at each successive heel-contact. Hence, the treadmill motion may have accentuated the inability of the individuals with Parkinson's disease to adapt to the instabilities that were present in the walking pattern.

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